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Title:

Exercises and corrected problems for Control of Electric Machines

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This Educational Support is intended for third-year Electrotechnics students (6th semester)

Introduction

This directed work in electrical engineering is crafted specifically for third-year Bachelor's degree students, focusing on the essential module of electric machine control. As part of the LMD (Licence-Master-Doctorat) reform, this material adheres to the official curriculum and is designed to equip students with the foundational knowledge and practical skills necessary for effectively managing various types of electrical machines.

The document is structured into two comprehensive chapters. The first chapter covers the control of **DC motors**, emphasizing the fundamental principles, control methods, and applications that underpin their operation. Students will explore the various techniques used to manipulate the performance of DC motors, making it a critical area of study for applications requiring precise control over speed and torque.

In the second chapter, we shift our focus to **AC motors**, specifically discussing the control mechanisms for both **synchronous** and **asynchronous motors**. This chapter aims to provide students with insights into the distinctive characteristics and control strategies associated with these types of machines. Understanding AC motor control is essential in industrial settings, where these motors are commonly utilized for a wide range of applications.

Through this directed work, students will gain valuable theoretical and practical insights into the control of electrical machines, preparing them for challenges in the field of electrical engineering and enhancing their readiness for future professional endeavors.

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Chapter I: *Control of DC motors*

Exercise 01:

We are given an electric motor with the following real and positive parameters:

(a, b, and c).

$$\begin{cases} T_m = a.\Omega - b \\ T_r = c.\Omega^2 \end{cases}$$

Tasks:

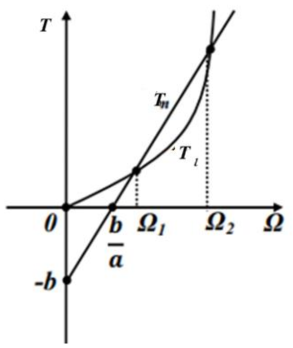
1. Determine the operating points (points of operation) of the system.
2. Discuss the stability of the operating points.

Solution of Exercise 01:

1) Determination of the operating points:

We equate the motor torque Tm and the load torque

$$T_m = T_l \Rightarrow c \cdot \Omega^2 - a.\Omega + b = 0$$



The discriminant is

$$\Delta = (-a)^2 - 4cb = a^2 - 4cb$$

For real solutions, the discriminant must satisfy $\Delta > 0$. Thus, the two operating points for the motor speed Ω are:

$$\begin{cases} \Omega_1 = \dfrac{a - \sqrt{\Delta}}{2c} \\ \Omega_2 = \dfrac{a + \sqrt{\Delta}}{2c} \end{cases}$$

2- Stability analysis of the operating points:

The condition for stability is :

$$\frac{dC_m}{d\Omega} - \frac{dC_r}{d\Omega} < 0$$

$$\frac{d(C_m - C_r)}{d\Omega} < 0$$

Differentiating the torque difference:

$$\frac{d(C_m - C_r)}{d\Omega} = \frac{d(a\Omega - b - c\Omega^2)}{d\Omega} = a - 2c\Omega$$

- For $\Omega = \Omega_1 = \frac{a-\sqrt{\Delta}}{2c}$ we have: $\boldsymbol{a} - 2c\Omega_1 = \sqrt{\Delta}$ Since, $\sqrt{\Delta} > 0 \Rightarrow$ the point Ω1 is unstable
- For $\boldsymbol{\Omega} = \boldsymbol{\Omega}_2 = \frac{\boldsymbol{a}+\sqrt{\Delta}}{2\boldsymbol{c}}$ we have: $\boldsymbol{a} - 2\boldsymbol{c}\boldsymbol{\Omega}_1 = -\sqrt{\Delta}$ Since $-\sqrt{\Delta} > 0$ the point Ω2 is stable.

Exercise 02:

A DC motor, with an appropriate control circuit, develops a torque expressed by the following relation**:** $\boldsymbol{T_m(\omega) = a\ \omega + b}$

where:

- $\boldsymbol{T_m(\omega)}$ is the torque generated by the motor as a function of speed ω,
- a and b are positive constants.

The motor drives a load with a torque described by: $\boldsymbol{T_l(\omega) = c\ \omega^2}$**+d**

where:

- $\boldsymbol{T_l(\omega)}$ is the load torque as a function of speed ω,
- c and d are positive constants.

The total moment of inertia of the system is denoted by J.

Tasks:

1. Determine the relationship between a, b, c, and d for the motor to start with its load and reach an equilibrium point.
2. Calculate the speed at the equilibrium point.
3. Assess the stability of the system at the equilibrium speed.
4. Determine the initial acceleration of the system.
5. Find the maximum acceleration of the system.

Solution of Exercise 02:

a) Condition for the motor to start with its load:

At ω=0, the motor torque Tm=b, and the load torque T_l=d. The motor can start if: :

$$\boldsymbol{b > d}$$

At the equilibrium point, Tm= T_l :

$$a * \omega + b = c * \omega^2 + d \Rightarrow c\omega^2 - a\omega - (b - d) = 0$$

The solution for **ω** is:

$$\omega_{1,2} = \frac{a \mp \sqrt{a^2 + 4c(b-d)}}{2c}$$

Since

$$b > d \Rightarrow a^2 + 4c(b-d) > 0$$

a) The positive solution for **ω** is:

$$\omega = \frac{a + \sqrt{a^2 + 4c(b-d)}}{2c}$$

The negative solution is rejected because **ω** must be positive, meaning:

$$a > \sqrt{a^2 + 4c(b-d)}$$

$$a^2 > a^2 + 4c(b-d)$$

$$4c(b-d) < 0$$

$$c < 0$$

This is not possible since **c** is a positive constant. Therefore, the equilibrium point is ω1 with

$$\sqrt{a^2 + 4c(b-d)} > 0$$

b) Calculation of equilibrium speed:

The equilibrium speed is:

$$\omega = \frac{a + \sqrt{a^2 + 4c(b-d)}}{2c}$$

c) Stability of the system at equilibrium:

For the system to be stable at the equilibrium point:

$$\frac{dT_l}{d\omega} > \frac{dT_m}{d\omega}$$

This leads to the condition:

$$2c\,\omega > a$$

From part b), we have:

$$2c\omega = a + \sqrt{a^2 + 4c(b-d)} > a$$

Thus, the equilibrium point is stable for the given driving mode.

d) Initial acceleration of the system:

The torque difference determines the acceleration:

$$J\frac{d\Omega}{dt} = T_m - T_l$$

Initially, at t=0, $T_m = b$ and $T_l = d$, so the initial acceleration is:

$$\left(\frac{d\Omega}{dt}\right)_{t=0} = \frac{b-d}{j}$$

e) Maximum acceleration of the system:

From the equation:

$$J\frac{d\Omega}{dt} = T_m - T_l = -c\omega^2 + a\omega + (b-d)$$

The acceleration is given by:

$$A = \frac{d\Omega}{dt} = \frac{-c\omega^2 + a\omega + (b-d)}{J}$$

The acceleration is maximum when:

$$\frac{d\,\mathrm{A}}{d\omega} = 0$$

$$\frac{a - 2c\omega}{J} = 0$$

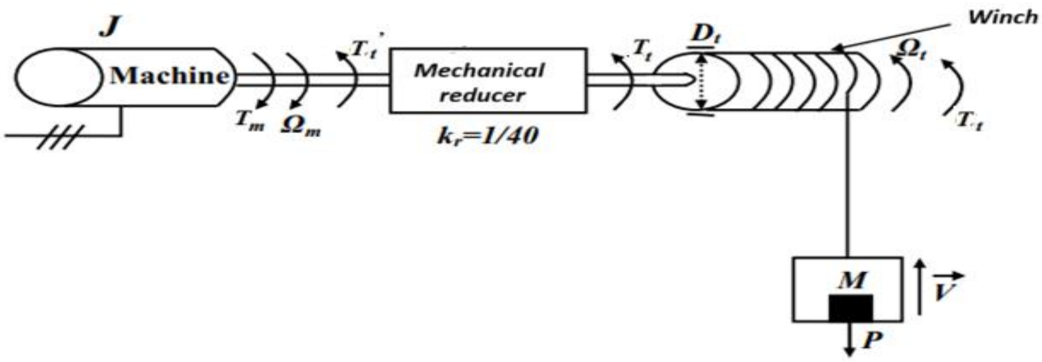
$$\omega = \frac{a}{2c}$$

Substituting this into the acceleration equation gives the maximum acceleration:

$$A_{\max} = \frac{a^2 + 4c(b-d)}{4cJ}$$

Exercise 03:

We are provided with the following parameters of an electric drive system:

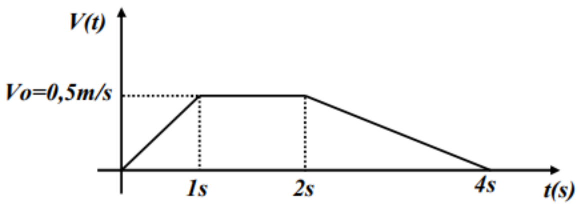


- P=M.G: Weight of the load

- V: Translation speed
- Ω: Rotation speed
- J=0.13 kgm^2: Moment of inertia
- D_t=30 cm: diameter
- M=2500 kg: Mass of the load
- kr= $\frac{\Omega_t}{\Omega_m} = \frac{1}{40}$: Mechanical reduction ratio
- η=1: Efficiency of mechanical reduction
- g=10 m/s^2: Gravitational acceleration

Questions:

1. Calculate the hoist torque T.
2. Calculate the torque transferred to the motor shaft $\grave{T}_t$.
3. Calculate the motor rotation speed $\mathbf{\Omega_m}$ as a function of the translation speed V.
4. Show that the variation of motor torque is given by $\boldsymbol{T_m}$=**a**$\boldsymbol{\frac{dV}{dt}}$**+b** Calculate **a** and **b**.
5. Calculate and plot the variation of motor torque Tm as a function of the translation speed V.



Solution of Exercise 03:

1) Calculating the lifting torque T_t :

The lifting torque T_t is calculated as:

$$T_t = P \cdot \frac{D_t}{2}$$

$$T_t = M.g \cdot \frac{D_t}{2}$$

$$T_t = 3750\text{Nm}$$

2) Calculating the torque transferred to the motor shaft $\grave{T}_t$ **:**

Using the mechanical reduction ratio $k_r = \frac{\Omega_t}{\grave{\Omega}_t}$, the torque at the motor shaft is:

$$T_t \Omega_t = T_t' \Omega_t'$$

$$\grave{T}_t = T_t \cdot \frac{\Omega_t}{\grave{\Omega}_t}$$

$$\grave{T}_t = T_t k_r$$

Substituting the values:

$$\grave{T}_t = 3750 \cdot \frac{1}{40}$$

$$\grave{T}_t = 93.75\text{Nm}$$

3) Calculating the motor speed $\Omega_m$as a function of translation speed V:

The rotation speed of the lifting drum Ω_t is:

$$\Omega_t = \frac{\boldsymbol{V}}{\left(\frac{D_t}{2}\right)}$$

The motor speed Ω_m is related to Ω_t t by the reduction ratio:

$$\Omega_m = \frac{\Omega_t}{k_r} = \frac{2V}{D_t k_r}$$

4) Demonstrating that the variation of $T_m$follows the form dc :

$$T_m = a\frac{dv}{dt} + b\textbf{:}$$

The torque difference equation for acceleration is:

$$J\frac{d\Omega_m}{dt} = T_m - \grave{T}_t$$

$$T_m = J\frac{d\Omega_m}{dt} + \grave{T}_t$$

$$T_m = \frac{2J}{k_\mathrm{r} D_t}\frac{dV}{dt} + \grave{T}_t$$

$$T_m = 34{,}66 \cdot \frac{dV}{dt} + 93{,}75$$

Where

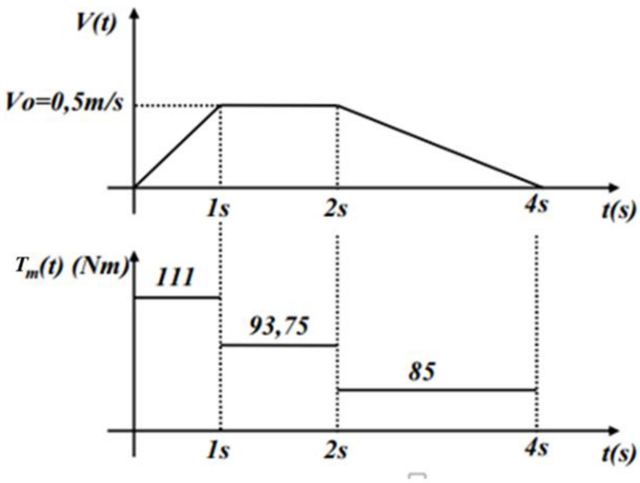
$$\begin{cases} a = 34{,}66 \\ b = 93{,}75 \end{cases}$$

5) Calculating and plotting the variation of motor torque T_m with respect to translation speed V:

The variation of T_m is linear. Using the equation:

$$T_m = 34{,}66 \cdot \frac{dY}{dt} + 93{,}75$$

For different speeds V, we calculate the torque based on the acceleration $\frac{dV}{dt}$ The plot of T_m versus V would show a linear relationship.



Exercise 04:

Consider a separately excited DC motor with the following parameters:

P_n=24.2 kW

- Uan=440 V ,
- Ufn=220 V
- Nn=1470 rpm
- Ra=0.37 Ω
- Rf=66 Ω
- K=0.108

1. Determine, in the case of nominal torque, the value of the resistance to be added in series with the armature to obtain a speed of N=1000 rpm
2. How much should the armature voltage be reduced to achieve a speed of 1000 rpm at nominal torque?

Solution of Exercise 04:

1. **Value of the Resistance to be Added:**

Calculate the nominal torque:

$$T_n = \frac{P_n}{\Omega_n}$$

$$T_n = \frac{24{,}2 \times 10^3}{1470 \times \frac{2\pi}{60}}$$

$$T_n = 157.28\ Nm$$

Calculate the angular velocity for N=1000 rpm:

$$\Omega = 1000 \times \frac{2\pi}{6}$$

$$\Omega = 104{,}67 \text{ rad/s}$$

The torque equation for the motor is given by:

$$T = \frac{K\phi V_a}{R} - \frac{(K\phi)^2}{R}\Omega$$

$$T = \frac{kI_{\text{ex}} V_a - \left(KI_{\text{ex}}\right)^2 \Omega}{R}$$

Réarrangions gives:

$$R = \frac{kI_{\text{ex}} V_a - \left(KI_{\text{ex}}\right)^2 \Omega}{T}$$

Calculate the excitation current Iex:

$$I_{\text{ex}} = \frac{V_f}{R_f}$$

$$I_{\text{ex}} = \frac{220}{66}$$

$$I_{\text{ex}} = 3.33A$$

Substitute values into the resistance equation:

$$R = \frac{0{,}108 \times 3{,}33 \times 440 - (0{,}108 \times 3{,}33)^2 \times 104{,}67}{157{,}285}$$

$$R = 0{,}92\Omega$$

Total resistance R is:

$$R = R_a + R_h$$

The resistance to be added in series with the armature to achieve a speed of 1000 rpm is approximately

$$R_h = 0{,}55\Omega$$

2) Required Armature Voltage for 1000 rpm at Nominal Torque:

The current Ia at nominal torque can be calculated:

$$T_n = \text{K Iex } I_a$$

$$I_a = 437{,}34\text{A}$$

The armature voltage equation:

$$V_a = R_a T_a + E$$

The armature voltage equation:

$$E = kI_{ex}\Omega$$

$$V_a = R_a I_a + K I_{ex} \Omega$$

Substitute the known values:

$$V_a = 0{,}37 \times 37{,}34 + 0{,}103$$

$$V_a = 168{,}84 \text{ V}$$

The armature voltage required to achieve a speed of 1000 rpm at nominal torque is approximately **168.84 V**.

Exercise 05:

A separately excited DC motor with constant excitation current has an armature voltage that can vary from 0 to 600V. At the maximum armature voltage, the motor rotates at a speed of 1600 rpm. The motor losses are negligible.

1. Calculate the armature current when the load torque is 420 N.m.
2. Calculate the power supplied by the source.
3. If the armature voltage is kept constant at 600V and the excitation current is reduced until the motor speed reaches 4000 rpm, determine the torque delivered by the motor at this speed.

Solution of Exercise 05:

1. Armature current when the load torque is 420 N.m:

Given:

- Load torque T_l=420 N.m, We have

$$\sum \text{p} = 0$$

$$\text{T} = T_l = 420 \text{ N} \cdot \text{m}$$

- Speed N=1600 rpm,
- Armature voltage V=600 V.

Mechanical power is calculated as:

$$P = T\,\Omega = 420 \times 160 \times \frac{2\pi}{60}$$

Electrical power is:

$$P = V I_a$$

Solving for Ia

$$I_a = \frac{P}{V} = \frac{P}{600}$$

$$I_a = 117.3 \text{ A}$$

2. Power supplied by the source:

The power supplied is the same as the motor's mechanical power:

$$P = 600 \times 117{,}3$$

$$P = 70.37\text{kw}$$

3. Torque delivered by the motor at 4000 rpm:

When the excitation current is reduced and the motor speed increases to 4000 rpm, the motor operates at constant power. So, the mechanical power remains the same:

$$P = T\Omega$$

Substituting the new speed:

$$P = T \cdot 4000 \times \frac{2\pi}{60}$$

Solving for T:

$$T = 168\ \text{N} \cdot \text{m}$$

Thus, the torque delivered by the motor at 4000 rpm is 168 N.m

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Exercise 06:

A separately excited DC motor has the following specifications:

- Power: 15 hp
- Voltage: 220 V
- Speed: 2000 RPM
- The motor drives a load requiring a torque C_l=45 N·m at a speed of 1200 RPM.
- Armature resistance: Ra=0.25 Ω
- Field resistance: Rf=147 Ω
- Motor constant: Kv=0.7032 **VA(rad/s)** $^{-1}$
- Excitation voltage: Vf=220 V
- Friction coefficient and losses are negligible.

Determine:

1. The back electromotive force (F.E.M) E.
2. The armature voltage Va.
3. The rated armature current Ia.

Solution of Exercise 06:

1. Back Electromotive Force (F.E.M)

First, we need to convert the speed from RPM to radians per second:

$$\Omega = 1200 \times \frac{2\pi}{60} = 125{,}6 \text{ rad/s}$$

Calculating the Field Current If:

$$I_f = \frac{V_f}{R_f}$$

$$I_f = \frac{2.20}{147}$$

$$I_f = 1{,}497 \text{ A}$$

Calculating the Back EMF E:

$$E = kv\, I_f \Omega$$

Substituting the values:

$$E = 132{,}22 \text{ V}$$

2. Armature Voltage Va

Using the torque equation:

$$T = KI_f I_a = 45 \text{ Nm}$$

We can solve for Ia:

$$I_a = \frac{45}{0{,}7032 \times 1{,}497}$$

$$I_a = 42{,}75 \text{ A}$$

Calculating the Armature Voltage Va:

$$V_a = R_a I_a + \text{E}$$

Substituting the known values:

$$V_a = 0{,}25 \times 42{,}75 + 132{,}22$$

$$V_a = 142{,}91V$$

3. Rated Armature Current Ia

To calculate the rated armature current, we first convert the motor power from horsepower to watts:

$$1\text{hp} = 746W$$

Now we can find the rated armature current:

$$I_{an} = \frac{P_n}{V_n}$$

$$I_{an} = \frac{15 \times 746}{220}$$

$$I_{an} = 50{,}17 \text{ A}$$

Exercise 07:

A separately excited DC motor with a constant excitation is powered by a constant voltage U=38 V, and its armature resistance is R=0.20 Ω. At nominal load, the armature current is I=5 A, and the motor rotates at a speed of 1000 RPM.

1. **For nominal operation**:

   a) Calculate the electromotive force E of the armature.

   b) Calculate the electromagnetic torque .

   c) Show that E can be expressed as a function of the rotational speed n, following the relation: E=k·n.

2. **With a change in load**, the armature current becomes I′=3.8 A. Calculate:

   a) The new electromagnetic torque .

   b) The new rotational speed.

