# Fractal Image Compression using Rational Numbers

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***Abstract*—Fractal image compression has been developed in the recent years due to its clear advantage compared to other compression mechanisms. Many approaches in the fractal domain have been successfully implemented such as hierarchical compression and neural network classification. In this paper, we present a new compression method based on rational number theory in order to represent a fractal sub-image. Our approach is based on finding a rational number whose corresponding nominator and denominator are of order of magnitude shorter than the original sub-image. In addition, we integrated a genetic algorithm method to optimize the search for the optimal pair of nominator and denominator that does not harm the original image quality. The preliminary results of our method show competitive results compared to other peer techniques.**

***Index Terms*—Fractal image, lossy compression, rational numbers, genetic algorithms.**

## I. Introduction

Images in their raw form usually require a huge amount of space to be stored and a large bandwidth to be transmitted [1]. In fact, the explosion of multimedia in the internet (audio, images, and videos) has surpassed the evolution of mass-storage and communication resources [2]. Therefore, many approches that aim to reduce the size of raw images and videos have been proposed in the literature. Digital image compression is considered to be a mature branch in image processing area, where several researches have already focused on developing algorithms that produce good compression rates with adequate image quality [3]. Image compression methods are of different approaches, but most of them focus on the characteristics of image data to produce good compression results. Lossless compression techniques like "Run Length Encoding" focus on fully restoring the original file at the decompression phase. On the other hand, lossy techniques like JPEG, TIF, and GIF, which are widely used nowadays, focus on trading off the quality to gain more compression ratio. Despite that lossy compressions may lose some quality of the original image, humans can tolerate the quality loss due to the fact that the human eye cannot distinguish all the $2^{24}$ color range of pixel values. This virtue created an enormous appetite to find smart ways to efficiently compress multimedia files with lighter restrictions on saving the characteristics of the original image.

The rest of this paper is organized as follows. Section 2 presents of wavelet based image compression. In Section 3, we outline a brief description of a neural net application of image compression. Our proposed technique of rational number based compression is shown in Section 4. The preliminary results of our compression technique are shown in Section 6. Section 7 is the conclusion.

## II. Wavelet Based Image Compression

Modern image compression uses transform encoding mechanisms as shown in Fig. 1 [4]. These transform coders start by applying an invertible transform to the image in order to de-correlate the signal of adjacent pixels. The famous used transforms in image compression are the discrete cosine transform, and the discrete wavelet transform, and their performance usually depends on how well the transform decorrelates the signal. The result of decorrelation is formed of coefficients that require some precision. The lesser the precision the lower quality; yet with low precision, the compressing mechanism can achieve higher compression ratio. A well decorrelated signal consists mainly of coefficients close to zero. After the transform step, the coefficients are quantized, i.e., expressed using symbols from a finite alphabet, and entropy coded, using as little space or bandwidth as possible. The three steps of an image coder have an direct impact on the image quality and the compression ratio. The tradeoff between quality and compression ratio is always performed and usually distinguished good compressors.

Two-dimensional wavelet transform uses scaling in order to compress an image as shown in Fig.2 [5]. If the scaling and wavelet functions are separable, the compression process can be decomposed into two stages: along the x-axis and then the y-axis. The two dimensional signal (usually image) is divided into four bands: LL(left-top), HL(right-top), LH(left-bottom) and HH(right-bottom). The HL band indicated the variation along the x-axis while the LH band shows the y-axis variation. The signal power is more compact in the LL band. In the point of coding, we can spend more bits on the low frequency band and less bits on the high frequency band or even set them to zero for higher compression ratio. In addition to decompose the image in two axis, Yu-Si Zhang [6] proposed a method to decompose the image along the natural edge of the image.