Solution of Exercise 07:

1. **For nominal operation**:

   a) **Electromotive Force E**:

$$E = U - R_a * I$$

Substituting the values:

$$\text{E} = 38 - 0.2 \times 5$$

$$\text{E} = 37 \text{ V}$$

b) Electromagnetic Torque

The angular speed Ω is calculated using:

$$\Omega = \text{N}\frac{2\pi}{60}$$

Substituting N=1000 RPM:

$$\Omega = 1000\frac{2\pi}{60}$$

$$\Omega = 104.66\; rad/s$$

The electromagnetic torque is given by:

$$T_{\text{em}} = \frac{EI}{\Omega}$$

Substituting the values:

$$T_{\text{em}} = \frac{37\ .\ \ 5}{104.66}$$

$$T_{\text{em}} = 1.76\ \text{N.m}$$

c) **Relation between E and n**: Since the excitation is constant, Φ=constant, and we can express the back EMF as:

$\text{E} = \text{K}\Phi\ \Omega$

We know that:

$$\Omega = \frac{2\pi}{60}\ \text{N}$$

So:

$$\text{E} = \text{K}\Phi\ \frac{2\pi}{60}\ \text{N}$$

$$E = kN$$

Where: $k = \text{K}\Phi\ \frac{2\pi}{60}$

2-Variation de la charge $I' = 3.8$ A

With a load change:

a) **New Electromagnetic Torque** :

Since the torque is proportional to the current, we can express the new torque as:

$$T\grave{}_{\text{em}} = T_{\text{em}} * \frac{I'}{I}$$

Substituting the values:

$$T\grave{}_{\text{em}} = 1.76 * \frac{3.8}{5}$$

$$T\grave{}_{\text{em}} = 1.3\text{N.m}$$

b) **New Rotational Speed**

The new speed is proportional to the new back EMF:

$$N' = E' * \frac{N}{E}$$

$$N' = 1006\text{tr/mn}$$

Exercise 08:

The motor of a crane, with a constant independent excitation, rotates at 1500 RPM when it applies a force of 30 kN to lift a load at a linear speed of V1=15 m/min . The armature resistance is Ra=0.4 Ω. The motor is connected to a gearbox, and the combined mechanical and magnetic losses of the motor and gearbox result in the useful power being 83% of the electromagnetic power transformed by the machine. The electromagnetic torque of the motor is proportional to the armature current I, with Tem=1.35I.

1.

a) Calculate the useful power and the electromagnetic torque.

b) Calculate the armature current, the electromotive force (EMF), and the voltage U applied to the armature.

c) Given that the excitation power consumed is Pe=235 W, calculate the total absorbed power and the system efficiency.

2. When lowering the unchanged load, the charge drives the rotor, and the DC motor operates as a generator. The excitation, gearbox ratio, and mechanical efficiency (motor + gearbox) remain unchanged. The goal is to limit the descent speed of the load to V2=12 m/min.

a) Calculate the angular speed of the rotor.

b) Calculate the electromagnetic power provided by the generator.

c) Calculate the resisting torque of this generator and the current through the additional resistance.

d) Determine the resistance R.

Solution: of Exercise 08:

1.

a) **Useful Power**: The useful power is calculated using:

$$P_{\mathrm{u}} = F * V$$

Substituting F=30 kN and V1=15 m/min:

$$P_{\mathrm{u}} = 30 * 15$$

$$P_{\mathrm{u}} = 450\ KNm/min$$

This is the **useful power** in kilonewton meters per minute (kNm/min).

Convert to kilowatts (kW): Since

$$\frac{1KNm}{min} = \frac{1}{60}KW$$

We convert 450 kNm/min to kilowatts:

$$P_u = 7.5KW$$

Electromagnetic Torque:

The electromagnetic power is related to the useful power by the efficiency η:

$$P_{em} = \frac{P_u}{\eta}$$

$$P_{em} = \frac{7.5}{0.83}$$

$$P_{em} = 9.04\ KW$$

The torque is given by:

$$T_{em} = \frac{P_{em}}{\Omega}$$

First, convert the rotational speed to angular velocity:

$$\Omega = 1500\frac{2\pi}{60}$$

$$\Omega = 157.08\frac{rad}{s}$$

Now calculate the torque:

$$T_{em} = \frac{9040}{157.08}$$

$$T_{em} = 57.57\ Nm$$

b) **Armature Current**:

The current is related to the electromagnetic torque by:

$$I = \frac{T_{em}}{1.35}$$

Substituting the torque:

$$I = \frac{57.57}{1.35}$$

$$I = 42.61\ \text{A}$$

Electromotive Force (EMF):

The EMF is given by:

$$E = \frac{P_{em}}{I}$$

Substituting the values:

$$E = \frac{9040}{42.61}$$

$$E = 212.16\,V$$

Voltage Applied to the Armature:

The voltage applied to the armature is:

$$U = E + R_a * I$$

Substituting the values:

$$U = 212.16 + 0.4 * 42.61$$

$$U = 229.2\text{ V}$$

c) **Total Absorbed Power**:

The total absorbed power is the sum of the electromagnetic power, the power losses in the armature, and the excitation power:

$$P_{\text{total}} = P_{\text{em}} + p_{\text{jr}} + p_{\text{e}}$$

Where:

$$p_{\text{jr}} = R_a * I^2$$

$$p_{\text{jr}} = 0.4 * (42.61)^2$$

$$p_{\text{jr}} = 726.88W$$

Thus

$$P_{\text{total}} = 9040 + 726.88 + 235$$

$$P_{\text{total}} = 9976\text{ W}$$

2.

a) **Angular Speed**: The angular speed during descent can be calculated as:

$$n_2 = \text{n}_1 * \frac{\text{V}_2}{\text{V}_1}$$

Substituting the values:

$$n_2 = 1500 * \frac{12}{15}$$

$$= 1200\,rmp$$

b) **Electromagnetic Power**:

The electromagnetic power provided by the generator is:

$$P_{\text{a}G} = \eta * F * V$$

Substituting the values:

$$P_{\text{a}G} = 0.83 * 30 * 12$$

$$= 4980\text{ W}$$

c) **Resisting Torque**:

The resisting torque is:

$$T_{\text{em}} = \frac{P_{\text{em}}}{\Omega}$$

The angular speed Ω is:

$$\Omega = 1200\,\frac{2\,\pi}{60}$$

$$\Omega = 125.66\ rad/s$$

Thus, the resisting torque is:

$$T_{\text{em}} = \frac{4980}{125.66}$$

$$T_{\text{em}} = 39.63\ \text{Nm}$$

Current through the Additional Resistance:

The current is:

$$I = \frac{T_{\text{em}}}{1.35}$$

$$I = \frac{39.63}{1.35}$$

$$\text{I} = 29.35\ \text{A}$$

d) **Resistance R**: The resistance is:

$$R = \frac{U}{I}$$

$$\text{R} = \frac{(E - R_a * I)}{I}$$

$$\text{R} = 5.7\Omega$$

Exercise 09:

An extraction machine is driven by a separately excited DC motor. The field winding is powered by a constant voltage u=600 V and carries a constant excitation current of J=30 A. The armature with resistance R=0.012 Ω is supplied by a voltage source providing a variable voltage from 0 V to the nominal value Un=600 V. The armature current has a nominal value of In=1.5 kA, and the nominal rotational speed is nn=30 RPM.

1. Starting

a) Write the relation between U, E, and I. Deduce the starting voltage Us to apply for $I_s = 1.2 \times \ln(I)$.

2. Nominal Operation during Load Lifting

a) Express the power absorbed by the armature of the motor and calculate its numerical value.

b) Express the total power absorbed by the motor and calculate its numerical value.

c) Express the total power lost by Joule effect and calculate its numerical value.

d) Given that other losses amount to 27 kW, express and calculate the useful power and the motor's efficiency.

e) Express and calculate the useful torque Tu and the electromagnetic torque Tem.

3. Operation During No-Load Lifting

a) Show that the electromagnetic torque Tem is proportional to the current I in the armature: Tem=K·I . Assume that during no-load lifting, the electromagnetic torque Tem′ is 10% of its nominal value and remains constant during the entire lifting process.

b) Calculate the armature current I′ during no-load lifting.

c) With the voltage U remaining equal to Un, express and calculate the motor's new electromotive force E′.

d) Express the new rotational speed n′ in terms of E′, I′ , and Tem′ . Calculate its numerical value.

Solution of Exercise 09:

1. Starting

a) Voltage Relation:

The voltage applied to the armature is related to the EMF and current by:

$$U = E + R_{\mathrm{a}} \cdot I$$

At startup, the EMF E=0 (since the speed is zero), so the starting voltage Us is:

$$U_s = R_a \cdot I_s$$

Substituting

$$I_s = 1.2 \cdot 1.5\mathrm{KA}$$

$$I_s = 1.8KA$$

$$U_s = 0.012 \quad \cdot 1800$$

$$U_s = 21.6\ \mathrm{V}$$

2. Nominal Operation

a) Power Absorbed by the Armature:

The power absorbed by the armature is:

$$P_{a_{\mathrm{ar}}} = U \cdot I$$

$$P_{a_{\mathrm{ar}}} = 600 \times 1{,}5, 10^3$$

$P_{a_{ar}} = 900\text{KW}$

b) Total Power Absorbed by the Motor:

The total power absorbed by the motor includes both the armature and field powers:

$$P_t = P_{a_{ar}} + P_{a\,ex}$$

Given that $P_{a\,ex}$=18 kWP, the total power is:

$$P_t = 900 + 18$$

$$P_t = 918\text{KW}$$

c) Joule Losses:

The total Joule losses are:

$$P_{jt} = R_a I^2$$

$$P_{jt} = 45KW$$

d) **Useful Power and Efficiency**:

The useful power is:

$$P_\text{u} = P_{a_{ar}} - P_{jt} - other\ losses$$

Substituting:

$$P_\text{u} = 900 - 27 - 27$$

$$P_\text{u} = 846\text{KW}$$

The efficiency is:

$$\eta = 0.92$$

e) **Useful Torque Tu and Electromagnetic Torque** T_{em}:

The useful torque is given by:

$$T_u = \frac{P_u}{\Omega}$$

The angular velocity Ω is:

$$\Omega = 30\,\frac{2\,\pi}{60}$$

$$\Omega = 3.14\,rad/s$$

Thus:

$$T_u = \frac{846000}{3.14}$$

$$T_u = 269.3KN.m$$

The electromagnetic torque is:

$$T_{em} = \frac{P_{aind}}{\Omega}$$

$$T_{em} = \frac{900000}{3.14}$$

$$T_{em} = 286.6 \text{ KN}m$$

3. No-Load Lifting

a) Proportionality of Torque to Current:

The electromagnetic torque T_{em} can be expressed as:

$$T_{\text{em}} = K_e \cdot \Phi \cdot I$$

Since Ke·Φ is a constant, we can simplify this to:

$$T_{\text{em}} = K \cdot I$$

This shows that the torque is directly proportional to the armature current I when the magnetic flux Φ is constant.

No-Load Lifting Condition:

During no-load lifting, the load on the motor is much smaller than during regular operation. According to the problem, the **electromagnetic torque** T_{em}' under no-load conditions is **10% of the nominal torque**:

$$T_{\text{em}}' = 0.1. T_{\text{em}}$$

$$T_{\text{em}}' = 0.1. K \cdot \text{I}$$

This torque value remains constant throughout the no-load lifting process.

Since the torque is proportional to the current in the armature, the current under no-load conditions will also be **10% of the nominal current**:

$$T_{\text{em}}' = K \cdot I'$$

So

$$K \cdot I' = 0.1. K \cdot \text{I}$$

$$I' = 0.1. I$$

$$I' = 150 \text{ A}$$

c) **New Electromotive Force E′**:

The new EMF is:

$$E' = U - R_a \cdot I'$$

Substituting U=600 V and I′=150 A :

$$E' = 598.2 \text{ V}$$

d) **New Rotational Speed n′**:

The rotational speed is proportional to the EMF:

$$T_{em} = E' \cdot I' \cdot \frac{30}{\pi} \cdot n'$$

$$n' = E' \cdot I' \cdot \frac{30}{\pi} \cdot T'_{\text{em}}$$

$$n' = 30.83\text{tr/mn}$$

Exercise 10:

A separately excited DC motor has the following specifications:

- Voltage: 220 V
- Current: 500 A
- Speed: 600 RPM
- Armature resistance: Ra=0.02 Ω
- Field resistance: Rf=0.03 Ω
- Motor constant: Kv=15.27 **m VA(rad/s)$^{-1}$**

The load torque is given by:

Cl=2000−2·N(in Nm, with N in RPM)

Speed variation is achieved by adjusting the armature voltage when N<Nn, and by adjusting the field excitation when N>Nn.

1. Calculate the armature current and voltage when the motor is running at 450 RPM.
2. Calculate the field current, the percentage reduction in flux (Δϕ/ϕe), and the field voltage when the motor is running at 750 RPM. The nominal field voltage is Vex=220 V.

Solution of Exercise 10:

1. Armature current and voltage when the motor runs at 450 RPM:

Torque at 450 RPM:

$$T = T_l$$

$$T = 2000 - 2 \cdot \text{N}$$

Substitute N=450 rmp:

$$\text{T} = 2000 - 2 \times 450$$

$$T = 1100 \text{ N} \cdot \text{m}$$

Since $= 450$ rmp $< N_n$, speed control is achieved by adjusting the armature voltage, so the field current remains constant:

$$I_f = I_{fn} = 500A$$

Armature current Ia:

$$T = KI_fI_a$$

$$I_a = \frac{T}{KI_f}$$

$$I_a = \frac{1100}{0{,}01527 \times 500}$$

$$I_a = 144{,}07 \text{ A}$$

Armature voltage Va:

$$V_a = R_aI_a + E$$

First, calculate the back EMF E. The angular velocity Ω at 450 RPM is:

$$\Omega = 250 \times \frac{2\pi}{60} = 47{,}1 \text{ rad/s}$$

Now, calculate E:

$$E = K_vI_f\Omega$$
$$E = 0.01527 \times 500 \times 47{,}1$$
$$\text{E} = 359{,}61 \text{ V}$$

Finally, calculate Va:

$$V_a = 0.02 \ \times 144.07 + 359.6$$
$$V_a = 362.49 \ V$$

2. Field current, flux reduction percentage Δϕ/ϕ, and field voltage when the motor runs at 750 RPM:

Torque at 750 RPM:

T=2000−2×750=500 Nm

Since N=750 RPM>Nn , the speed control is achieved by adjusting the field current If.

Back EMF E: The angular velocity Ω at 750 RPM is:

$$\Omega = 750 \times \frac{2\pi}{60} = 78.54 \text{ rad/s}$$

Now, calculate E:

Now, calculate E:

$$E = K_v I_f \Omega$$
$$E = 0.01527 \times 500 \times 78.54$$
$$\mathrm{E} = 599.59 \text{ V}$$

Field current If: Using the torque equation

$$T = KI_{\text{an}} I_f$$

And solve for If:

$$I_f = \frac{500}{0{,}01522 \times 142{,}07}$$
$$I_{\text{f}} = 228.28 \text{ A}$$

Flux reduction percentage Δϕ/ϕe : To calculate the percentage reduction in flux, we use the flux expressions at different conditions:

$$C_1 = 1100 = KI_1\phi_1$$

$$\phi_1 = \frac{C_1}{KI_a}$$

$$C_2 = 500 = KI_{\text{a}}\phi_2$$

$$\phi_2 = \frac{C_2}{KI_{\text{a}}}$$

Now, the flux reduction ratio:

$$\frac{\Delta\phi}{\phi} = \frac{\phi_2 - \phi_1}{\phi_1}$$

$$\frac{\Delta\phi}{\phi} = \frac{\frac{C_2}{KI_a} - \frac{C_1}{KI_a}}{\frac{C_1}{KI_a}}$$

$$\frac{\Delta\phi}{\phi} = \frac{C_2 - C_1}{C_1}$$

$$\frac{\Delta\phi}{\phi} = \frac{500 - 1100}{1100}$$

$$\frac{\Delta\phi}{\phi} = -0{,}52\%$$

Exercise 11:

The nameplate of a separately excited DC machine indicates the following nominal values:

U_{Na}=90V, I_{Na}=10A, P_{outN}=750W, n_N=1500tr/min, Ra=0.4Ω.

For nominal operation, calculate:

1. The electromotive force.
2. The useful torque .
3. The electromagnetic torque .

When the motor is operating nominally and the load is removed (the motor runs at no-load), the speed changes:

- Does the rotational speed increase or decrease?
- To bring the speed back to its nominal value , which parameter must be adjusted and in what direction (increase or decrease)? Justify.

The motor is powered by an armature voltage U that is adjustable between 0 and 90V. The load exerts a constant resisting torque T*l*=547 N.m .

- In these conditions, are the electromagnetic torque and the armature current Ia constant or variable? Justify and provide their values.
- Demonstrate that one can write:

  Ω=183·U−73

  Then deduce that:

  n=175·U−70

  where n is the rotational speed in rpm.
- Determine the motor's starting voltage .

Solution of Exercise 11:

I. Nominal Operation:

1-Electromotive Force E_N:

$$E_N = U_n - R_a I_{\text{an}}$$

Substituting the values:

$$E_N = 90 - 0{,}4 \cdot 10$$

$$E_N = 86 \text{ V}$$

2-Useful Torque :

$$T_{outN} = \frac{P_{out\ N}}{\Omega_N}$$

First, calculate the angular velocity Ω_N:

$$\Omega_n = 1500 \times \frac{2\pi}{60} = 157\text{rad/s}$$

Now, calculate the torque:

$$T_{outN} = \frac{750}{157}$$

$$T_{outN} = 4{,}77 N \cdot m$$

3-Electromagnetic Torque : $T_{\text{em}\,N}$

$$T_{\text{em}\,N} = \frac{P_{\text{em}\,N}}{\Omega_n}$$

$$T_{\text{em}\,N} = \frac{E_N \cdot I_N}{\Omega_n}$$

Substituting the values:

$$T_{\text{em}\,N} = \frac{86 \times 10}{157}$$

$$T_{\text{em}\,N} = 5{,}47 N \cdot n$$

II. No-Load Operation:

1. **At no load, the rotational speed increases.**
2. **At no load, the EMF is approximated as**:

$$E_0 = U - R_a I_{ao} \simeq U$$

Since:

$$K\phi\Omega \approx \text{U}$$

To bring the speed back to its nominal value, the armature voltage U_a at the motor terminals must be decreased.

III. Armature Voltage Control:

1. **Torque and Armature Current**:

This control is also known as constant torque control, so:

$$T_{out} = T_{\text{l}} = 5{,}47\text{Nam}$$

The electromagnetic torque is given by:

$$T = \text{K}\emptyset I_a$$

With constant flux ϕ, we get:

$$T = aI_a$$

$$E = k\phi\Omega$$

$$E = a\,\Omega$$

$$E_N = a\Omega_n$$

$$a = \frac{E_N}{\Omega_n}$$

$$a = \frac{86}{157} = 0,547$$

Therefore, the armature current is constant:

$$I_a = \frac{T}{a}$$

Substituting the values

$$I_a = \frac{5.47}{0,547} = 10A$$

2-Rotational Speed:

The equation for the back EMF is:

$$\mathrm{E} = a\Omega$$

Therefore:

$$\Omega = \frac{E}{a}$$

Substituting the values:

$$\Omega = \frac{U - R \cdot I_a}{a}$$

$$\Omega = \frac{U - 4}{0,547}$$

$$\Omega = 1,83\,U - 7,3$$

The rotational speed N in RPM is:

$$N = \Omega \times \frac{60}{2\pi}$$

$$\mathrm{N} = (1,83U - 7,3) \times \frac{60}{2\pi}$$

$$\mathrm{N} = 17,5U - 70$$

3-Starting Voltage Us: At the start

$$\Omega = 0$$

$$E_S = 0$$

Therefore, the starting voltage is:

$$U = R_a I_{aN}$$

$$U_S = 4V$$

Exercise 12:

We consider a separately excited DC motor, where both the armature and the field windings are powered by two single-phase full-bridge thyristor rectifiers. The following parameters are given:

- Ra=0.2 Ω (armature resistance),
- Rex=130 Ω (field resistance),
- $Kv=1.2VA^{-1}(rad/sec)^{-1}$
- The supply voltage for the rectifiers is 240V.

1. The motor is powered by an armature voltage of 180V, running at 1000 rpm, and drawing an armature current of 10 A. Calculate:

a-The field current and the firing angle of the thyristor rectifier supplying the field winding α_{ex},

b-The power factor.

2. Calculate the firing angle αa of the rectifier supplying the armature for the motor to produce the same torque at a speed of 100 rpm.

3. A freewheeling diode is added across the armature terminals. Sketch the waveform of the load voltage and calculate the new speed.

Solution of Exercise 12:

Given:

$$V_s = 240 \Rightarrow V_m = V_s.\sqrt{2}$$

$$V_\text{m} = 339{,}41 \text{ V}$$

1) Calculate Iex and αex

We know:

$V_a = 180 \text{ V}, N = 1000 \text{ rpm} = 104{,}67 \text{rad/s}$ and $I_a = 10 \text{ A}$

First, calculate E:

$$V_a = R_a I_a + E$$

$$E = V_a - R_a I_a$$

$$\text{E} = 180 - 0{,}2 \times 10$$

$$E = 178 \text{ V}$$

Next, calculate the excitation current I_{ex}:

$$E = K_v I_{ex} \Omega$$

$$I_{ex} = \frac{E}{K_v \Omega}$$

$$I_{ex} = \frac{178}{1,2 \times 104,67}$$

$$I_{ex} = 1,42 \text{ A}$$

Now, we calculate α_{ex}

$$V_{ex} = R_{ex} I_{ex}$$

$$V_{ex} = 130 \ \times 1,42$$

$$V_{ex} = 184,6 \text{ V}$$

In the PD2 rectifier, the output voltage is:

$$V_{ex} = \frac{2V_m}{\pi} \cos \alpha_{\text{ex}}$$

$$\cos \alpha_{\text{ex}} \ = \frac{\pi \cdot V_{ex}}{2V_m}$$

$$\cos \alpha_{\text{ex}} \ = \frac{3,14 \times 184,6}{2 \times 339,41}$$

$$\cos \alpha_{ex} = 0,85389$$

$$\alpha_{\text{ex}} = 32°$$

b) **Power Factor Calculation**: $F_p = \frac{p}{s}$

Power P:

$$P = V_a I_a + V_{ex} I_{\text{ex}}$$

$$P = 180 \times 10 + 184.6 \times 1,42$$

$$P = 2062,132 \text{ W}$$

Now, the apparent power S:

$$S = V_s I_s \ \text{ and } V_s = 240 \text{ V}$$

Where

$$I_S = \sqrt{I_a^2 + I_{ex}^2}$$

$$I_S = 10,10\text{A}$$

$$S = 240 \times 10,10$$

$$S = 2424 \ VA$$

The power factor Fp:

$$F_p = \frac{2062,13}{2424}$$

$$F_p = 0,85$$

2) Calculateα_a for $\mathrm{N} = 100\ \mathrm{rpm} = 10{,}27\mathrm{rad/s}$

When we vary αa and apply the control, the variation of the armature voltage in this part of the control results in a constant torque.

We calculate the firing angle αa for 100 rpm. The current remains the same:

- $I_{\mathrm{ex}} = cs\mathrm{t} = 1{,}42\mathrm{A}$ and $I_a = \mathrm{cst} = 10\mathrm{A}$

First, calculate the back EMF E:

$$E = K_v \cdot I_{\mathrm{ex}} \cdot \Omega$$

$$E = 1{,}2 \times 1{,}42 \times 10{,}47$$

$$E = 17{,}84\ \mathrm{V}$$

Now calculate the armature voltage Va:

$$V_a = R_a I_a + E$$

$$V_a = 0{,}2 \times 10 + 17{,}84$$

$$V_a = 19{,}84\ \mathrm{V}$$

Using the PD2 rectifier formula:

$$V_a = \frac{2V_m}{\pi} \cos \alpha_a$$

$$\cos \alpha_a = \frac{\pi V_a}{2V_m}$$

$$\cos \alpha_a = \frac{3{,}14 \times 19{,}84}{2 \times 339{,}41}$$

$$\alpha_a = 84,83°$$

3) Adding a Freewheeling Diode:

In this case, the armature voltage becomes:

$$V_a = \frac{V_m}{\pi}(1 + \cos \alpha_{\mathrm{a}})$$

$$V_a = \frac{339{,}41}{3{,}14}(1 + \cos(84,83°))$$

$$V_a = 117{,}83\ \mathrm{V}$$

Now, we calculate the new speed Ω using the back EMF equation:

$$E = V_a - R_a I_a$$

$$E = K_v \cdot I_{\mathrm{ex}} \cdot \Omega$$

$$\Omega = 67{,}5\mathrm{rad/s} = 652\ \mathbf{rpm}$$

So, the new speed is approximately **652 rpm**

Exercise 13:

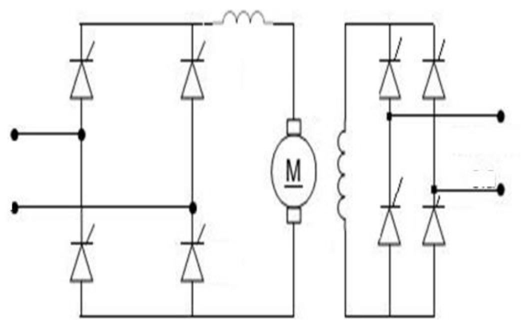
We consider a separately excited direct current (DC) motor whose armature and field windings are powered by two **complete single-phase thyristor bridge**s. Given parameters are:

- Ra=0.25 Ω (Armature resistance)
- Rf=175 Ω (Field resistance)
- Kv=1.4 **VA^{-1}(rad/sec)$^{-1}$.** (Motor constant)
- The supply voltage for the bridges is Vs=440 V
- The firing angle of the bridge supplying the field is αf=0°
- The firing angle of the bridge supplying the armature is αa=**π/3**
- The motor consumes an armature current of Ia=45 A

Questions:

1. Calculate the torque developed by the motor as well as its speed.
2. Calculate the power supplied by the network and the power factor.

Solution of Exercise 13:



1. Torque Developed by the Motor and Speed

Torque Calculation:

The torque developed by the motor is given by:

$$T = KI_a I_f$$

First, we need to calculate the field current I_f

$$V_f = \frac{2V_m}{\pi}\cos\alpha_f$$

The peak voltage Vm is:

$$V_m = 440 \times \sqrt{2}$$

$$V_m = 622{,}25\ \text{V}$$

Therefore:

$$V_f = \frac{2 \times 622{,}25}{\pi}$$

$$V_f = 396{,}14 \text{ V}$$

Calculating the Field Current I_f:

$$I_{ex} = \frac{V_f}{R_f} = \frac{396{,}14}{175}$$

$$\Rightarrow I_{\text{ex}} = 2{,}26 \text{ A}$$

Calculating the Torque T:

$$T = 1{,}4 \times 45 \times 2{,}26$$

$$T = 142{,}4 \text{ N} \cdot \text{m}$$

Calculating the Speed Ω:

The back EMF E is calculated using:

$$E = KI_{\text{ex}}\,\Omega$$

For the armature voltage:

$$V_a = \frac{2V_m}{\pi}\cos 60°$$

$$V_a = \frac{2 \times 622{,}25}{\pi}\cos 60°$$

$$V_a = 198{,}07 \text{ V}$$

Now, using:

$$V_a = RI_a + E$$

We rearrange to find E:

$$E = V_a - RI_a$$

$$E = 198{,}07 - 45 \times 0{,}25$$

$$E = 186{,}82V$$

Finally, the speed Ω is:

$$\Omega = \frac{E}{\text{K}I_{ex}} = \frac{186{,}82}{1{,}4 \times 2{,}26}$$

$$\Omega = 56{,}05\text{rd}/s$$

$$\Omega = 564 \text{ rmp}$$

2. Power Supplied by the Network and Power Factor

Calculating Power P:

$$\text{P} = V_a I_a + V_f I_f$$

Substituting the known values:

$$P = 198{,}07 \times 45 + 396{,}14 \times 2{,}26$$

$$P = 9808{,}4W$$

Calculating the Power Factor PF:

The apparent power S is calculated as:

$$S = V_S I_S$$

Where:

$$V_S = 440V$$

$$I_S = \sqrt{{I_a}^2 + {I_f}^2}$$

$$I_S = 45.06A$$

Thus, the apparent power:

$$S = 19826{,}4\ VA$$

Finally, the power factor PF is:

$$\text{PF} = \frac{P}{S}$$

$$F_p = \frac{9808{,}4}{19826{,}4}$$

$$F_P = 0{,}495$$

Exercise 14:

We consider a separately excited direct current (DC) motor whose armature and field windings are powered by two complete single-phase thyristor bridges. Given parameters are:

- Ra=0.2 Ω(Armature resistance)
- Rf=130 Ω (Field resistance)
- Kv=1.2 **$VA^{-1}(rad/sec)^{-1}$** (Motor constant)
- The supply voltage for the bridges is Vs=240 V

1. The motor powered by an armature voltage of Va=180 V, runs at a speed of N=1000 RPM and absorbs an armature current of Ia=10 A. Calculate:
   - The excitation current If and the firing angle αf of the bridge supplying the field.
   - The power factor PF.
2. Calculate the firing angle αa of the bridge supplying the armature to develop the same torque at a speed of 100 RPM.

3. A free-wheeling diode is added across the armature terminals of the motor. Sketch the charging voltage waveform and calculate the new speed.

Solution of Exercise 14:

1. Excitation Current and Firing Angle of the Field Bridge

Calculating the Back EMF E:

Using the formula:

$$V_a = R_a I_a + E$$

Substituting the known values:

$$E = V_a - R_a I_a$$

$$E = 180 - 0{,}2 \times 10$$

$$E = 178 \text{ V}$$

$$E = k_v I_\mathrm{f} \Omega$$

Calculating the Excitation Current I_f:

Using:

$$I_\mathrm{f} = \frac{E}{\mathbf{K}_{\boldsymbol{V}}\ \Omega}$$

$$I_\mathrm{f} = \frac{178}{1{,}2 \times 104{,}67}$$

$$I_\mathrm{f} = 1{,}42 \text{ A}$$

Calculating the Firing Angle α_f :

The field voltage V_f is given by

$$V_f = R_f I_f$$

$$V_f = 130 \times 1{,}42$$

$$V_f = 184{,}6A$$

Using the relationship for the firing angle:

$$V_f = \frac{2V_m}{\pi} \cos \alpha_f$$

Calculating α_f :

$$\cos \alpha_f = \frac{\pi V_f}{2V_m} = \frac{\pi \times 184{,}6}{2 \times 339{,}41}$$

$$\cos \alpha_f = 0{,}85389$$

$$\alpha_f = 32°$$

b) Power Factor Calculation

Calculating Power P:

$$P = V_a I_a + V_f I_f = 180 \times 10 + 184{,}6 \times 1{,}42$$

$$P = 2062{,}132 \text{ W}$$

Calculating Apparent Power S:

$$S = V_S I_S$$

$$V_S = 240$$

$$I_s = \sqrt{\mathrm{I}_a^2 + I_\mathrm{f}^2}$$

$$I_s = \sqrt{10^2 + 1{,}42^2}$$

$$I_s = 10{,}10 \text{ A}$$

$$S = 240 \times 10{,}10$$

$$\mathrm{S} = 2424\ VA$$

Calculating Power Factor PF:

$$\mathrm{PF} = \frac{P}{S}$$

$$PF = \frac{2062{,}13}{2424}$$

$$PF = 0{,}85$$

2. Firing Angle αa for Same Torque at 100 RPM

To maintain the same torque at N=100 RPM

$$N = 100 \text{ rmp } \Rightarrow \Omega = 100 \frac{\times 2\pi}{00} = 10{,}47 \text{rad/s}$$

Since the torque is constant, the field current remains If=1.42 A and the armature current Ia is constant.

Calculating Back EMF E:

$$E = kI_{\mathrm{ex}}\,\Omega$$

$$E = 1{,}2 \times 1{,}42 \times 10{,}47$$

$$E = 17{,}84$$

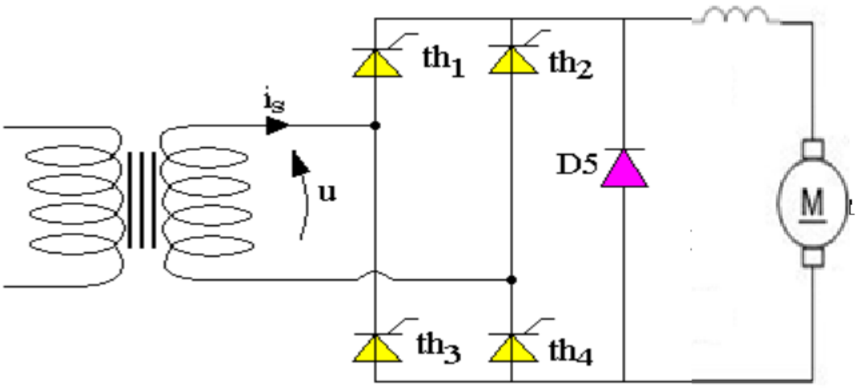
Calculating Armature Voltage Va:

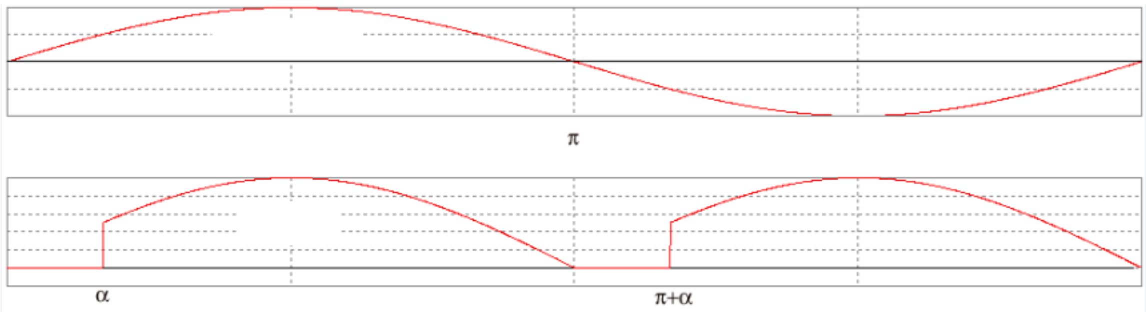
$$V_a = \frac{2V_m}{\pi} \cos \alpha_a$$

Using:

$$\cos \alpha_a = \frac{\pi V_a}{2V_m}$$

3. Free-Wheeling Diode and New Speed





Voltage Across Armature:

The voltage across the armature with the free-wheeling diode can be expressed as

$$V_a = \frac{V_m}{\pi}(1 + \cos \alpha_a)$$

Calculating New Voltage Va:

Assuming αn is the new firing angle (for example, let's take α_a=84.83°:

$$V_a = \frac{339{,}21}{3{,}121}(1 + \cos 84{,}83)$$

$$V_a = 117{,}83 \text{ V}$$

Calculating Back EMF with New Voltage:

$$V_a = R_a I_a + E$$

$$E = V_a - R_a I_a$$

Calculating New Speed Ω:

$$E = k_v I_f \Omega$$

$$\Omega = 68.45 \frac{\text{rad}}{\text{s}}$$

Converting to RPM:

N=652 rmp

Exercise 15:

We consider a separately excited DC motor with a power rating of 20 hp, 300V, and a speed of 1800 RPM, controlled by a **three-phase converter**. The excitation current is also controlled by a three-phase converter and is set to the maximum possible value. The converter input is 208 V, 60 Hz Y-connected. The armature resistance Ra=0.25 Ω , the field resistance Rex=245, and Kv=1.2 **VA^{-1}(rad/sec)$^{-1}$**

Determine:

a) The firing angle αa of the converter feeding the armature, if the motor provides nominal power at nominal speed.

b) The no-load speed if the firing angle is the same as in part **a**, and the armature current at no-load is 10% of the nominal value.

c) The speed regulation.

Solution to Exercise 15:

Given:

Nominal speed $N = 1800\text{rmp}$

Angular speed $\Omega = 188{,}51\ \text{rad/s}$

Input voltage U=208 V (Y-connected)

$$V = \frac{U}{\sqrt{3}} = \frac{208}{\sqrt{3}} = 120\ \text{V}$$

$$V_m = 120\sqrt{2} = 169{,}7\ \text{V}$$

Power: 1 hp=746 W, so the nominal power

$$P_n = 20 \times 746 = 14920\ \text{W}$$

a) Calculating the firing angle α_a when the motor provides nominal power P_n at the nominal speed N_n:

The nominal armature current:

$$I_{an} = \frac{P_n}{V_n}$$

$$I_{an} = \frac{14920}{300}$$

$$I_{an} = 49{,}73\ \text{A}$$

For the excitation circuit (PD3), the excitation voltage is given by:

$$V_{\text{ex}} = \frac{3\sqrt{3}V_n}{\pi}\cos\alpha_{\text{ex}}$$

Since the converter is set to the maximum value:

$$V_{\text{ex}} \text{ max} \Rightarrow \cos\alpha_{ex} = 1$$

$$\alpha_{ex} = 0°$$

$$V_{ex} = \frac{3\sqrt{3}}{\pi}V_m$$

$$V_{ex} = \frac{3\sqrt{3}}{\pi}169{,}7$$

$$V_{ex} = 280{,}7 \text{ V}$$

The excitation current:

$$I_{\text{ex}} = \frac{V_{ex}}{R_{ex}}$$

$$I_{\text{ex}} = \frac{280{,}7}{245}$$

$$I_{\text{ex}} = 1{,}146 \text{ A}$$

- At nominal power, the armature current $I_a = I_{an} = 49{,}73$ A
- For the back EMF E:

$$E = K_v I_{\text{ex}}\,\Omega$$

$$E = 1{,}2 \times 1{,}146 \times 188.5$$

$$\Rightarrow E = 259{,}2\ V$$

Now, the armature voltage Va:

$$V_a = R_a I_a + E$$

$$V_a = 0{,}25 \times 49{,}73 + 259{,}2$$

$$V_a = 271{,}63 \text{ V}$$

For the armature circuit (PD3):

$$V_a = \frac{3\sqrt{3}}{\pi}V_m\cos\alpha_a$$

Solving for αa :

$$\cos\alpha_a = \frac{3{,}14 \times 271{,}63}{3\sqrt{3} \times 169{,}7}$$

$$\alpha_a = 14{,}59°$$

b) No-load speed:

At no-load,

$$I_a = 10\% I_n$$

$$I_a = 4{,}973 \text{ A}$$

The back EMF at no-load E0:

$$E_0 = V_a - R_a I_a$$

$$E_0 = 271{,}63 - 0{,}25 \times 4{,}973$$

$$E_0 = 270{,}39 \text{ V}$$

The no-load speed Ω0 is:

$$E_0 = K_v I_f \Omega$$

$$\Omega_0 = \frac{E_0}{K_V I_f}$$

$$\Omega_0 = \frac{270{,}39}{1{,}2 \times 1{,}416}$$

$$\Omega_0 = 196{,}62 \text{ rad/s}$$

Converting to RPM: $N = 1877$ rmp

c) Speed Regulation:

Speed regulation is given by:

$$Speed\ Regulation = \frac{N_0 - N}{N}$$

$$Speed\ Regulation = \frac{1877 - 1800}{1800}$$

$$Speed\ Regulation = 0{,}043 = 4{,}3\%$$

Exercise 16:

A series-excited DC motor is powered by a series chopper with a 600V source.

Given:

- Armature resistance Ra=0.02 Ω
- Field resistance Rf=0.25 Ω
- Torque constant Kv=15. 27m $VA^{-1}(rad/sec)^{-1}$.
- Average armature current Ia=250 A
- Chopper duty cycle D=60%

Calculate:

A) Input power from the source

B) Equivalent input resistance of the chopper

C) Motor speed

D) Developed torque

Solution of Exercise 16:

a) Calculation of the input power from the source P_i

$$P_i = D \times V_s \times I_a$$

$$P_i = 0,6 \times 600 \times 250$$

$$P_i = 90 \text{ kW}$$

b) Calculation of the equivalent input resistance of the chopper

$$R_{eq} = \frac{V_s}{I_a D}$$

$$R_{eq} = \frac{600}{250 \times 0,6}$$

$$R_{eq} = 4\Omega$$

c) Calculation of the motor speed

$$V_a = RI_a + E$$

$$V_a = DV_s$$

$$V_a = 0,6 \times 600$$

$$V_a = 360 \text{ V}$$

Since this is a series-excited motor,

$$R = R_a + R_f$$

$$R = 0,02 + 0,03$$

$$R = 0,05\Omega$$

Thus:

$$E = V_a - RI_a$$

$$E = 360 - 0,05 \times 250$$

$$E = 347.5V$$

We also have:

$$E = K_v I_{ex} \Omega$$

$$\Omega = \frac{E}{K_v I_{ex}}$$

$$\Omega = \frac{347.5}{0.01527 \times 250}$$

$\Omega = 91.03\ ras/s$ N=869rmp

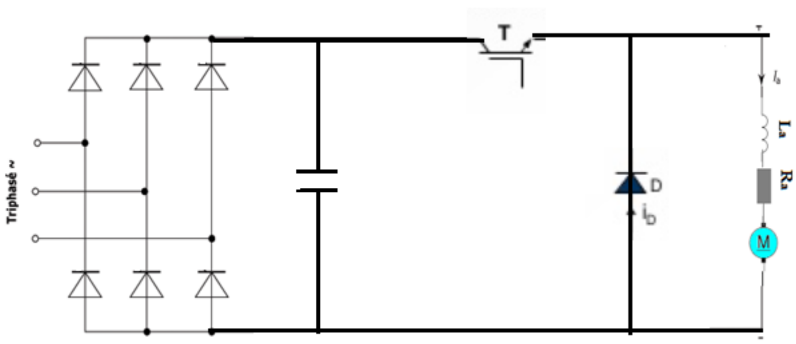
d) Calculation of the developed torque

$$T = K_v I_{ex} I_a = K_v I_a^2$$

T=954.38Nm

Exercise 17:

We consider a separately excited DC motor rated at 5 kW, 250V, 1500 RPM, powered by a three-phase 220V network through a series chopper using an IGBT with a frequency of 2 kHz



The motor's armature has a resistance of 400 mΩ, and its nominal current is 20A.