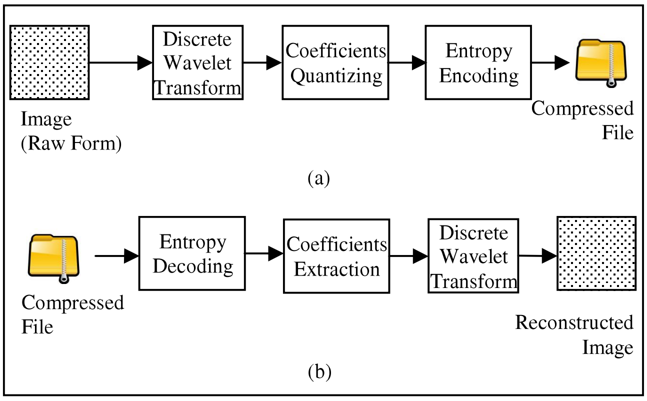


Fig. 1 : Wavelet based image compression (a) Compression (b) Decompression

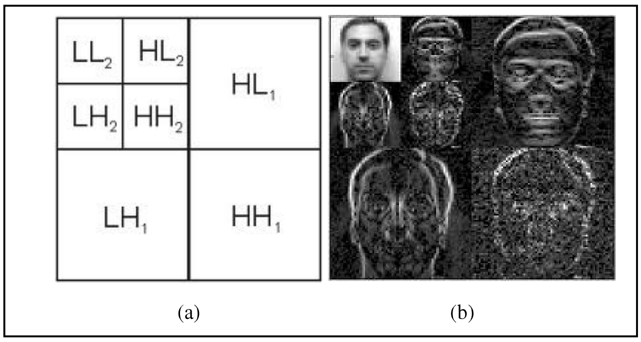


Fig. 2 : Band separation in wavelet transform (a) Separation diagram (b) Image separation example

## III. Neural Nets Image Compression

### A. AVQ

The Apadtive Vector Quantization theory (AVQ) is one of the most recent techniques used in the domain of image compression [7]. In the AVQ image compression-based approach, the input image is divided into equal size sections (subimages), where each section of $n^2$ pixels is considered as a vector in the encoding space $N^{nxn}$. As shown in Fig. 3, the neural net clusters the subimages into classes of similar subimages. Each class has a representative vector, the centroid, that represents any of its member subimages. All centroid representatives for an image are to be tabled in a lookup table or codebook. Then, each subimage is compressed into (i.e., encoded as) the index of its corresponding class representative in the codebook. Thus, the compressed image is a set of representative indices that represent (in order) its original subimages. The compression is realized because the byte size of the original subimage ($n^2$) is an order of magnitude larger than its corresponding centroid bit index.

### B. KSOFM

The Kohonen self-organizing feature map (KSOFM) is a structured network with a two-dimensional array of nodes in which an adaptive partner weight vector is associated with every node [11]. Based on the AVQ theory, the KSOFM initializes the weight matrix randomly. The input vectors are presented sequentially to the network [10]. During the training process, the nearest weight vector (winner) to the input vector is adaptively moved closer to such an input vector. In addition, adjacent centroid that belongs to certain neighborhood space, arround the winning centroid, are also trained [12]. Initially, the neiborhood radius is very large, then it decreases as the training progresses. Consequently, with every introduction of an input vector, the winning vector moves toward some theoretical class's center of mass (centroid) in its way to become the true representative of all input vectors that have won and so are similar. The input vector is associated with the class of its corresponding representative centroid. The input vectors are presented in epochs until no change of memberships is observed.

### C. Direct Classification

Since compression ratio and image quality are two very critical and conflecting factors, a convenient way to gracefully compromise between them is needed [8]. Soliman and Omari [7] proposed a new classification method (Direct Classification) based on Kohonen and ART (Adaptive Resonance Theory) neural nets. In the local codebook approach, each image will have its own codebook, even when the codebooks of same domain images overlap. Motivated by the great overhead of extra codebook per image and the codebooks overlap, Soliman and Omari introduced the universal codebook approach [9]. In the new mechanism, all images in the same domain will train one universal codebook, increasing the compression ratio. Unfortunately, the image quality will be affected, depending on the codebook quality. A rich universal codebook trained long and hard might result in the inclusion of the most essential centroids, for good recovered image quality.