1. Calculate the duty cycle K when:
   a. The motor is at a standstill, delivering a starting current of 20A.
   b. The motor is operating at full load.
   c. The motor develops its nominal torque at a speed of 400 RPM.
2. Calculate the active power absorbed by the converter when the motor develops its nominal torque at a speed of 400 RPM.

Solution of Exercise 17:

Part 1:

a-The motor is stationary, delivering a starting current of 20A.

At startup, the speed is zero Ω=0, meaning the back EMF E=0.

- The armature voltage Va is calculated using Ohm's law:

$$V_a = R_a\ I_a$$

$$V_a = 0.4 \times 20$$

$$V_a = 8V$$

- The output voltage of the three-phase rectifier is given by:

$$V_h = \frac{3\sqrt{3}}{\pi} \cdot V_m$$

Where Vm is the peak voltage:

$$V_h = \frac{3\sqrt{3}}{\pi} \cdot \sqrt{2}\, V$$

$$V_h = \frac{3\sqrt{3}}{\pi} \cdot \frac{\sqrt{2}\, U}{\sqrt{3}}$$

Substituting into the formula:

$$V_h = \frac{3\sqrt{2}}{\pi} \cdot U$$

$$V_h = \frac{3\sqrt{2}}{\pi} \cdot 220$$

$$V_h = 297 \text{ V}$$

- The duty cycle K is given by:

$$k = \frac{V_a}{V_h}$$

$$\text{k} = \frac{8}{297}$$

$$k = 0{,}027$$

b: The motor operates at full load.

At full load, the voltage at the motor terminals is the nominal voltage Va=250V.

- The duty cycle is

$$K = \frac{V_a}{V_h}$$

$$\text{k} = \frac{250}{297}$$

$$K = 0{,}84$$

c: The motor develops nominal torque at a speed of 400 RPM.

To develop the nominal torque at 400 RPM, the armature voltage is varied. Since the excitation current I_{ex} is constant, and the armature current Ia=Ian=20A , we can calculate as follows:

- At nominal conditions Van=250V, Ian=20A, N=1500RPM.
- The back EMF at nominal speed is:

$$E_n = V_a - R_a I_{an}$$

$$E_n = 250 - 400 \times 10^{-3} \cdot 20$$

$$E_n = 242V$$

- The back EMF at 400 RPM can be found using the proportional relationship between speed and back EMF:

$$E = K_\text{v}\, I_{ex} \cdot \Omega$$

$$E = B \cdot \Omega$$

Where B is constant.

$$B = K_\text{v}\, I_{ex}$$

$$E(1500\ \text{rmp}) = B\left(1500 \times \frac{2\pi}{60}\right)$$

$$E(400\ \text{rmp}) = \beta\left(400 \times \frac{2\pi}{60}\right)$$

$$E(1500\ \text{rmp}) = 242\text{V} \longrightarrow 1500\ rmp$$

$$E(400\ \text{rmp}) = ? \longrightarrow 400\ \text{rmp}$$

Using the proportionality:

$$\frac{E}{E_n} = \frac{N}{N_n}$$

Therefore:

$$E = 64{,}53\ \text{V}$$

The armature voltage Va at 400 RPM is:

$$V_a = R_a I_a + E$$

$$V_a = 400 \times 10^{-3} \cdot 20 + 64{,}53$$

Va=72,53 V

The duty cycle is:

$$K = \frac{V_a}{V_h}$$

$$\text{K} = \frac{72{,}53}{297}$$

$$K = 0{,}24$$

Part 2: Calculate the active power absorbed by the converter when the motor develops nominal torque at 400 RPM.

- The active power P absorbed by the converter is:

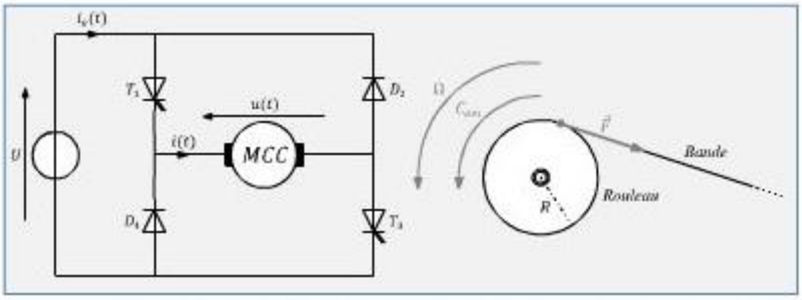
$$P = I_a V_a$$

$$\text{P} = 20 \times 72{,}53$$

$$P = 1{,}45\text{KW}$$

Exercise 18:

The setup in the adjacent figure is a chopper powering a permanent magnet DC machine used for motorizing a winder/unwinder. Transistors T1 and T2 are controlled over the interval [0,αT]with a chopping frequency f=50 Hz



- The source voltage is Us=120 V
- The machine's armature is modeled by an electromotive force (EMF) E=h·Ω, where the torque constant K=0.8 , and the armature resistance Ra is negligible.
- The winder/unwinder is operated under traction such that the force F is maintained constant at F=50 N.

Questions:

1. Is the system reversible? In how many quadrants can the DC machine operate? List them.
2. Using the fundamental principle of $j\frac{d\Omega}{dt} = T_{em}F.R$, calculate the value of the motor torque Tem for a radius R=15 cm. Then, deduce the value of the armature current.
3. For a duty cycle α, plot the voltage Va and the load current Ia over one period T. Specify the conduction sequences of the components.
4. Determine the expression for the average voltage Va as a function of α, then deduce the expression for the EMF E.
5. For a duty cycle α=0.6, calculate the speed and the electromagnetic power Pem.

Solution of Exercise 18:

1) Reversibility and Quadrants:

- The system is a reversible chopper in terms of voltage and can drive the motor in two quadrants: Quadrant 1 (motoring) and Quadrant 4 (regenerative braking).

2) Motor Torque Calculation:

- From the fundamental principle of dynamics

$$j\frac{d\Omega}{dt} = T_{\text{em}}F.R$$

$$\frac{d\Omega}{dt} = 0$$

Substituting F=50 N and R=0.15 m

$$T_{\text{en}} = F \cdot R = 50 \times 0{,}15$$

$$C_{em} = 7{,}5 \text{ N} \cdot \text{m}$$

- For a permanent magnet DC machine where the flux ϕ is constant

$$T_{em} = KI_a$$

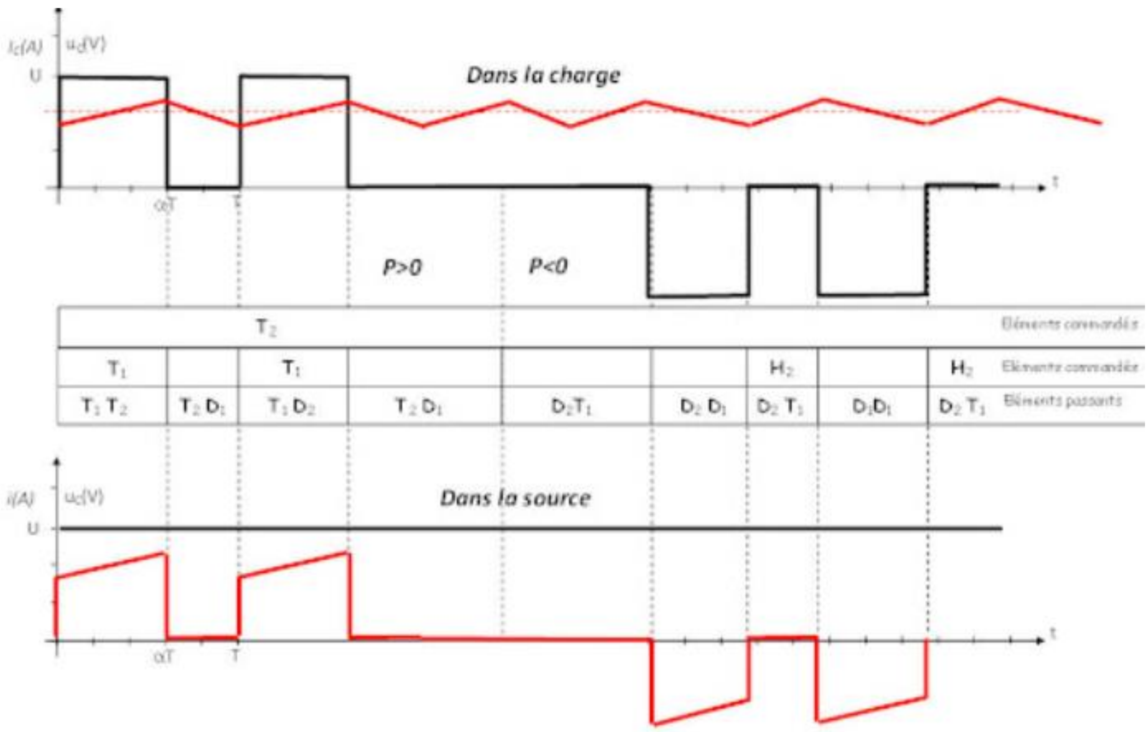
Solving for the armature current:

$$I_a = \frac{T_{em}}{K} = \frac{7{,}5}{0{,}8}$$

$$I_a = 9{,}38 \text{ A}$$

3) **Duty Cycle Plot**:

- For a given duty cycle α, plot the armature voltage Va and the load current Ia over one period T, specifying the conduction sequences of the transistors T1 and T2 during operation.



4) Average Voltage:

- The average voltage Va is given by

$$V_a = (2\alpha - 1)U_s$$

$$V_a = R_a\, I_a + E$$

Given that Ra is negligible

$$V_a = E$$

Therefore:

$$V_a = \mathrm{E} = (2\alpha - 1)U_s$$

5) Speed and Electromagnetic Power:

- The back EMF is related to the speed by

$$V_a = E = K\Omega$$

Solving for the angular velocity Ω:

$$\Omega = \frac{E}{K}$$

$$\Omega = \frac{(2\alpha - 1)U_s}{K}$$

For α=0.6, Us=120 V , and K=0.8:

$$\Omega = 30\ rad/s$$

The electromagnetic power is given by:

$$P_{em} = T_{em}\,\Omega$$

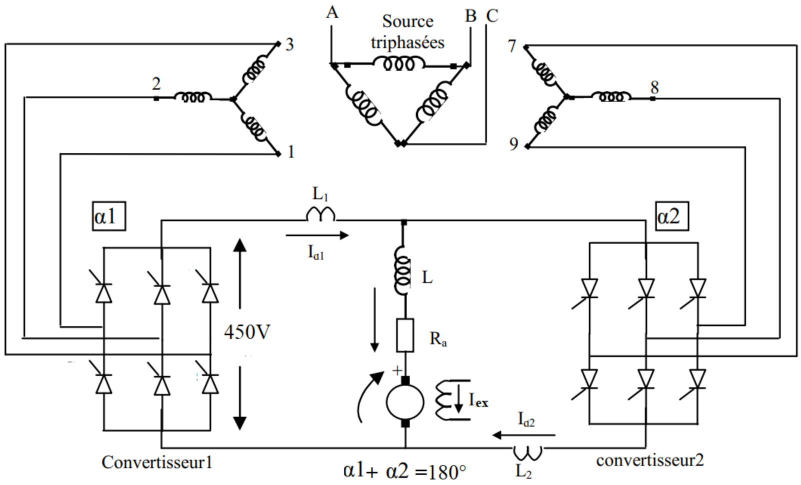
Substituting T_{em} =75 N.m and Ω=30 rad/s:

$$P_{em} = 7{,}5 \times 30$$

$$P_{em} = 225\ \mathrm{W}$$

Exercise 19:

An industrial process is powered by a DC motor as shown in the following figure:



- The nominal voltage across the motor's armature is 450V, and the armature current is 50A.
- Converter 1 supplies a current of Ia1=80 A , while converter 2 absorbs a current of Ia2=30 A . The three-phase voltage supplying the converters is U=380 V .

Calculate:

1. The firing angles of the converters.
2. The power associated with each converter.
3. The active power drawn from the AC network.
4. The reactive power drawn from the network.
5. The power factor.

Solution of Exercise 19:

1. **Firing angles of the converters:**
   - **For converter 1:**

$$V_a = 450V$$

The armature voltage is given by:

$$V_a = \frac{3\sqrt{3}V_m}{\pi}\cos\alpha_1 \qquad 0 < \alpha < \pi$$

Where

$$U = \sqrt{3}V$$

And

$$V_m = \sqrt{2}V$$

$$V_m = \sqrt{2}\frac{U}{\sqrt{3}}$$

$$V_m = 310.27V$$

Substituting into the equation

$$V_a = \frac{3\sqrt{3}}{\pi} 310.27 \times \cos\alpha_1 = 250$$

$$\cos\alpha_1 = 28.7°$$

For converter 2:

The firing angles α1 and α2 are related by:

$$\alpha_1 + \alpha_2 = 180°$$

Therefore:

$$\alpha_2 = 180 - 28{,}7$$

$$\alpha_2 = 151{,}3°$$

2-Power associated with each converter:

- **For converter 1**:

$$P_1 = I_{a1} \times V_a$$

$$P_1 = 80 \times 450$$

$$P_1 = 36\ \text{kW}$$

- **For converter 2**:

$$P_2 = I_{a2} \times V_n$$

$$P_2 = 30 \times 450$$

$$P_2 = 13{,}5\text{KW}$$

3-Active power drawn from the AC network:

The active power drawn is the difference between the power supplied by converter 1 and the power absorbed by converter 2:

$$P = P_1 - P_2$$

$$P = 22{,}5\ \text{kW}$$

4-Reactive power drawn from the network:

The reactive power can be calculated using the relation:

$$P = V \cdot I\cos\alpha$$

$$Q = VI\sin\alpha$$

$$\tan\alpha = \frac{Q}{P}$$

For converter 1:

$$Q = P \tan \alpha$$

$$Q_1 = P_1 \tan \alpha_1$$

$$Q_1 = 36.10^3 \times tan(28.7)$$

$$Q_1 = 19.3\text{KVAR}$$

For converter 2:

$$Q_2 = P_2 \tan \alpha_2$$

$$Q_2 = 13{,}5 \cdot 10^3 \times \tan(151{,}3°)$$

$$Q_2 = 7{,}4KVR$$

$$Q = Q_1 + Q_2$$

$$\text{Q} = 13{,}3 + 7{,}4$$

$$Q = 26{,}7KVR$$

Power factor: The power factor is given by:

$$PF = \frac{p}{s}$$

Where

$$\text{S} = \sqrt{P^2 + Q^2}$$

Substituting the values:

$$\text{S} = 34{,}9\text{KVA}$$

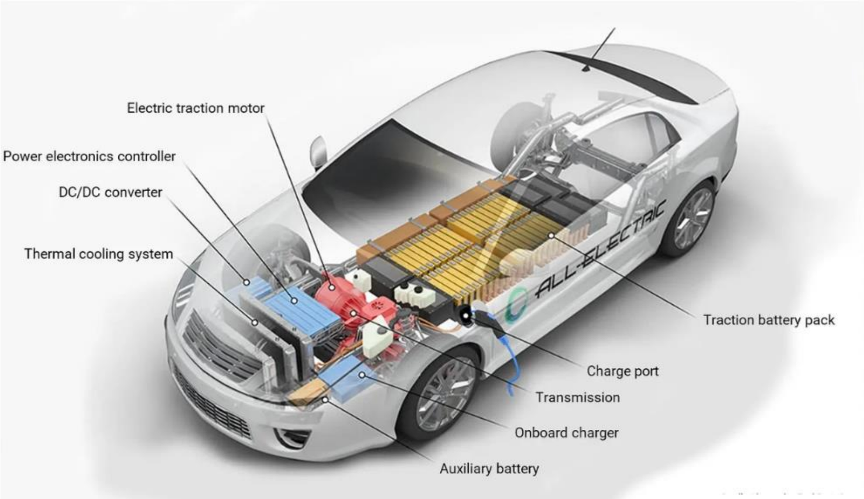
Therefore, the power factor is:

$$\text{PF} = 0{,}6$$

Exercise 20:

In this exercise, we study the principle of an electric car. The DC motor uses a simple power electronics system connected to the battery.

This is why some manufacturers decide to equip their first-generation cars with this type of motor.



Part A - The DC Motor

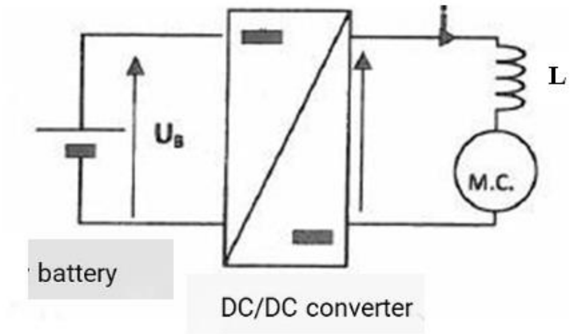
The motor's characteristics are as follows:

- **Independent excitation machine**: the electromotive force (EMF) E is proportional to the angular rotational speed Ω in rad/s: $E= K\o.\Omega$, with $K\o$ =1,31V/rad.s^{-1};
- **Armature circuit resistance**: R=0.15 Ω;
- **Constant armature voltage**: U=260 V. Constant losses are negligible.

1. The motor is driven by a current with an intensity I=170 A.
   - Draw the equivalent electrical model of the armature.
   - Calculate:
     - The electromotive force E of the motor;
     - The rotational speed N of the rotor in RPM;
     - The power losses dissipated by Joule heating in the armature;
     - The useful power Po;
     - The torque of the useful couple.
2. The motor drives the electric vehicle. Two preliminary tests were conducted on the motor:
   - **No-load test**: N0=1880 RPM ;
   - **Load test**: N=1700 RPM and Tu=234 N.m .
   - Plot the mechanical characteristic curve of the motor Tu(N) .

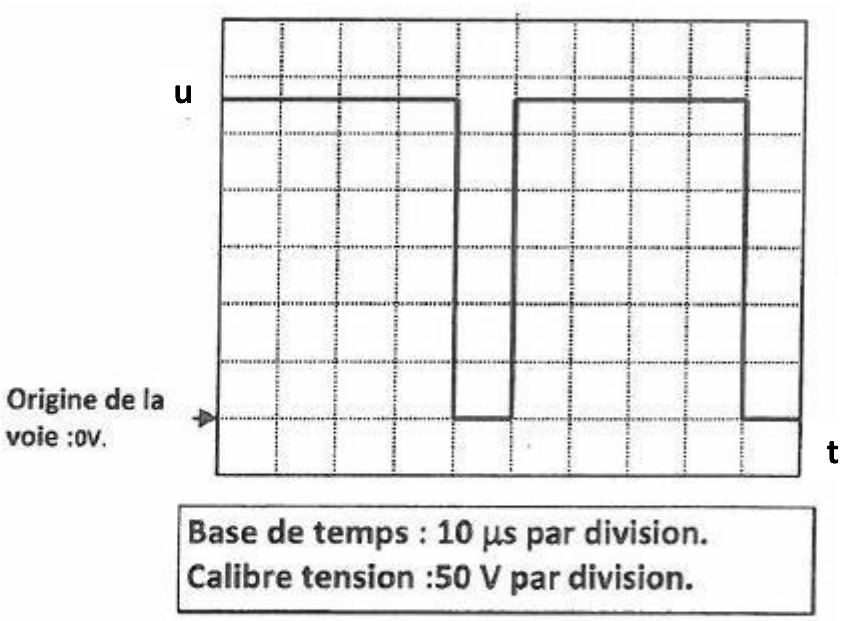
Part B - The Electronic System

The motor is powered via a DC-DC converter. The electrical energy is stored in a battery placed at the rear of the vehicle. The block diagram of the power circuit is as follows:



- What is the name of this DC-DC converter?
- What is the role of the inductor?

We observe the voltage u across the load. The oscillogram of the voltage u is provided below:



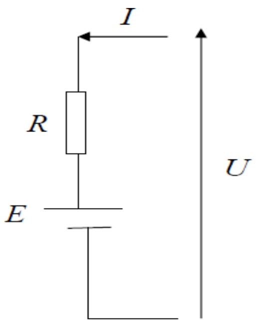
- Determine:
  - The frequency of the voltage u;
  - The duty cycle D;
  - The average value of the voltage.

Solution of Exercise 20:

Part A - The DC Motor

The motor is driven by a current with an intensity I=170 A.

- **Equivalent electrical model of the armature:**



Calculations:

- **Electromotive force E:**

$$E = U - R.I$$

$$E = 260 - 0{,}15 \times 170$$

$$E = 234{,}5V$$

- **Rotational speed N of the rotor in RPM:**

$$\mathrm{E} = K\Omega = 1{,}31 \times \Omega$$

$$\Omega = \frac{E}{1{,}31} = \frac{234{,}5}{1{,}31}$$

$$\Omega = 179\frac{\mathrm{rd}}{\mathrm{s}}$$

$$\mathrm{N} = 60 \times \frac{\Omega}{2\pi} = 60 \times \frac{179}{2\pi}$$

$$\mathrm{N} \approx 1710\mathrm{tr/min}$$

- **Joule losses in the armature:**

$$p_{cu} = RI^2$$

$$p_{cu} = 0{,}15 \times 170^2$$

$$p_{cu} = 4335\ \mathrm{W}$$

- **Useful power :**

$$P_o = P_{in} - p_{cu} - p_c$$

$$P_o = UI - p_{cu}$$

$$P_o = 260 \times 170 - 4335$$

$$P_o \approx 40000\ \mathrm{W}$$

$$P_o = 40\mathrm{KW}$$

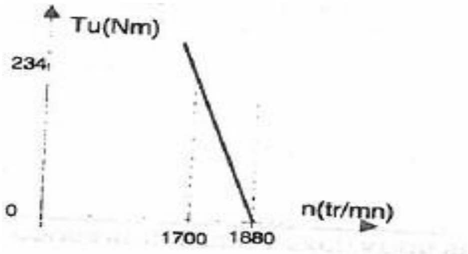
- **Useful torque :**

$$T_u = \frac{P_o}{\Omega}$$

$$T_u = \frac{40000}{179}$$

$$T_u = 223{,}5\text{Nm}$$

- **Mechanical characteristic of the motor Tu(N):** At no-load, N0=1880 RPM, and under load, N=1700 RPM with Tu=234 N.m.



Part B - The Electronic System

- **Name of the DC-DC converter:** Series chopper.
- **Role of the inductor:** The current in an inductance cannot undergo a sudden change. The inductance opposes variations in the current passing through it, particularly when:
  - L is large;
  - The voltage across the inductor is low.

An inductor smooths the current.

- **Frequency of the voltage U:** From the curve, one period corresponds to T $= 5.10\mu s = 50\mu s$, hence:

$$f = \frac{1}{T} = \frac{1}{50.10^{-6}}$$

$$f = 20\text{KHz}$$

- **Duty cycle D:** The duty cycle is defined as:

$$\alpha = \frac{\text{conduction time}}{\text{Période}}$$

$$\alpha = \frac{40}{50}$$

$$\alpha = 0{,}8$$

- **Average voltage**

$$U_{aver} = \alpha U = 0{,}8 \times 260$$

$$U_{aver} = 208 \text{ V}$$

Chapter II:

Control of AC motors

Exercise 01:

We are given an asynchronous machine :

- Nominal supply voltage Vsn=220V
- Rotor resistance Rr′=10Ω
- Number of pole pairs p=1
- Rotor inductance Lr′=0.02H
- Load torque Cl=3Nm

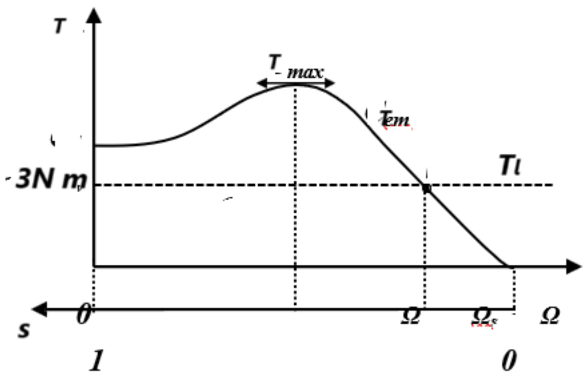
For Vs=Vsn and fs=50Hz

- Calculate the number of equilibrium points.
- Study the stability of these equilibrium points.

Solution of Exercise 01:

1. **Number of Equilibrium Points**: We are given the electromagnetic torque characteristic as a function of slip S and rotor speed Ω.

The maximum torque, Tmax , and the starting torque Ts need to be compared to the load torque T_l=3Nm



- Calculate Xr′=Lr′·ωs=0.02·314=6.28Ω
- Starting torque $\boldsymbol{T_S}$ is calculated using

$$T_s = \frac{3p}{\omega_s} \cdot R_r' \cdot \frac{V_s^2}{R_r'^2 + X_r'^2}$$

$$T_s = \frac{3}{314} \cdot 10 \cdot \frac{220^2}{10^2 + 6,28^2}$$

$$T_s = 33,163\text{Nm}$$

We observe that $\boldsymbol{T_l} < \boldsymbol{T_s}$. Therefore, there is a single intersection point between the electromagnetic torque (Tem and T_s, leading to only one operating point..

Stability of the Operating Point: The intersection between the electromagnetic torque and the load torque in the linear region (where slip S<Smax) confirms that the point of operation is stable.

The intersection of T_{em} withT_l occurs in the linear zone (where the linear part of T_{em} is valid for S<Smaxg).

Thus, we have**:**

$$\boldsymbol{T_{em}} = \frac{3 \cdot p \cdot V_l^2}{\omega_s \cdot R_r'} \cdot S$$

$$\boldsymbol{T_{em}} = \frac{3 \cdot 220^2}{314 \cdot 10} \cdot S$$

$$\boldsymbol{T_{em}} = 46{,}2 \cdot S$$

And since

$$\mathrm{S} = \frac{\Omega_s - \Omega}{\Omega_s}$$

$$T_{em} = 46{,}2 \cdot \frac{\Omega_s - \Omega}{\Omega_s}$$

Stability Condition:

The stability condition is given by:

$$\frac{dT_{em}}{d\Omega} - \frac{dT_l}{d\Omega} < 0$$

Since T_l is constant, we have:

$$\frac{dT_l}{d\Omega} = 0$$

$$\frac{dT_{em}}{d\Omega} < 0$$

$$\frac{dT_{em}}{d\Omega} = -46{,}2 < 0$$

Thus, the operating point is stable**.**

Exercise 02:

The nameplate of a slip-ring asynchronous motor indicates the following:

- Power: 37 kW
- Voltage: 220/380 V
- Frequency: 50 Hz
- Speed: 1455 rpm
- Efficiency (η): 0.91

- Power factor (cos φ): 0.85

1. What should its coupling be to operate on this network? What is the synchronous speed and how many poles does the machine have?
2. Calculate, for nominal operation:
   - the stator current
   - the slip
   - the useful torque
3. Perform a power balance assuming mechanical losses are very small. Determine approximately the value of the rotor current.

Solution of Exercise 02:

1. **Coupling, Synchronous Speed, and Number of Poles:**
   - **Coupling:** The motor should be coupled in **star (Y)** to operate on a **380 V** network.
   - **Synchronous Speed and Number of Poles:** The synchronous speed Ns is $=$ 1500 rmp
   - Given the actual speed of the motor is **1455 rpm** and the synchronous speed is approximately **1500 rpm**, the motor has **4 poles**.
2. **Stator Current, Slip, and Useful Torque:**

   Stator Current: The input power P_{in} can be calculated as:

$$P_{in} = \sqrt{3} U I_s \cos\varphi$$

The stator current Is is then

$$I_s = \frac{P_{in}}{\sqrt{3} U \cos\varphi}$$

$$\eta = \frac{P_{out}}{P_{in}}$$

$$P_{in} = \frac{P_{out}}{\eta}$$

$$P_{in} = \frac{37 \cdot 10^3}{0{,}91}$$

$$P_{in} = 40659{,}34 \text{ W}$$

$$I_s = \frac{P_{in}}{\sqrt{3}\, \text{U} \cos Q}$$

$$I_s = \frac{40659{,}34}{\sqrt{3} \times 380 \times 0{,}85}$$

$$I_s = 72{,}67 \text{ A}$$

Slip: The slip S is calculated as:

$$S = \frac{N_s - N_r}{N_s}$$

$$S = 0.03$$

Useful Torque:

The synchronous angular speed Ωs is:

$$\Omega_s = N_5 \times \frac{2\pi}{60} = 152{,}29\text{ad/s}$$

The useful torque is:

$$T_{out} = \frac{P_{out}}{\Omega_r}$$

$$T_{out} = \frac{37 \cdot 10^3}{152{,}29}$$

$$T_{out} = 242{,}96\ \text{N} \cdot \text{m}$$

3 Power Balance:

Assuming mechanical losses are negligible, the output power Pout is approximately equal to the mechanical power Pm:

$$p_{\text{mec}} = 0 \Rightarrow P_{out} = P_m$$

The electromagnetic power Pem is:

$$P_m = (1 - S)P_{em}$$

$$P_{em} = \frac{P_m}{(1 - S)}$$

$$P_{em} = \frac{37 \times 10^3}{1 - 0{,}03}$$

$$P_{em} = 38144{,}33\ \text{W}$$

- Rotor joule losses p_{jr} are:

$$p_{jr} = P_{em} \times g$$

$$p_{jr} = 1144{,}33w$$

Alternatively:

$$p_{j\text{r}} = P_{em} - P_{\text{m}}$$

$$p_{j\text{r}} = 1144{,}33W$$

- **Rotor Current:** The rotor current I_r can be approximated by:

$$p_{jr} = 3R_{\text{r}}I_r^2$$

$$I_r = \sqrt{\frac{p_{jr}}{3 \times R_r}} = \sqrt{\frac{1144{,}33}{3 \times \frac{0{,}08}{2}}}$$

$$I_r = 97{,}7 \text{ A}$$

- **Stator Copper Losses:** Stator copper losses p_{js} are:

$$p_{js} = 3R_sI_s^2$$

$$p_{js} = 3 \times \frac{0{,}1}{2} \times (72{,}67)^2$$

$$p_{js} = 792{,}131w$$

- **Iron Losses:** Iron losses p_f are approximated as:

$$P_{em} = P_{in} - p_f - p_{j_s}$$

$$p_f = P_{in} - P_{em} - p_{j_s}$$

$$p_f = 1722{,}87w$$

Exercise 03:

The nameplate of the asynchronous motor provides the following information:

3 ~ 50 Hz, 1455 RPM, cos ϕ = 0.80

Δ 220 V, 11 A

Y 380 V, 6.4 A

1. The motor is powered by a 50 Hz, 380 V three-phase network.
   What should be the winding configuration for normal operation?
2. What is the number of poles of the stator?
3. Calculate the nominal slip (in %).
4. A no-load test under nominal voltage gives:
   - Absorbed power: Pa = 260 W
   - Line current: I = 3.2 A
   - Mechanical losses are estimated at 130 W.
   - The resistance of one stator winding is measured at 0.65 Ω.
   - Deduce the iron losses.
5. For nominal operation, calculate:
   - Joule losses in the stator
   - Joule losses in the rotor
   - Efficiency
   - Useful torque Tu

Solution of Exercise 03:

1. The winding configuration should be star (Y).
2. The number of stator poles is 4.

$$2P = 4$$

3. **Nominal slip:**

$$S = 0{,}03 = 3\%$$

4) **Iron losses:** Under no-load: p_f

$$p_{js} = 3R_s I_s^2$$

$$p_{js} = 3 \times 0{,}65 \times (3{,}2)^2$$

$$p_{js} = 20 \text{ W}$$

$$p_f = P_{in} - p_{js} - p_{\text{mac}}$$

$$p_f = 260 - 130 - 20$$

$$p_f = 110 \text{ w}$$

3) For nominal operation:

- **Joule losses in the stator:**

$$p_{js} = 3R_s I_s^2$$

$$p_{js} = 3 \times 0{,}65 \times (6{,}4)^2$$

$$p_{js} = 80 \text{ W}$$

- **Joule losses in the rotor:**

$$P_{in} = \sqrt{3} \times \text{U} \times \text{I} \times \cos \phi$$

$$P_{in} = \sqrt{3} \times 330 \times 6{,}4 \times 0{,}8$$

$$P_{in} = 3370 \text{ W}$$

$$P_{\text{em}} = P_a - p_{js} - p_f$$

$$P_{\text{em}} = 3370 - 80 - 110$$

$$P_{\text{em}} = 3180 \text{ W}$$

$$p_{jr} = S\, P_{em} = 0{,}03 \times 3180$$

$$p_{jr} = 95W$$

- **Efficiency:** $\eta =$?

$$P_{out} = P_{\text{em}} - p_{\text{jr}} - p_{\text{mec}}$$

$$P_{out} = 3180 - 95 - 130$$

$$P_{out} = 2955 \text{ W}$$

$$\eta = \frac{2955}{3370}$$

$$\eta = 84.7\%$$

- **Useful torque Tu**

$$T_u = \frac{P_{in}}{\Omega_r}$$

$$T_u = \frac{2955}{1450 \cdot \frac{2\pi}{60}}$$

$$T_\mu = 19{,}4\ N\text{m}$$

Exercise 04:

We are studying a three-phase asynchronous motor with the following nameplate information:

- 50 Hz,
- 230/400V,
- 2A,
- 1450 rpm,
- 4 poles,
- cosϕ=0.8

1. What is the synchronous speed Ns (in rpm)?
2. Calculate the value of the nominal slip.
3. Draw a single-phase equivalent circuit of the machine. Explain the significance of the various elements introduced, knowing that the stator phase resistance is R=30mΩ.
4. A no-load test at nominal voltage gives the following values: P0=130W, I0=0.8A . Assume that mechanical losses and iron losses are of equal value. Calculate these losses in detail. Then, deduce the value of two elements introduced in the equivalent circuit.
5. Calculate the power consumed by the motor under nominal conditions: Pn .
6. Calculate the stator Joule losses: Pjs (assuming that the current passing through it is approximately equal to In).
7. Deduce the power received by the rotor Pr . Then calculate the Joule losses in the rotor . Deduce the value of the useful power provided by the machine .
8. Represent all the power values on a power flow diagram.
9. Calculate the value of the machine's nominal efficiency. What element could be neglected in this equivalent circuit?

10. Also, determine the value of the nominal reactive power consumed by the machine.
11. Calculate the value of all undetermined elements in the equivalent circuit.
12. Finally, calculate the value of the efficiency corresponding to a useful power equal to a quarter of that under nominal conditions and a speed of 1,475 rpm.

Solution of Exercise 04:

1) Synchronous Speed:

$$N_S = \frac{60f}{P} = \frac{60 \times 50}{2}$$

$$N_S = 1500 \text{ rmp}$$

2) Nominal Slip:

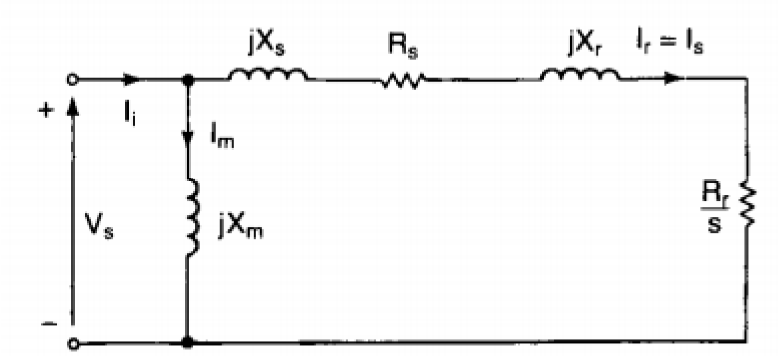
$$S_n = \frac{N_S - N_r}{N_S}$$

$$S_n = \frac{1500 - 1450}{1500}$$

$$S_n = 0{,}033$$

3) **Single-phase Equivalent Circuit:**

- The equivalent circuit represents the electrical behavior of the motor under different conditions. In this circuit, resistance Rs=30mΩ represents the stator resistance, and additional elements like inductance and rotor resistance reflect the motor's impedance and energy dissipation.



No-Load Test:

- Given $P_0 = 130$ W, $I_0 = 0{,}8$ A, and assuming equal mechanical and iron losses:
- $p_m = p_f$
- $P_0 = p_m + p_f$
- Mechanical losses: p_m=65W
- Iron losses: p_f=65W
- Negligible Joule losses:

$$p_{j_s} = 3R_s I_s^2$$

$$p_{j_s} = 3 \times 0{,}03 \times (0{,}8)^2$$

$$p_{j_s} = 0{,}058 \text{ W}$$

- From the equivalent circuit:

$$p_f = 3\text{VI}$$

$$p_f = 3\,V\,\frac{\text{V}}{R_m}$$

$$p_f = \frac{3V^2}{R_m}$$

$$R_m = \frac{3V^2}{R_m} = \frac{3(230)^2}{65}$$

$$R_m = 2{,}44\text{K}\omega$$

$$Q_0 = \sqrt{S_0^2 - P_0^2}$$

$$Q_0 = \sqrt{(3VI_0)^2 - P_0^2}$$

$$Q_0 = \sqrt{(3 \times 230 \times 0{,}8)^2 - 130^2}$$

$$Q_0 = 536.47 \text{ VAR}$$

We have:

$$Q_0 = \frac{3V^2}{L_m \cdot \omega}$$

Inductive reactance:

$$L_m = \frac{3 \cdot V^2}{Q_0 \cdot w}$$

$$L_m = \frac{3 \cdot (230)^2}{536{,}47 \cdot 2\pi 50}$$

$$L_m = 0{,}94 \text{ H}$$

$$X_m = 2\pi f\, L_m$$

5 Power Consumed by the Motor

$$P_n = 3V_n I_n \cos Q_n$$

$$P_n = 1104w$$

6 Stator Joule Losses:

$$p_{js} = 3RI_n^2$$

$$p_{js} = 0{,}36 \text{ W}$$

7 Power Received by the Rotor:

$$P_{em} = P_n - p_f - p_{js}$$

$$P_{em} = 1039$$

Rotor Joule Losses:

$$p_{j_r} = S\, P_{em}$$

$$p_{j_r} = 34.28 \text{ W}$$

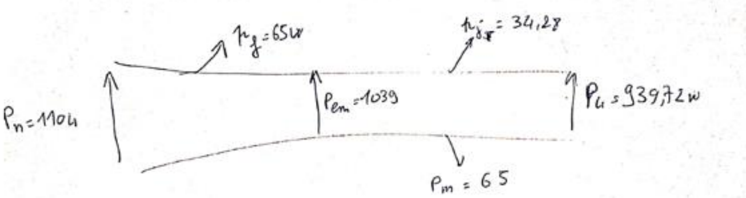
Useful Power:

$P_{out} = P_{\text{em}} - p_m - p_{j\text{r}}$

$P_{out} = 939{,}72 \text{ W}$

8. Power Flow Diagram:

- The diagram should illustrate the input power, Joule losses, mechanical losses, rotor losses, and the useful power delivered by the motor.



9. Nominal Efficiency:

$$\eta = \frac{p_u}{p_a}$$

$$\eta = 0{,}85$$

The stator resistance R_s could be neglected due to minimal Joule losses 0,360 W.