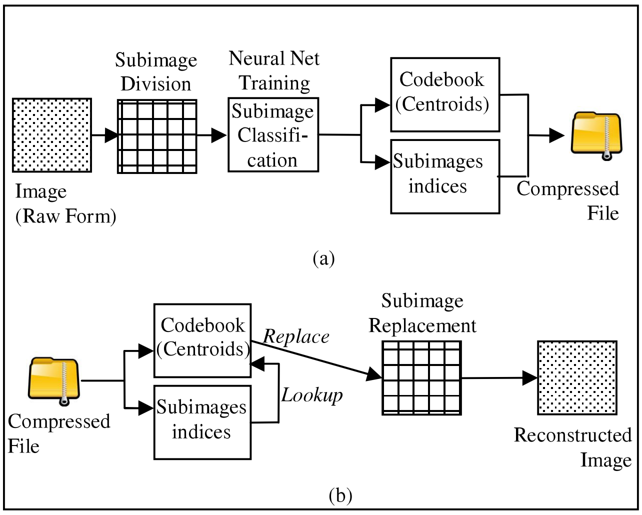


Fig. 3 : Neural net based image compression (a) Compression (b) Decompression

## IV. IMAGE COMPRESSION USING RATIONAL NUMBERS BASED ON GENETIC ALGORITHMS

### A. *Genetic Algorithms*

Genetic algorithms (GAs) are search procedures based on the mechanics of natural selection and natural genetics [13]. Genetic algorithms (GAs) were invented by John Holland in the 1960s and were developed by Holland and his students and colleagues at the University of Michigan in the 1960s and the 1970s [14]. In contrast with evolution strategies and evolutionary programming, Holland's original goal was not to design algorithms to solve specific problems, but rather to formally study the phenomenon of adaptation as it occurs in nature and to develop ways in which the mechanisms of natural adaptation might be imported into computer systems.

GA is based on moving from one population of "chromosomes" (e.g., strings of ones and zeros, or "bits") to a new population by using a kind of "natural selection" together with the genetics–inspired operators of crossover, mutation, and inversion. Each chromosome consists of "genes" (e.g., bits). The selection operator chooses those chromosomes in the population that will be allowed to reproduce, and on average the fitter chromosomes produce more offspring than the less fit ones. Crossover exchanges subparts of two or more chromosomes. Mutation randomly changes the allele values of some locations in the chromosome (Fig. 4).

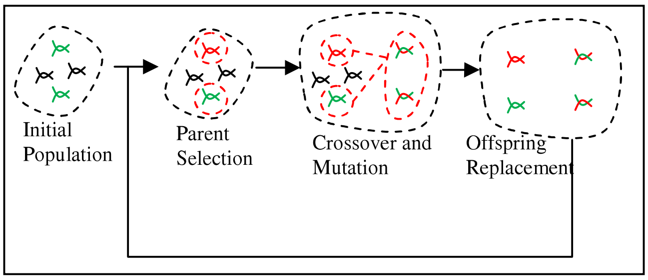


Fig. 4 : Genetic algorithms phases.

In GAs, each individual in the search space is coded as a chromosome, which consists of the characters (genes) 1's and 0's [13]. So first we choose randomly a set of chromosomes to constitute an initial population. Then, each chromosome is evaluated based on a fitness function that defines optimality. After that, a set of basic operations is applied to the population of the chromosomes. Selection is an operation which selects the chromosomes according to their fitness values. So, if a chromosome has higher fitness value, it will have a high chance to produce offsprings into the next generation. In the crossover operation, the selected parents are crossed to create new chromosomes by swapping genes between parents. Mutation is the last operation which alters the gene of some chromosomes (usually with low probability). The GA phases are repeated until an optimization solution is found or a termination criterion is fulfilled.

### B. *Rational Numbers and Image Compression*

Rational and irrational numbers are forms of representing long string by few digits. A rational number is defined formally as follows:

$$(x \text{ is rational}) \equiv (\exists\, p, q \in Z / x = \tfrac{p}{q}) \qquad (1)$$

So, if we consider a rational number x is of high precision, whereas its corresponding pair of integers p and q (based on (1)) are of short form, then we can claim sort of compression from a fractional form to nominator-denominator form.

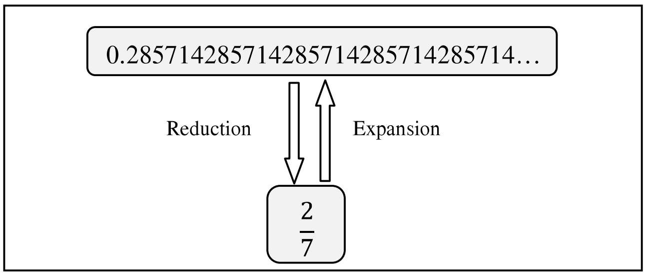


Fig. 5 : Reduction/expansion of rational numbers.