10) Reactive Power:

$$Q_n = 3V_nI_n \sin\varphi_n$$

$$Q_n = 3 \times 230 \times 2 \times \sin\varphi_n$$

$$Q_n = 828\text{VAR}$$

11) Equivalent Circuit Parameters: $X_s + X_r = X$ and Rr

$$Q_n = Q_0 + 3L\,\omega\, {I_n}^2$$

$$L = \frac{Q_n - Q_0}{3wI_n{}^2}$$

$$L = 77mH$$

$$X = 2\pi f\mathrm{L}$$

$$X = 40\Omega$$

Rotor resistance Rr can be calculated from the rotor Joule losses:

$$p_{\mathrm{jr}} = 3R_r I_2{}^2$$

$$S_2 = 3VI_2$$

$$S_2 = \sqrt{P_{en}^2 + (Q_n - Q_0)^2}$$

$$S_2 = \sqrt{103y^2 + (828 - 536{,}47)^2}$$

$$S_2 = 1078\mathrm{VA}$$

$$I_2 = \frac{S_2}{3\ \mathrm{V}}$$

$$I_2 = \frac{1078}{3.230}$$

$$I_2 = 1{,}56\ \mathrm{A}$$

$$R_r = \frac{p_{\mathrm{jr}}}{3\ I_2{}^2}$$

$$R_r = 4.69\Omega$$

12) Efficiency at 1,475 rpm:

$$P_{out}^{`} = \frac{1}{4} P_{out}$$

$$P_{out}^{`} = \frac{1}{4}\ 939.72$$

$$P_{out}^{`} = 234.8W$$

Slip

$$S = \frac{1500 - 1475}{1500}$$

$$S = 0{,}016$$

Recalculate power and efficiency:

$$P_{em} = P_{out} + p_m + p_{jr}$$

$$P_{em} = P_{out} + P_m + SP_{em}$$

$$P_{em} = 304.6\mathrm{W}$$

$$p_{jr} = 4.87W$$

$$P_{in} = 369.47W$$

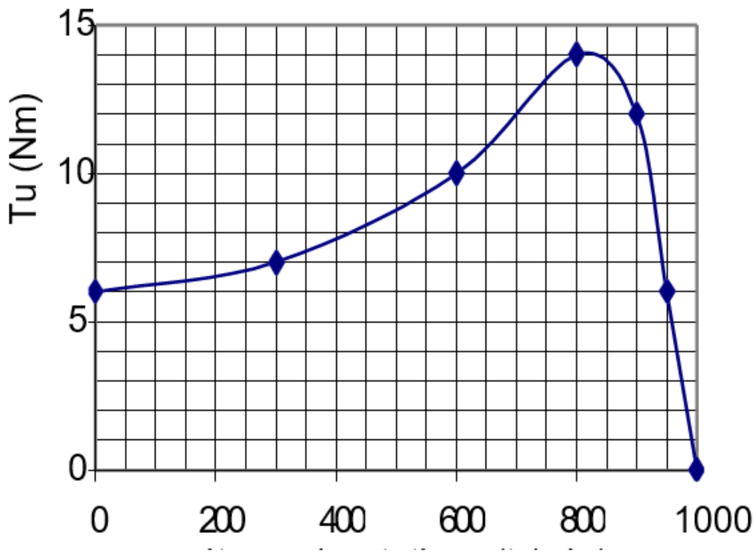
$$\eta = \frac{P_{out}}{P_{in}}$$

$$\eta = \frac{236,8}{374,47}$$

$$\eta = 0,62$$

Exercise 05:

The mechanical characteristic of an asynchronous motor is given below:



This motor drives a compressor with a constant resisting torque of 4 Nm.

1-1) Is it possible to start the motor under load?

1-2) In the useful zone, verify that Tu=−0.012n+120

1-3) Determine the steady-state rotational speed of the system.

1-4) Calculate the power transmitted to the compressor by the motor.

The motor is now used to drive a pump whose resisting torque is a function of the rotational speed given by the equation: $T_l = 10^{-5} n^2$

with T_l in Nm and n in rpm.

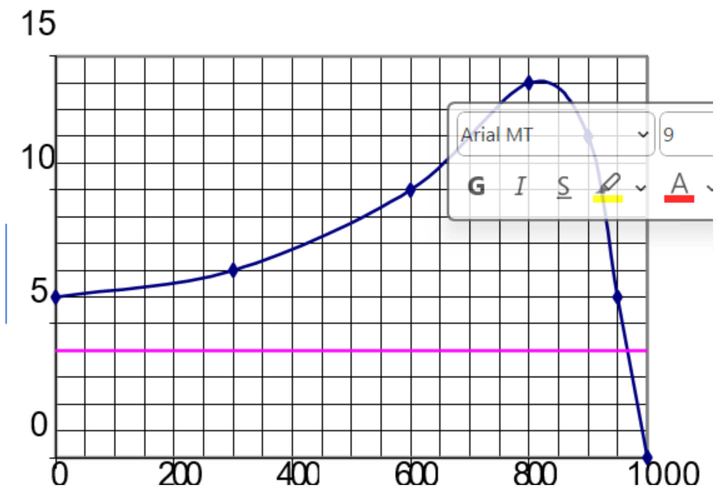
2-1) Plot the curve of T_l (n) on the same graph as the previous one.

2-2) In steady-state, determine the rotational speed of the system and the useful torque of the motor.

Solution to Exercise 05:

1-1) Is it possible to start the motor under load?

Yes, because the motor's starting torque (6 Nm) is greater than the resisting torque (4 Nm).



1-2) Verifying Tu=−0.012n+120:

In the useful zone, the characteristic is a straight line, so the equation is linear.

For n=1000 rpm, Tu=0 Nm.

For n=950 rpm, Tu=6 Nm.

Thus, the equation is verified.

1-3) Determine the steady-state rotational speed:

In steady-state, the useful torque exactly balances the resisting torque:

$$Tu = T_l$$

$$Tu = -0.012n + 120 = Tr = 4 \text{ Nm},$$

Solving gives n=967 rpm.

1-4) Calculate the power transmitted to the compressor by the motor:

This is also the useful power of the motor:

$$P = T\,\Omega$$

$$P = \frac{T\,n\,2\pi}{60}$$

$$P = \frac{4 \times 967 \times 2\pi}{60}$$

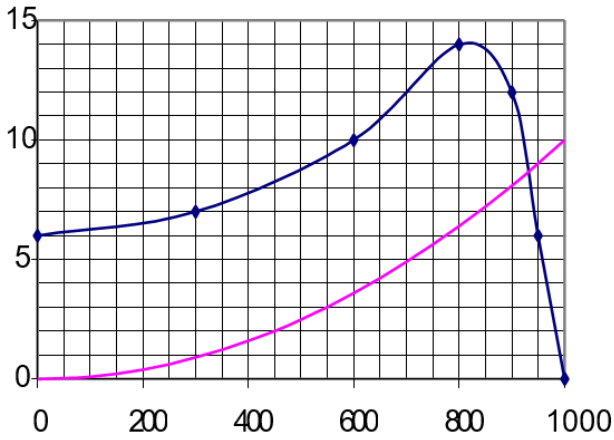
$$P = 405$$

2-1) Plot the curve of $T_l(n)$:

The equation for the resisting torque is given as:

$$T_l = 10^{-5}n^2$$

This curve can be plotted on the same graph as the motor's mechanical characteristic.



2-2) Determine the steady-state rotational speed and the useful torque of the motor:

In steady-state:

$$T_u = T_l$$

$$-0.012n + 120 = 10^{-5}n^2$$

Rearranging:

$$10^{-5}n^2 + 0.012n - 120 = 0$$

This quadratic equation has two solutions, and the physically acceptable one is:

$$n = \frac{-0.012 + \sqrt{0.012^2 + 4 \times 10^{-5} \times 120}}{2 \times 10^{-5}}$$

$$n = 928\ rmp$$

Thus, the steady-state speed is 928 rmp.

The corresponding useful torque is:

$$T_u = T_l = 10^{-5} \times 928^2$$

$$T_u = 862\ Nm$$

Exercise 06:

An induction motor with a squirrel cage is powered by a three-phase network with a frequency of 50 Hz and a phase-to-phase voltage of 380 V. It has been subjected to the following tests:

No-load:

- Power absorbed: P_0=360
- Line current: I_0=36 A
- Rotational speed: n_0=2995 rpm

Under load:

- Power absorbed: P=4560 W
- Line current: I=81 A
- Rotational speed: n=2880 rpm
- The stator windings are connected in a star configuration, and each windings has a resistance of 0.75 Ω . The iron losses are estimated at 130 W.

1. What is the synchronous speed? Deduce the slip under load.
2. **For no-load operation:**
   - Calculate the Joule losses in the stator.
   - Justify that the Joule losses in the rotor are negligible. Deduce the mechanical losses.
3. **For load operation:**
   - Calculate the Joule losses in the stator and rotor.
   - Calculate the useful power and the torque Tu
   - Determine the efficiency of the motor.
4. The motor now drives a pump whose resisting torque T_l is proportional to the rotational speed and equals 18 Nm at 3000 rpm. In the useful zone, the mechanical characteristic Tu(n) of the motor can be approximated by a straight line. Determine the rotational speed of the motor-pump system.

Solution of Exercise 06:

1. **What is the synchronous speed?**

The synchronous speed is:

$$n_s = 3000\, rmp$$

Determine the slip under load:

$$s = \frac{n_s - n}{n_s}$$

$$s = \frac{3000 - 2880}{3000}$$

$$s = 4\%$$

3. No-Load Operation

a) Calculation of Stator Copper Losses

The stator copper losses P_{su} are given by the formula:

$$P_{su} = 3 \times R_s \times I_V^2$$

Where I_V=36 A is the no-load line current.

Thus, the stator copper losses are:

$$p_{su} = 3 \times 0{,}75 \times (3{,}6)^2$$

$$p_{su} = = 29 \text{ W}$$

b) Justification that Rotor Copper Losses Are Negligible

At no-load, the slip **S** is very small, which means that the power transmitted to the rotor is minimal. The rotor copper losses are given by $p_{ru} = s\ P_{em}$, where P_{em} is the power transmitted to the rotor. Since **S** is negligible, the rotor copper losses can also be neglected.

c) Mechanical Losses

The total input power at no-load is P_{in}=360 W. We subtract the stator copper losses $p_{su} = 29W$ and the core losses $p_c = 130$ W to find the mechanical losses:

$$p_m = P_0 - p_{su} - p_c$$

$$p_m = 360 - 29 - 130$$

$$p_m = 210W$$

4. Loaded Operation

a) Calculation of Stator and Rotor Copper Losses

For stator copper losses under load:

$$p_{su} = 3 \times R_s \times I_V^2$$

$$p_{su} = 3 \times 0.75 \times 81^2$$

$$p_{su} = 148\ W$$

For rotor copper losses:

$$p_{ru} = s(P_{in} - p_{su} - p_c)$$

$$p_{ru} = 0.04(4560 - 148 - 130)$$

$$p_{ru} = 171\,W$$

b) Useful Power and Torque

The useful power is:

$$P_{out} = P_{in} - p_{su} - p_{ru} - p_c - p_m$$

$$P_{out} = 4560 - 148 - 171 - 130 - 210$$

$$P_{out} = 3901\,W$$

The angular speed is:

$$\Omega = \frac{2880 \times 2\pi}{60}$$

$$\Omega = 301{,}44\,rad/s$$

The useful torque is:

$$T_u = \frac{P_{out}}{\Omega}$$

$$T_u = \frac{3901}{301{,}44}$$

$$T_u = 12.94\,Nm$$

c) Motor Efficiency

The efficiency η is the ratio of useful power to input power:

$$\eta = \frac{P_{out}}{P_{in}}$$

$$\eta = \frac{3901}{4560}$$

$$\eta = 85.6\%$$

5. Rotation Speed of the Motor-Pump Assembly

The pump's resistant torque T_l is proportional to the rotational speed:

$$T_l = 0.0006\,n$$

The motor's torque characteristic Tu(n) is approximated by a straight line between two points:

- T_u = 0 Nm at 3000 tr/min
- T_u = 13,0 Nm at 2880 tr/min

Hence, the motor torque can be expressed as:

$$T_u = 324 - 0.108n$$

At the operating point, $T_u = T_l$:

$$324 - 0.108n = 0.0006\,n$$

Solving for n

n = 2842 rmp

Thus, the motor-pump assembly will rotate at approximately 2842 rpm.

Exercise 07:

A three-phase, four-pole squirrel cage induction motor with 220 V / 380 V ratings is powered by a 220 V phase-to-phase network, 50 Hz.

A no-load test at a rotational speed very close to synchronism gave the following results for the absorbed power and power factor: P_0=500 W, cos φ_0 = 0,157

A load test gave the following results:

- Absorbed current: I=122 A
- Slip:S =6%
- Absorbed power: P_{in}=3340 W

The resistance of a stator winding is R_s=1 Ω .

1.1 Which of the two voltages on the nameplate can a stator winding support?

1.2 Deduce the stator connection on the 220 V network.

2. For no-load operation, calculate:

2.1 The rotational speed n_0, assumed to be equal to the synchronous speed.

2.2 The line current I_0

2.3 The value of stator copper losses p_{su}.

2.4 The value of core losses p_c, assumed to be equal to the mechanical losses p_m.

3. For loaded operation, calculate:

3.1 The rotational speed (in rpm).

3.2 The power transmitted to the rotor $P_{em}$and the electromagnetic torque Tem .

3.3 The useful power $P_{out}$and the efficiency η.

3.4 The useful torque Tu.

4. The motor drives a machine whose resistant torque (in Nm) is given as a function of the rotational speed n (in rpm) by the relation:

$$T_r = 8 \cdot 10^{-6} n^2$$

The useful part of the motor's mechanical characteristic is approximated by a straight line. Determine the relation between Tu and n (assume Tu=175 Nm for n=1410 rpm). Deduce the rotational speed of the system and calculate the useful power of the motor.

Solution of Exercise 07:

1.1 Stator voltage:

The voltage supported by a stator winding is **220 V**.

1.2 Stator connection:

The stator connection for the 220 V network is a **delta connection**.

2. No-load operation:

2.1 **Rotational speed** n_0:

The synchronous speed for a four-pole motor is:

$$n_s = \frac{60\,f}{p}$$

$$n_s = \frac{60\ \times 50}{p}$$

$$n_s = 1500\,rmp$$

Thus, n_0=1500 rpm

2.2 Line current I_0:

$$I_0 = \frac{P_0}{\sqrt{3}U\cos\varphi_0}$$

$$I_0 = \frac{500}{\sqrt{3}\,220 \times 0.157}$$

$$I_0 = 8.36A$$

2.3 Stator copper losses $p_{su\,0}$

$$p_{su\,0} = 3R_s I_{ph\,0}^2$$

For a delta connection, the phase current $I_{ph}$is related to the line current I by:

$$I_{ph} = \frac{I}{\sqrt{3}}$$

So the phase current $I_{ph\,0}$ in a delta connection is:

$$I_{ph\,0} = \frac{I_0}{\sqrt{3}}$$

$$I_{ph\,0} = \frac{8.36}{\sqrt{3}}$$

$$I_{ph\,0} = 4.826 \text{ A}$$

Now, using this in the formula for stator copper losses:

$$p_{su\,0} = 3 \times 1 \times (4.826)^2$$

$$p_{su\,0} = 70\,W$$

2.4 core losses p_c and **mechanical losses** p_{mec}

Power balance:

$$P_0 = p_{su\,0} + p_c + p_{mec}$$

$$p_c + p_{mec} = P_0 - p_{su\,0}$$

$$p_c + p_{mec} = 500 - 70$$

$$p_c + p_{mec} = 430\,W$$

And $p_c = p_{mec}$ so:

$$p_c = p_{mec} = \frac{430}{2}$$

$$p_c = p_{mec} = 215\,W$$

3. Loaded operation:

3.1 Rotational speed:

$$\Omega = \Omega_s(1 - S)$$

Ω =1500(1- 0,06)
Ω = 1410 tr/min

3.2 Power transmitted to the rotor Pem and **electromagnetic torque Tem**:

$$P_{em} = P_{in} - p_{su} - p_c$$

$$P_{em} = 3340 - 150 - 215$$

$$P_{em} = 2975W$$

The electromagnetic torque is:

$$T_{em} = \frac{P_{em}}{\Omega_s}$$

$$T_{em} = \frac{2975}{1500\frac{2\pi}{60}}$$

$$T_{em} = 18.95\ Nm$$

3.3 Useful power P_{out} and **efficiency η**:

The useful power is defined as the power that is actually converted into mechanical energy for work. It can be calculated using the formula:

$$P_{out} = P_{in} - p_{su} - p_c - p_{ru} - p_{me}$$

Or

$$P_{out} = P_{em} - p_{ru} - p_{me}$$

Calculate Rotor Copper Losses

Rotor copper losses can be calculated using the slip and the power transmitted to the rotor P_{em}:

$$p_{ru} = S\ P_{em}$$

$$p_{ru} = 0.06\ \times 2975$$

$$p_{ru} = 183.3\ W$$

The useful power is

$$P_{out} = 2975 - 183.3 - 215$$

$$P_{out} = 2580W$$

Efficiency η:

$$\eta = \frac{P_{out}}{P_{in}}$$

$$\eta = \frac{2580}{3340}$$

$$\eta = 77.3\%$$

3.4 Useful torque Tu:

$$T_u = \frac{P_{out}}{\Omega_r}$$

$$T_u = \frac{2580}{1410\,\frac{2\pi}{60}}$$

$$T_{em} = 17.5\ Nm$$

4. Relation between Tu and n:

The resistant torque is given by:

$T_l = 8 \cdot 10^{-6}\ n^2$.

At n=1410 rpm, Tu=175 Nm.

At n=1500 rpm, T

The relation between Tu and n is a straight line:

$T_u = -\ 0{,}1944\ n + 291{,}7$

Rotational speed of the system:

At equilibrium Tu=Tr :

$-\ 0{,}1944\ n + 291{,}7 = 8 \cdot 10^{-6}\ n^2$

Solving this quadratic equation gives n=1417 rpm.

Motor's useful power:

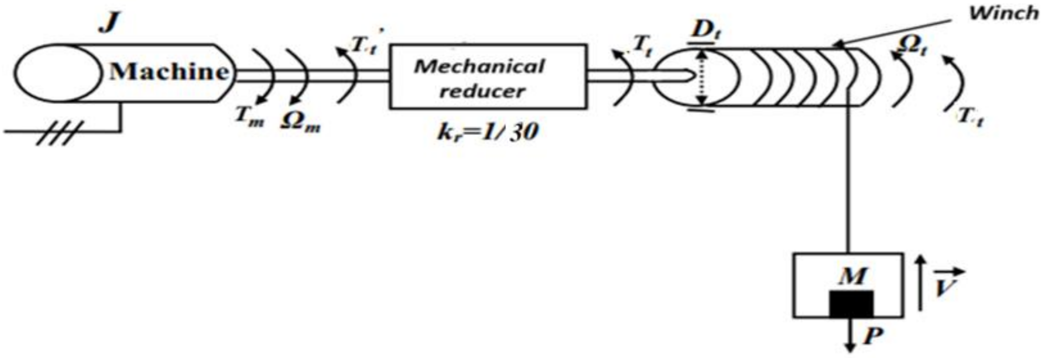
$T_r = T_u = 8 \cdot 10^{6} \cdot 1417^2$

$T_u = 16{,}1\ Nm$

$P_u = 2385\ W$

Exercise 08:

We are given the following electric drive system: A winch driven by an asynchronous motor (MAS) through a mechanical speed reducer. The winch lifts a load vertically (with a mass M=300 kg) at a constant speed v=0.5 m/s.



System Details:

- **MAS parameters**: ***R_r' =2Ω , P =1, L_r' = 0.01H, J= 0.01 Kg.m^2*** (Magnetic and mechanical friction losses are neglected).
- **Reducer parameters**: Reduction ratio Kr = 1/30..
- **Winch diameter**: Dt=20 cm.
- **Gravity**: *g= **10 m/s^2***.
- **Efficiency of each component**: 1.

Questions:

1. Justify the use of the mechanical reducer in the drive system.
2. If the stator supply frequency of the MAS is fs=30 , calculate the stator supply voltage Vs.
3. Calculate the stator current Is absorbed by the AS.
4. What is the value of the motor torque Tm required to accelerate the winch speed to v=1 m/s within 2 seconds?
5. Is the motor capable of lifting a mass of M=900 kg
   - If not, what is the solution to lift this load?

Solution of Exercise 08:

1. Justification for using the mechanical reducer:

The mechanical reducer is used in the drive system for the following reasons:

- **To increase the driving torque**: The mechanical reducer multiplies the torque transmitted to the load, which is essential for applications involving heavy lifting.
- **To reduce the translation speed**: It reduces the motor's rotational speed to the required winch speed, which is necessary for controlled lifting.
- **To downsize the motor**: The reducer allows for the use of a smaller motor by reducing the required motor power, which in turn decreases the motor's size, weight, and the cross-section of the wires needed.