Unlike irrational numbers, rational numbers include a redundant pattern in their fractional form (the sequence 285714 is repeated in Fig. 5). So in order to extract the short form of nominator-denominator that represents a rational number, we can perform a series of transformations as follows. Let us consider a rational number x:

$$x = 0.285714285714285714285714\ldots$$

so, $x.10^6 = 285714.285714285714285714285714\ldots$

and then, $x.10^6 - x = 285714$

so, $x.(10^6 - 1) = 285714$

Therefore, $x = \frac{285714}{999999}$

Now, we should find the great common divisor GCD between the nominator and the denominator in order to reduce the digit representation of x:

$$GCD(285714, 999999) = 142857$$

Hence,

$$x = \frac{285714 \div 142857}{999999 \div 142857} = \frac{2}{7} \qquad (2)$$

In other words, the long sequence of the fractional representation of x was reduced to a short representation of two digits. In order to make use of this reduction, we proposed a simple mechanism that converts a subimage (2x2, 4x4, …) into a long rational number. Subsequently, we reduce the latter into a short-digit nominator and denominator, which results in compression from a subimage to two digits as shown in Fig. 5.

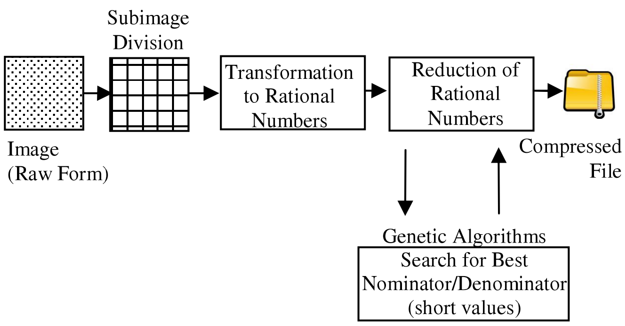


Fig. 5 : Image compression using rational numbers

In lossless compression, the compressed data is fully recovered at the decompression phase. This mode of compression is necessary when the loss of data is not tolerated (e.g., text files, bank transactions, …). On the other hand, lossy compression is widely used in the multimedia domain due to the fact that the human visual system can tolerate some loss of data that constitute images, sounds and videos. Lossy image compression is used to gain higher compression ratio with the advantage of saving the same high quality as with lossless compression.

For that matter, we propose a technique that gathers all low significant bits of subimage pixels, and apply a genetic algorithm as follows:

1- Divide the input image into a set of subimages ($S_1$, $S_2$, … $S_n$).
2- For each subimage $S_i$ of $n \times n$ pixels extract the corresponding rational number $R$ which is equivalent to the concatenation of all bytes corresponding to its pixels in the RGB representation, i. e., $R$ is of length $3n^2$ bytes.
3- If we consider a number of least significant bits ($1 \leq LSB \leq 8$), then $R$ should belong to a search space S of size $(2^{LSB})^{3n^2}$, i.e., R lays between $R_{min}$ and $R_{max}$. Our goal in this step is to find an equivalent rational number $\grave{R}$ where the latter can be reduced into a shorter form compared to $R$.

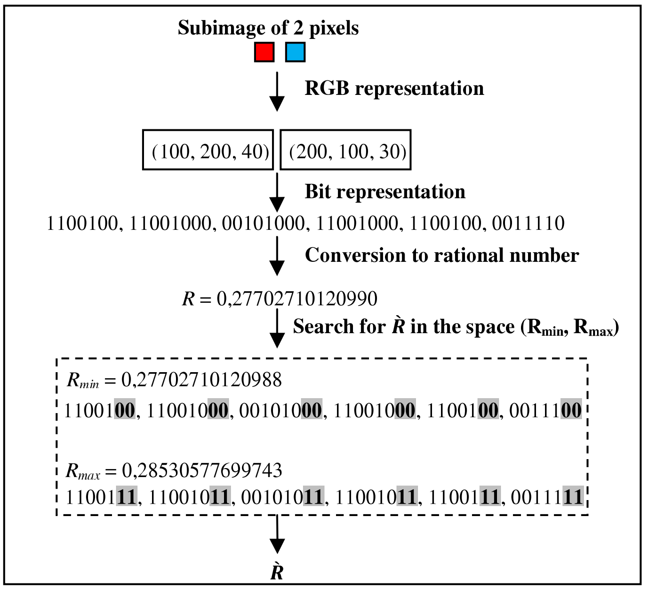


Fig. 6 : Image compression using genetic algorithms

4- Generate an initial population of m chromosomes ($C_1$, $C_2$, …, $C_m$) where $R_{min} \leq C_i \leq R_{max}$ and $C_i$ only differs than R in the LSB bits of each bytes. For evaluation purposes, we set the fitness function of a chromosome based on the shortest nominator-denominator form of each chromosome.
5- For each $C_i$, reduce it to the shortest form by applying the same method as shown in (2).
6- Select the best parents based on the defined fitness function.
7- Apply the simple one-point crossover between each two parents with the cross point at a fixed position.
8- Apply a random mutation by flipping a bit on a selected chromosome with certain probability.
9- Among the set of parents and the set of offsprings, select the best individuals based on the above defined fitness function.
10- Repeat Steps 4-9 until a number of epochs is elapsed.

## V. Preliminary Results

The application of our proposed image compression model was done using the Java Development Kit (JDK). JDK provides an easy platform to handle different types of input images (jpeg, png, bmp, …). In addition, JDK libraries facilitate the bit operations needed in our model in order to construct new chromosomes. Our preliminary results show a promising achievement in compressing simple images with graphs and documents (compression ratio up to 100), with high quality of recovered image. This work is still at experimentation phase especially with landscape images and personal photos. Fig. 7 shows a set of different selected images that we chose for our genetic-algorithm image compression.

## VI. Conclusion

Multimedia compression is in the rise recently especially with the huge mass of images and videos that migrate in the Internet between users. Fractal image compression is widely used in the domain of lossy compression due to its simple way of quantizing sub-frames inside an image. In this paper, we proposed a new compression idea that is based on reducing rational numbers into nominator-denominator form. Moreover, our technique utilizes the efficiency of genetic algorithms to enhance the time search in order to find better rational numbers with shorter reduced form. Our preliminary results show very promising achievements compared to the peer image compression techniques. As a future work, we will investigate the possibility of classifying subimages based on their corresponding rational numbers' characteristics in terms of great common divisors.

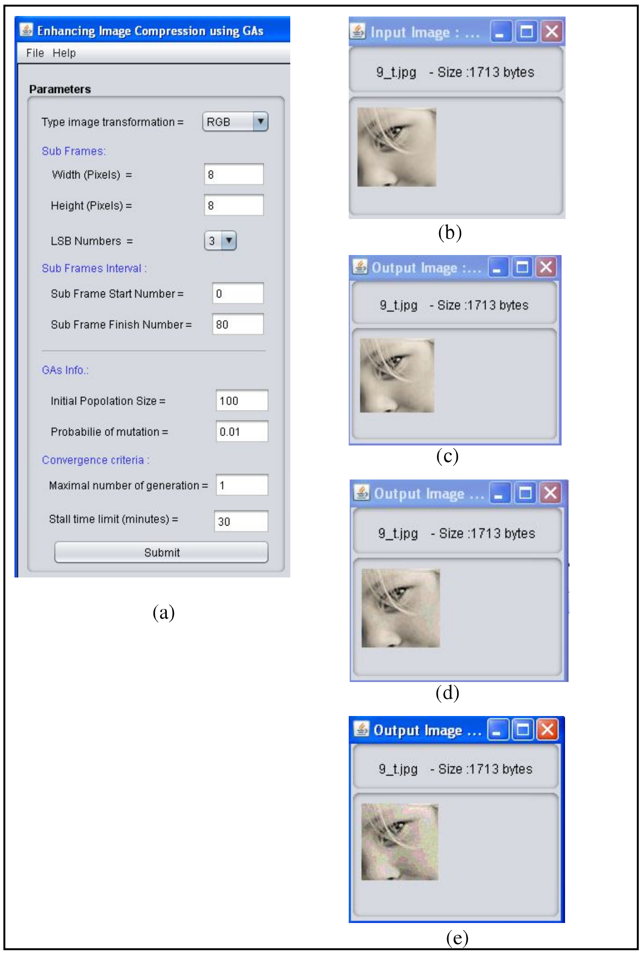


Fig. 7 : Preliminary results of image compression: (a) image compression interface (b) original image (c) compression with LSB = 3 (d) compression with LSB = 4 (e) compression with LSB = 5

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