2. Calculation of the stator supply voltage Vs:

From the torque curve Tem=f(Ω), and for S<Smax (the stable linear operating region):

$$T_{em} = \frac{3 \cdot p \cdot V_s^2}{\omega_s \cdot R_r'} \cdot S$$

we can solve for Vs:

$$V_s = \sqrt{\frac{T_{em} \cdot \omega_s \cdot R_r'}{3 \cdot p \cdot S}}$$

Where:

- Tem=Tm (neglecting constant losses),
- The load weight is M·g=300×10=3000
- Winch torque Tt=M·g·Dt2=3000·0.2/2=300 Nm C
- Motor torque Tm=Tt·Kr=300/30=10 Nm

The angular speed of the winch is:

$$\Omega_t = \frac{v}{\left(\frac{D_t}{2}\right)}$$

$$\Omega_t = \frac{0,5}{0,1}$$

$$\Omega_t = 5 \text{ rad/s}$$

Thus, the motor angular speed is:

$$\Omega_m = \frac{\Omega_t}{\mathrm{K}_r}$$

$$\Omega_m = \frac{5}{\frac{1}{30}}$$

$$\Omega_m = 150 \, {}^{rad}/_{s}$$

With ωs=2πfs=2π×30=188.4 rad/s ,

slip S=0.2.

Finally, substituting into the voltage equation:

$$V_s = \sqrt{\frac{10 \cdot 2 \cdot 3{,}14 \cdot 30 \cdot 2}{3 \cdot 0{,}2}}$$

$$V_s = 79{,}25\text{Volt}$$

3. Calculation of the stator current Is:

The transmitted power is Ptr=Cm·Ωs=10·188.4=1884 W.

Using the power equation:

$$P_{tr} = 3\left(\frac{R_r'}{S}\right) I_s^2$$

$$I_s = \sqrt{\frac{P_{tr} \cdot S}{3 \cdot R_r'}}$$

$$I_s = \sqrt{\frac{1884 \cdot 0{,}2}{3 \cdot 2}}$$

$$I_s = 7{,}92 \text{ A}$$

So, the stator current is Is=7.92 A.

4. Calculation of the motor torque Tm needed to accelerate the load's speed to 1 m/s in 2 seconds:

Using the relation

$$T_m = \frac{2 \cdot J}{K_r \cdot D_t} \cdot \frac{\Delta v}{\Delta t} + T_0$$

Where:

- Δv=1−0.5=0.5 m/sec ,
- Δt=2−0=2 sec,
- J=0.01 kg.m^2.

Substituting values:

$$T_m = \frac{2 \cdot 0{,}01}{\left(\frac{0{,}2}{30}\right)} \cdot \frac{(1 - 0{,}5)}{(2 - 0)} + 10$$

$$T_m = 10{,}75\text{Nm}$$

5. Is the motor capable of lifting a mass of 900 kg?

For a mass M=900 kg:

$$T_m = M \cdot g \cdot \frac{D_t}{2} \cdot K_r$$

$$T_m = 900 \cdot 10 \cdot \frac{0{,}2}{2} \cdot \frac{1}{30}$$

$$T_m = 30\text{Nm}$$

The maximum motor torque is:

$$T_{max} = \frac{3p}{2 \cdot L'_r} \cdot \left(\frac{V_s^2}{\omega_s^2}\right)$$

$$T_{max} = \frac{3 \cdot 1}{2 \cdot 0{,}01} \cdot \left(\frac{79{,}25}{188{,}4}\right)^2$$

Since Tm=30 Nm>Tmax=26.54 Nm , **the motor is not capable** of lifting the 900 kg mass.

6. To lift the 900 kg load, the motor must generate a maximum torque $T_{max} > T_m$

From the equation:

$$C_{max} > 30\text{Nm}$$

$$\frac{3p}{2 \cdot L'_r} \cdot \left(\frac{V_s^2}{\omega_s^2}\right) > 30$$

$$V_s^2 > 30 \cdot \frac{2 \cdot L'_r \cdot \omega_s^2}{3 \cdot p}$$

$$V_s > \sqrt{30 \cdot \frac{2 \cdot L'_r \cdot \omega_s^2}{3 \cdot p}}$$

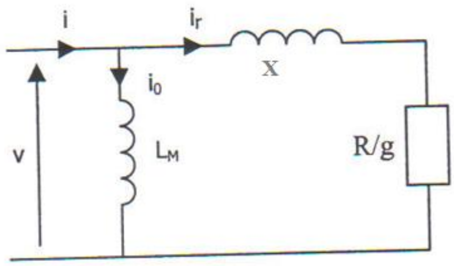
$$V_s > \sqrt{30 \cdot \frac{2 \cdot 0{,}01 \cdot 188{,}4^2}{3 \cdot 1}} = 119.15\, V$$

Thus, the machine must be supplied with a voltage: ***V_s > 119, 15 Volts***

Exercise 09:

The figure below shows a simplified equivalent circuit in the form of an inductance L for which we have:

- R=66mΩ,
- l=1mH (X=$2\pi f l$),
- Vn=220V,
- P=2,



1) Derive that the electromagnetic torque Tem can be expressed as:

$$T_{em} = \frac{3p}{2\pi}\left(\frac{V}{f}\right)^2 \frac{R \cdot f_r}{R^2 + 4\pi^2 l^2 f_r^2}$$

Where: $f_r = S \cdot f$

1) Show that for small values of f_r , we can write:

$$T_{em} = A\left(\frac{V}{f}\right)^2 f_r$$

Give the expression of A.

2) Knowing that for V=Vn and $f = f_n$, show that the electromagnetic torque can be written as

$$T_{em} = B.\ f_r$$

Provide the numerical value of B.

Solution of Exercise 09:

1. **Deriving the expression for Tem:**

$$T_{em} = \frac{3p}{2\pi}\left(\frac{V}{f}\right)^2 \frac{R \cdot f_r}{R^2 + 4\pi^2 l^2 f_r^2}$$

We start with the mechanical power expression for the electromagnetic torque:

$$T_{em} = \frac{P_{em}}{\Omega_s}$$

Where:

- Pem is the electromagnetic power
- Ωs is the synchronous angular speed.

From the power in the rotor:

$$P_{em} = \frac{p_{jr}}{s}$$

Where:

- $p_{jr} = 3RI_r^2$ (rotor Joule losses)
- S is the slip.

Thus, the torque becomes:

$$T_{em} = \frac{3R}{S\,\Omega_s} I_r^2$$

From the equivalent circuit, the rotor current Ir is given by:

$$I_r = \frac{V}{\sqrt{\left(\frac{R}{s}\right)^2 + x^2}}$$

$$I_r = \frac{V}{\sqrt{\left(\frac{R}{s}\right)^2 + (2\pi f l)^2}}$$

Substituting this expression of Ir into the torque equation:

$$T_{\text{em}} = \frac{3RV^2}{S\Omega_s\left[\left(\frac{R}{S}\right)^2 + (2\pi f l)^2\right]}$$

Now, using the relation for synchronous speed Ωs:

$$N_{\text{s}} = \frac{60f}{P}$$

$$\Omega_s = \frac{2\pi N_{\text{s}}}{60} = \frac{2\pi}{60}\frac{60\,f}{P}$$

$$\Omega_s = \frac{2\pi f}{P}$$

Thus, we can rewrite the torque equation as:

$$T_{\text{em}} = \frac{3PRV^2}{\frac{2\pi f}{S}[R^2 + (2\pi f l)^2 \cdot S^2]}$$

$$T_{\text{em}} = \frac{3PRV^2 \cdot S}{2\pi f[R^2 + 4\pi^2 f^2 l^2 S^2]}$$

and $f_r = S \cdot f$

$$T_{\text{em}} = \frac{3PRV^2 \cdot f_r}{2\pi f\left[R^2 + 4\pi^2 {f_r}^2 l^2\right]}$$

Which simplifies to:

$$T_{em} = \frac{3p}{2\pi}\left(\frac{V}{f}\right)^2 \frac{R \cdot f_r}{R^2 + 4\pi^2 l^2 f_r^2}$$

This is the required expression.

2. For small values of f_r:

When f_r is small, the term $4\pi^2 l^2 f_r^2$ becomes negligible compared to R^2. Thus, the expression simplifies to:

$$T_{em} = \frac{3p}{2\pi}\left(\frac{V}{f}\right)^2 \frac{R \cdot f_r}{R^2}$$

Which simplifies further to:

$$T_{em} = \frac{3p}{2\pi R}\left(\frac{V}{f}\right)^2 f_r$$

So, we can write:

$$A = \frac{3p}{2\pi R}$$

3. or V=Vn and *f=fn* :

When V=Vn and $f=f_n$, the torque expression becomes:

$$T_{em} = B.\ f_r$$

Where

$$B = \frac{3p}{2\pi R}\left(\frac{V_n}{f_n}\right)^2$$

Exercise 10:

A 3-phase squirrel cage induction motor with a power rating of 11.2 kW, 1750 RPM, 4 poles, Y configuration, 460V, 60Hz has the following parameters:

- Rs is negligible,
- Rr=0.38 Ω,
- Xs=1.140 Ω,
- Xr=1.71 Ω,
- Xm=33.2 Ω.

The motor is controlled by a frequency inverter. If the braking torque is 35 Nm, calculate:

A. The supply frequency

B. The motor speed

Solution of Exercise10:

In this exercise, we apply frequency variation control.

A. Calculating the supply frequency

The synchronous speed:

$$N_S = \frac{60f}{p} = 60 \times \frac{60}{2}$$

$$N_s = N_b = 1800 \text{ rmp}$$

The synchronous angular speed:

$$\Omega_s = N_s \times \frac{2\pi}{60} = 1800 \times \frac{2\pi}{60}$$

$$\Omega_s = \Omega_b = 188{,}4 \text{ rad/s}$$

We calculate the mechanical torque Tu:

$$T_u = \frac{P_\mu}{\Omega_r} = \frac{11,2 \cdot 10^3}{1750 \times \frac{2\pi}{60}} = \frac{11,2 \cdot 10^3}{183 \cdot 167}$$

$$T_u = 61,146 \text{ N} \cdot \text{m}$$

Now, we calculate β,

$$\beta = \sqrt{\frac{T_{em_{\max b}}}{T_{em_{\max}}}}$$

$$\beta = \sqrt{\frac{61,146}{35}} = 1.32$$

Thus, the new supply frequency is:

$$f_2 = \beta \times f_b$$

$$f_2 = 1,32 \times 60$$

The supply frequency is : $f_2 = 79.2$ Hz

B. Calculating the motor speed

The synchronous angular speed for the new supply frequency is:

$$\Omega_s = \beta \times \Omega_b$$

$$\Omega_s = 1,32 \times 188,4$$

For : $f_2 = 79.2$ Hz we have: $\Omega_s = 248,69 \text{rad/s}$

The torque corresponding to the slip at maximum torque $T_\text{f} = 35$ N · m :

the slip at maximum torque$S_{\text{T max}}$ is calculated using:

$$S_{\text{T max}} = \frac{Rr}{\beta(x_s + x_\text{r})}$$

$$S_{\text{T max}} = \frac{0,38}{1,32(1,42 + 171)}$$

$$S_{\text{T max}} = 0,1$$

Thus, the rotor angular speed is:

$$\Omega_r = (1 - g)\Omega_s = (1 - 0,1)248,69$$

$$\Omega_r = 223,82 \text{ rad/s}$$

Finally, the motor speed in RPM is:

$$N_\text{r} = \Omega_r \times \frac{60}{2\pi}$$

The motor speed is $N_r = 2138$ rmp

Exercise 11:

A 3-phase squirrel cage induction motor with a power rating of 11.2 kW, 1750 RPM, 4 poles, Y configuration, 460V, 60Hz has the following parameters:

- Rs=0.66 Ω,
  Rr=0.38 Ω,
  Xs=1.14 Ω,
  Xr=1.71 Ω,
  Xm=33.2 Ω.

The motor is controlled by a current converter. If the input current is maintained at 20A, the frequency at 40Hz, and the torque at 55 Nm, calculate:

A – The slip at maximum torque and the value of Tmax.
B – The slip and motor speed.
C – The phase voltage.
D – The power factor.

Solution of Exercise 11:

Given:

$I = 20$ A,

$f = 40$ Hz ,

$\mathrm{T} = 55\ \mathrm{N \cdot m}$

A- A. Calculating the slip for maximum torque and the value of T_{max}

- Given frequency $f = 40\text{Hz}.$

$$\Omega_s = \frac{2\pi f}{P} = \frac{2\pi \times 40}{2}$$

$$\Omega_s = 125{,}6\ \mathrm{rad/s}$$

$$x = 2\pi f_b \mathrm{L} \Rightarrow \mathrm{L} = \frac{x}{2\pi f_b}$$

$$x' = 2\pi f_2 L = 2\pi f_2 \frac{x}{2\pi f_2}$$

$$x' = \frac{x f_2}{f_b} = x.\beta$$

Next, we calculate the new reactances at the reduced frequency using the scaling factor

$$\beta = \frac{f_2}{f_b}$$

$$\beta = \frac{40}{60}$$

$$\beta = 0{,}67$$

The reactances are scaled with frequency:

$$x_s' = 1{,}14 \times \beta \Rightarrow x_s' = 0{,}76\Omega$$

$$x_r' = 1{,}71 \times B \Rightarrow x_r' = 1{,}14\Omega$$

$$x_m' = 33{,}2 \times \beta \Rightarrow x_m' = 22{,}13\Omega$$

Now, calculate the slip at maximum torque:

$$S_{\mathrm{T\,max}} = \frac{R_r}{x_m' + x_r'} = \frac{0{,}38}{22{,}13 + 1{,}12}$$

Or

$$S_{T\,max} = \frac{R_r}{\sqrt{R_s^2 + (x_s' + x_r' + x_m')^{\,2}}}$$

$$S_{T\,max} = \frac{0.38}{\sqrt{0.66^2 + (0{,}76 + 1{,}14 + 22{,}13)^{\,2}}}$$

$$S_{T\,max} = 0.0163$$

To calculate the maximum torque **T_{max}**

$$T_{\mathrm{max}} = \frac{3X_m^2 I_s^2}{2\Omega_s(x_m' + X_r')}$$

$$T_{\mathrm{max}} = \frac{3 \times (22{,}13)^2 \times (20)^2}{2 \times 125{,}6(22{,}13 + 1{,}14)}$$

$$T_{\mathrm{max}} = 100{,}54\ \mathrm{N \cdot m}$$

B. Slip and Motor Speed

For the torque T_{em}=55 Nm, the slip S is calculated from the torque equation for current-controlled motors:

$$T_{em} = \frac{3\,\mathrm{Rr}(x_m I_s)^2}{S.\Omega_s\left[\left(R_s + \frac{\mathrm{R_r}}{S}\right)^2 + (x_m + x_s + x_\mathrm{r})^2\right]} = 55$$

$$\frac{3\left(\frac{Rr}{s}\right)(x_m I_s)^2}{\Omega_s\left[\left(R_s + \frac{R_r}{s}\right)^2 + (x_m + x_s + x_1)^2\right]} = 55$$

This results in the quadratic equation:

$$\left(\frac{R_s}{s}\right)^2 - 84{,}08\ \frac{R_s}{s} + 577{,}88 = 0$$

$$\frac{R_r}{s} = x$$

$$x^2 - 84{,}88x + 577{,}88 = 0$$

$$\Delta = (84{,}08)^2 - 4(577{,}88) = 4758{,}2464$$

$$\sqrt{\Delta} = 68{,}98$$

Solving for x:

$$x_1 = 7{,}55 \text{ ou } x_2 = 76{,}53$$

And so:

$$\frac{R_r}{s} = 7{,}55 \text{ or } 76{,}58$$

Solving for S: $S = 0{,}05$ ou . S =0,0049

The motor operates with higher slip in the stable region, so we select

$$S = 0{,}05$$

The motor speed is then:

$$\Omega_r = (1 - S)\Omega_s$$

$$\Omega_r = (1 - 0{,}05) \times 125{,}6$$

$$\Omega_r = 119{,}32 \text{ rad/s}$$

$$N_{\mathrm{r}} = 1140\ rmp$$

C. Phase Voltage

The equivalent impedance Ze is calculated as:

$$Z_e = \frac{-x_m(x_s + x_{\mathrm{r}}) + jx_m\left(R_s + \frac{R_r}{\mathrm{s}}\right)}{\left(R_s + \frac{R_r}{s}\right) + j(x_m + x_s + x_{\mathrm{r}})}$$

$$Z_e = \frac{-42{,}047 + j182{,}79}{8{,}26 + j24{,}03}$$

Substituting the values:

$$Z_e = \frac{-42{,}047 + j182{,}79}{8{,}26 + j24{,}03} \times \frac{(8126 - j24{,}03}{(8{,}26 - j24{,}03}$$

$$Z_e = 6{,}26 + j3{,}9$$

The magnitude of Ze:

$$\| \bar{Z}_e \| = 7{,}38\Omega \quad \text{and} \quad \phi = 31{,}9^\circ$$

The phase voltage is:

$$V_e = Z_e I = 7{,}38 \times 20 \Rightarrow V_e = 147{,}6 \text{ V}$$

D – Power Factor

The power factor is:

$$\text{Power Factor:} = \cos(31{,}9) = 0{,}849$$

Exercise 12:

A three-phase squirrel-cage induction motor (220V-380V, 50Hz, 4 poles) is powered by a variable voltage using a dimmer. The simplified equivalent circuit for one phase is represented by an L circuit with:

- Rr=1.2 Ω
- Xs+Xr=2.5 Ω

1. **Show that the numerical expression (in N.m) for the useful torque of the machine is:**

$$T(S)=\frac{2.29\times10^{-3}\times V_S^2}{6.25\times S\,\frac{1.44}{S}}$$

where S is the slip at the operating point of the motor-load system.

2. **Give the expressions for the starting torque and the maximum torque.** Derive the values of the starting torque and maximum torque for V=220V and V=70V.
3. The machine is driving a constant load torque T_l=70 Nm. Is the use of a dimmer suitable in this case for starting? What happens?
4. Knowing that in its useful range, the torque-slip characteristic can be approximated by a straight line with the equation:

$$T(S)=19\times10^{-3}\times V_s^2\times S$$

And the motor drives a load with a resistant torque given by:

$T_l(n)=60\times(\frac{n}{n_s})^2$ where n is the rotor speed.

4.1 **Determine the operating point for Vs=220V and Vs=70V**

4.2 **Is the dimmer usable in this case?**

Solution of Exercise 12:

1- Demonstration of the Torque Expression

We want to demonstrate that the electromagnetic torque can be expressed as:

$$T(S)=\frac{2.29\times10^{-3}\times V_S^2}{6.25\times S\,\frac{1.44}{S}}$$

The general expression for the electromagnetic torque is:

$$T_{em}=\frac{P_{em}}{\Omega_s}=\frac{3\,R_r\,{I_r}^2}{S\,\Omega_s}$$

The rotor current Ir is given by:

$$I_r = \frac{V_s}{\sqrt{\left(\frac{R_r}{S}\right)^2 + x_T^2}}$$

Where $x_T = x_s + x_r$ is the total reactance.

Substituting $I_r{}^2$ into the torque equation:

$$T_{em} = \frac{3R_r V_s{}^2}{S\Omega_s\left[\left(\frac{R_r}{S}\right)^2 + x_T^2\right]}$$

Simplifying, and using the known values for Rr, Xs+Xr, and Ωs, we get:

$$T_{em} = \frac{3 \times 1{,}2\ V_s^2}{157 \cdot S\left[\frac{1{,}44}{S^2} + 6{,}25\right]}$$

This leads to the required expression:

$$T_{em} = \frac{2{,}29 \cdot 10^{-2} \times V_s^2}{6{,}25 \cdot S + \frac{1{,}44}{S}}$$

2) Starting Torque and Maximum Torque

Starting Torque $S = 1$

$$T_S = \frac{2{,}29 \cdot 10^{-2} \times V_s^2}{6{,}25 + 1{,}44}$$

$$T_S = 29.81 \times 10^{-4} \times V_s^2$$

Maximum Torque occurs at

$$S_{\text{T max}} = \frac{R_r}{X_T} = \frac{1{,}2}{2{,}5}$$

$$S_{\text{T max}} = 0{,}48$$

$$T_{\max}, \frac{2{,}29 \cdot 10^{-2}}{6{,}25 \cdot 0{,}48 + \frac{1{,}44}{6{,}44}} \cdot V_s^2$$

$$T_{\max} = 3.81 \cdot 10^{-3} V_s^2$$

For V=220V:

- Starting torque Ts=144,28Nm
- Maximum torque$T_{\max} = 184.97$Nm

For V=70V:

- Starting torque Ts=14.5 Nm
- Maximum torque $T_{\max} = 18.72$Nm

Since the load torque T_l=70 Nm, the motor cannot start because the starting torque T_{start}=14.5 Nm is less than the load torque.

3. Feasibility of Using the Dimmer

The use of a dimmer is not suitable for starting the motor in this case because the motor's starting torque is insufficient to overcome the load torque.

4. Determination of the Operating Point

The torque characteristic can be approximated as:

$$T(S) = 19 \times 10^{-3} \cdot V_s^2 \times S.$$

The load torque is given by:

$$T_l(\mathrm{N}) = 60 \cdot \left(\frac{N}{N_s}\right)^2$$

Where

$$S = \frac{N_s - N}{N_s} \Rightarrow N = (1 - S)N_s$$

$$\frac{N}{N_s} = (1 - g)$$

Thus, the load torque becomes:

$$T_l = 60 \times (1 - g)^2$$

4.1 Operating Point for Vs=220V and Vs=70V:

The operating point is found by equating the motor torque and load torque:

$$T_l = T$$

$$19 \times 10^{-3} \times V_s^2 \times S = 60 \times (1 - S)^2$$

For V=220V :

- $\Delta = 296{,}77$, solutions $S_1 = 0{,}0579$, $S_2 = 17{,}28$
- Valid slip: 0<S<1 , $S_1 = 0{,}0579$
- Rotor speed: $N = 1433\mathrm{rmp}$
- Torque: $\mathrm{T} = 53{,}24\ \mathrm{Nm}$

For r V=70V :

- $\Delta = 8{,}626$, solutions $S_1 = 0{,}31, S_1 = 3{,}24$
- Valid slip:$S_1 = 0{,}31$
- Rotor speed:$N = 1035\mathrm{rmp}$
- Torque:$\mathrm{T} = 28{,}56\ \mathrm{Nm}$

4.2 Is the Dimmer Usable?

No, the dimmer is not suitable because the motor cannot generate sufficient starting torque to overcome the load at reduced voltages.

Exercise 14:

Using Scalar Control

We are given an asynchronous machine (ASM):

- Vsn=220 Volts
- Rr′=5 ohm
- p=1p
- Lr′=0.03 H (Note: Magnetic losses and friction are neglected.)

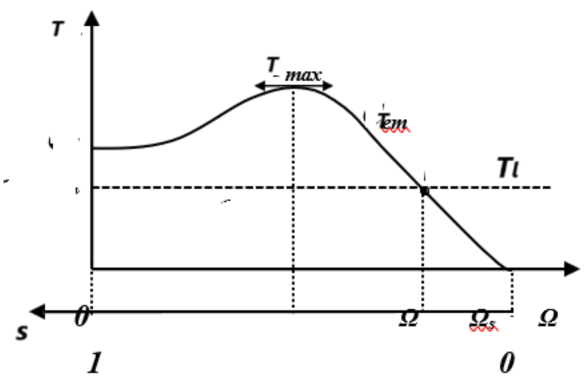
Given Vs=150 V and the frequency fs=50 Hz:

1. Calculate the starting current Is and the maximum torque Tmax .
2. For a constant load torque Tl=2 Nm, calculate the rotor speed Ω (rad/sec) and the slip S.
3. If T*l*=4 Nm , calculate ωsmax

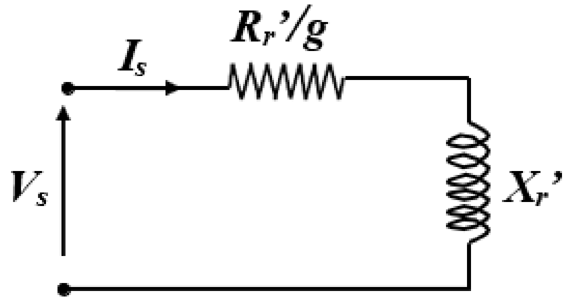
Solution of Exercise 14:

1. Calculation of Starting Current Is and Maximum Torque Tmax

We are given the characteristic of electromagnetic torque as a function of slip (s) and rotational speed (Ω).



Assuming magnetic losses and friction are negligible, we obtain the equivalent circuit of the ASM.



At startup: S=1

Then:

$$V_s = I_d\sqrt{R_r'^2 + X_r'^2}$$

$$I_s = \frac{V_s}{\sqrt{R_r'^2 + X_r'^2}}$$

$$I_s = \frac{150}{\sqrt{5^2 + (0{,}03.314)^2}}$$

$$I_s = 14 \text{ A}$$

The starting current I_s=14 A.

Next, the maximum torque Tmax is calculated as follows:

$$\boldsymbol{T}_{\max} = \frac{3p}{2\omega_s} \cdot \frac{V_s^2}{\bar{x}_r'}$$

$$\boldsymbol{T}_{\max} = \frac{\mathbf{3}}{\mathbf{2.314}} \cdot \frac{\mathbf{150^2}}{\mathbf{9,42}}$$

$$\boldsymbol{T}_{\max} = 11{,}41\text{Nm}$$

Thus, the maximum torque Tmax=11.41 Nm .

2. Calculation of Rotor Speed Ω (rad/sec) and Slip S forT_l ==2 Nm

Assuming $S \ll S_{\max}$:

$$T_{em} = \frac{3 \cdot p \cdot V_s^2}{\omega_s \cdot R_r'} \cdot S$$

In equilibrium:

$$T_{em} = T_l = 2\ Nm$$

$$S = \frac{T_l \cdot \omega_s \cdot R_r'}{3 \cdot p \cdot V_s^2}$$

$$S = \frac{2 \cdot 314 \cdot 5}{3 \cdot 1 \cdot 150^2}$$

$$\text{S} = 0{,}0465$$

Thus, the slip S=0.0465 .

We have:

$$S = \frac{\Omega_s - \Omega}{\Omega_s}$$

$$\Omega = \Omega_s(1 - S)$$

$$\Omega = \frac{\omega_s}{p}(1 - S)$$

$$\Omega = 314(1 - 0{,}0465)$$

$$\Omega = 299{,}399\text{rad/sec}$$

Therefore, the rotor speed Ω=299.4 rad/sec.

3. Calculation of $\boldsymbol{\omega_{s\,max}}$or $\boldsymbol{T_l}$ ==4 Nm

According to the principle of scalar control, to maintain the flux (Φs=constant), the ratio ***V_s/ω_s*** must remain constant since ***$\Phi_s = V_s/\omega_s$***

we increase ***ω_s*** to vary the speed, we must also increase ***V_s*** in parallel to keep ***Φ_s***=constant until ***$V_s = V_{sn} = 220V$***.

In this case, if ***$V_s = V_{sn}$*** , we increase ***ω_s*** with ***$V_s = V_{sn}$*** to decrease ***Φ_s*** and increase speed Ω.

Thus, we observe a decrease in ***T_{max}*** until ***T_{max} (min) = T_l = 4 Nm***.

Therefore, for ***$T_{max} = T_l$*** and ***$V_s = V_{sn}$*** , we have ***$\omega_s = \omega_{smax}$*** .

Since if we increase ωsω_sωs too much, we obtain ***$T_{max} < T_l$*** (which is unacceptable).

We find:

$$T_{\max} = T_l$$

$$\frac{3p}{2\omega_{s\max}} \cdot \frac{V_{sn}^2}{X_r'} = \frac{3p}{2 \cdot L_r'} \cdot \left(\frac{V_{sn}^2}{\omega_{s\max}^2}\right)$$

$$\omega_{s\max}^2 = \frac{3pV_{sn}^2}{2L_r'C_r}$$

$$\omega_{s\max} = \sqrt{\frac{3pV_{sn}^2}{2L_r'C_r}}$$

Calculating gives:

$$\omega_{s\max} = \sqrt{\frac{3.1 \cdot 220^2}{2 \cdot 0{,}03 \cdot 4}}$$

$$\omega_{s\max} = 777{,}8\text{rad/sec}$$

Exercise 15:

Using Scalar Control

We are given an asynchronous machine (ASM):

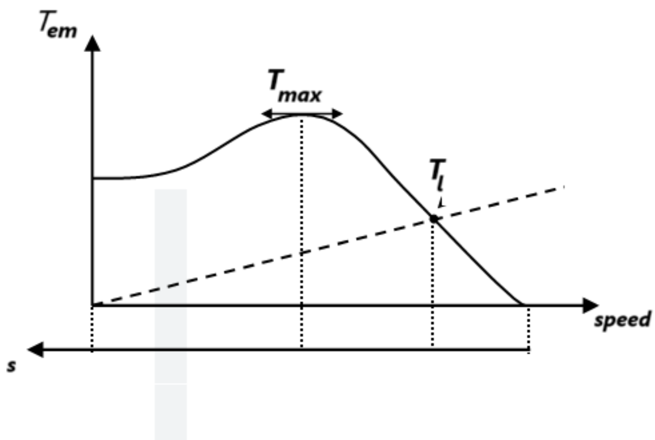
- Vsn=220 Volts
- Rr′=10 ohm
- p=1
- Lr′=0.02 H
- ω_{smax}=1200 rad/s (maximum allowable speed)
- The load torque varies as a function of speed T_l=k·Ω C with k=0.02
- Friction is neglected.

1. For Vs=Vsn and frequency fs=50 Hz : Calculate the rotational speed Ω.
2. For Ω=200 rad/sec and V=Vsn : Calculate the supply frequency fs.
3. For Φsn=0.5 Weber: Calculate the maximum speed Ωmax .

Solution of Exercise 15:

1. Calculation of Rotational Speed Ω for Voltage Vs=Vsn and Frequency fs = 50 ,

The following figure shows the characteristic Tem=f(Ω)



On the curve ***T_{em}=f(Ω)***, for a slip ***S < S_{max}*** (the linear operating zone):

$$T_{em} = \frac{3 \cdot p \cdot V_S^2}{\omega_s \cdot R_r'} \cdot S$$

$$T_{em} = \frac{3 \cdot 1 \cdot 220^2}{314 \cdot 10} \cdot S$$

Thus:

$$T_{em} = 46.242\, S$$

And:

$$S = \frac{\Omega_s - \Omega}{\Omega_s}$$

$$\Omega = \Omega_s(1 - S)$$

We also have:

$$T_l = k \,.\, \Omega$$

$$T_l = 0{,}02 \,.\, \Omega$$

$$T_l = 0{,}02 \,.\, \Omega_s \,.(1 - S)$$

At equilibrium $\boldsymbol{T_{em} = T_l}$, we get:

$$46.242\, S = 0.02\, \Omega_s\, (1 - S)$$

$$\mathrm{S} = \frac{(0{,}02 \cdot \Omega_s)}{46{,}242 + (0{,}02 \cdot \Omega_s)}$$

$$\Omega_s = \frac{\omega_s}{p}$$

Where $p=1$

$$\Omega_s = \omega_s$$

$$\mathrm{S} = \frac{(0{,}02 \cdot \omega_s)}{46{,}242 + (0{,}02 \cdot \omega_s)}$$

$$\mathrm{S} = \frac{(0{,}02 \cdot 314)}{46{,}242 + (0{,}02 \cdot 314)}$$

$$\mathrm{S} = 0{,}12$$

Thus, the slip S=0.12 , and the rotational speed is:

$$\Omega = \Omega_s \,.(1 - g)$$

$$\Omega = 314 \,.(1 - 0{,}12)$$

$$\Omega = 276{,}32 \text{ rad/sec}$$

The rotational speed Ω=276.32 rad/sec.

2. Calculation of the Supply Frequency fs for Rotational Speed Ω=200 rad/sec and Voltage V=Vsn

We know:

$$\Omega_s = \omega_s$$

Also,

$$T_{\mathrm{em}} = \frac{3 \cdot p \cdot V_s^2}{\omega_s \cdot R_r'} \cdot S$$

$$T_{\mathrm{em}} = \frac{14520}{\omega_s} \cdot S$$

$$T_{\mathrm{em}} = 0{,}02 \cdot \Omega$$

Thus:

$$\frac{14520}{\Omega_s} \cdot \frac{\Omega_s - \Omega}{\Omega_s} = 0{,}02 \cdot \Omega$$

$$\frac{14520}{\Omega_s^2} \cdot (\Omega_s - \Omega) = 0{,}02 \cdot \Omega$$

Replacing Ω=200 rad/sec:

$$-14520 \,.\, \Omega_s + 14520 \,.\, \Omega + 0{,}02 \,.\, \Omega_s^2 = 0$$

This simplifies to:

$4. \Omega^2 - 14520 . \Omega + 2904000 = 0$

We calculate the discriminant:

$$\Delta = 164366400 \quad \rightarrow \quad \sqrt{\Delta} = 12820{,}546$$

This gives two solutions:

- $\boldsymbol{\Omega_{s1} = 3417{,}57}$ ***rad/sec*** $\boldsymbol{\omega_{s1} = \Omega_{s1} > \omega_{smax}}$ (this is an overestimate and not acceptable).
- $\boldsymbol{\Omega_{s2} = 212{,}5}$ ***rad/sec*** $\boldsymbol{\omega_{s2} = \Omega_{s2} < \omega_{smax}}$ (this is an acceptable value).

Thus:

$$\boldsymbol{\omega_s = \omega_{s2} = 212{,}5 \text{ rad/sec}}$$

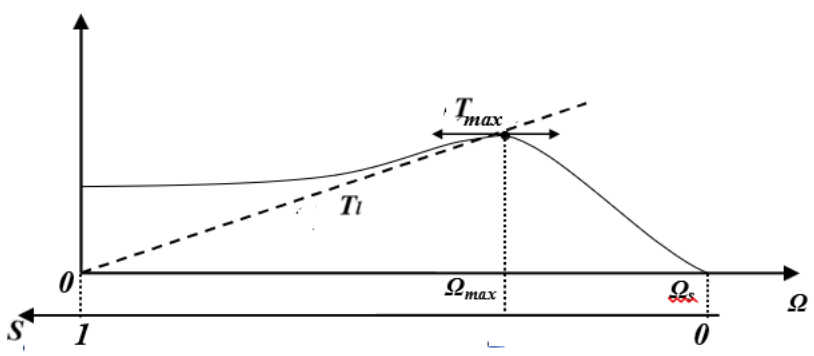
The supply frequency is:

$$\boldsymbol{f_s = \omega_s / (2 \,.\, 3{,}14) = 33{,}84 \text{ Hz}}$$

So, the supply frequency $\boldsymbol{f_s = 33{,}84}$ ***Hz***

3. Calculation of the Maximum Rotational Speed Ωmax for Stator Flux ($\boldsymbol{\Phi_s=\Phi_{sn}=0{,}5}$ ***weber*** **)**

1) As shown in the figure, at Ω=Ωmax , we have $\boldsymbol{T_{max} = T_l}$



$$T_{em} = T_l$$

$$\frac{3 \cdot p \cdot \Phi^2}{\omega_s \cdot L_r'} = \mathrm{k}\,\Omega = 0.02\,\Omega$$

Therefore:

$$\Omega_{max} = \frac{3 \cdot p \cdot \Phi_{sn}{}^2}{2 \cdot \ \mathrm{k} \cdot L'_r}$$

$$\Omega_{max} = \frac{3 \cdot 1 \cdot 0.5^2}{2 \cdot \ 0.02 \cdot 0.02}$$

Ω_{max} =937.5rad/sec

Thus, the maximum rotational speed Ωmax=937.5 rad/sec.

Exercise 16:

A three-phase, 460V, 60-Hz, six-pole, Y-connected cylindrical rotor synchronous motor has the following data:

- **Synchronous reactance,** Xs=2.5 Ω
- **Negligible armature resistance**
- **Load torque,** T_L=398 Nm at 1200 rpm
- **Power factor** is maintained at unity by field control
- The **voltage-to-frequency ratio** is kept constant at the rated value.

If the inverter frequency is 36 Hz and the motor speed is 720 rpm, calculate the following:

1. The input voltage Vs
2. The armature current Ia
3. The excitation voltage Vf
4. The torque angle δ
5. The pull-out torque Tp

Solution of Exercise 16:

1-Power factor:

$$\mathrm{PF} = \cos\theta_m = 1.0$$

$$\theta_m = 0$$

2-Rated Voltage:

The line-to-neutral voltage at rated conditions is:

$$V_{\mathrm{a(rated)}} = V_{\mathrm{b}} = \frac{460}{\sqrt{3}}$$

$$V_{\mathrm{a(rated)}} = 265.58\ \mathrm{V}$$

Motor speed and synchronous speed:

For a six-pole motor running at 1200 rpm, the synchronous speed is:

$p = 6$

$$\omega = 2\pi \times 60 = 377\text{rad/s} \text{ or } 1200 \text{ rpm}$$

(synchronous angular velocity at 60 Hz)

The mechanical speed in radians per second is:

$$\omega_b = \omega_s = \omega_m = 2 \times 377/6 = 125.67\text{rad/s}$$

Voltage-to-frequency ratio:

Since the voltage-to-frequency ratio is kept constant, we have:

$$d = \frac{V_f}{\omega_b} = 265.58/125.67 = 2.1133$$

At 720 rpm:

The load torque is proportional to the square of the speed, so:

$$T_L = 398 \times \left(\frac{720}{1200}\right)^2$$

$$T_L = 143.28 \text{ N} \cdot \text{m}$$

The mechanical angular speed at 720 rpm is:

$$\omega_{,} = \omega_m = 720 \times \frac{\pi}{30} = \frac{75.4\text{rad}}{\text{s}}$$

The power output is:

$$P_0 = 143.28 \times 75.4$$

$$P_0 = 10{,}803 \text{ W}$$

(a) **Input Voltage Vs:**

Using the voltage-to-frequency ratio, the input voltage at 36 Hz is:

$$V_s = d\omega_m = 2.1133 \times 75.4 = 159.34 \text{ V.}$$

(b) **Armature Current Ia:**

The power output is related to the armature current by the following equation:

$$P_0 = 3V_a I_a \text{PF}$$

$$P_0 = 10.803$$

Solving for Ia:

$$I_a = \frac{10{,}803}{3 \times 159.34}$$

$$I_a = 22.6 \text{ A}$$

(c) **Excitation Voltage Vf:**

The excitation voltage is found from the voltage equation:

$$\bar{\mathbf{v}}_f = V_a\angle 0 - \overline{\mathrm{I}_a}\,(\mathrm{R}_a + j\,\mathrm{X}_s)$$

Since Ra is negligible

$$\bar{\mathbf{v}}_f = 159.34 - 22.6(1 + j0)(j2.5)$$

Since Ra is negligible

$$\bar{\mathbf{v}}_f = 169.1\angle - 19.52°$$

(d) **Torque Angle δ:**

The torque angle is the phase angle between Va and Vf, which is:

$$\delta = -19.52°.$$

(e) **Pull-out Torque Tp:**

The pull-out torque is given by the equation :

$$T_p = \frac{3\,V_a V_f}{X_s \omega_s}$$

Substituting the values:

$$T_p = \frac{3 \times 159.34 \times 169.1}{2.5 \times 75.4}$$

$$T_p = 428.82 \text{ N} \cdot \text{m}$$

Exercise 17:

A three-phase, 230-V, 60-Hz, four-pole, Y-connected reluctance motor has the following data:

- **Direct axis reactance,** Xd=22.5 Ω
- **Quadrature axis reactance,** Xq=3.5 Ω
- **Negligible armature resistance**
- **Load torque,** T_L=12.5 Nm
- **Voltage-to-frequency ratio** is maintained constant at the rated value.

Determine the following:

1. **Torque angle, δ**
2. **Line current, Ii**
3. **Input power factor, PF**

Solution of Exercise 17:

1. **Load Torque**:

$$T_L = 12.5\ \text{N} \cdot \text{m}$$

The line-to-neutral voltage at rated conditions is:

$$V_{\text{a(rated)}} = V_b = \frac{230}{\sqrt{3}} = 132.79\ \text{V}$$

The motor has four poles:

$$p = 4$$

The synchronous angular velocity at 60 Hz is:

$$\omega = 2\pi \times 60 = \frac{377\text{rad}}{\text{s}},$$

The mechanical angular velocity for four poles is:

$$\omega_\text{b} = \omega_s = \omega_m = 2 \times \frac{377}{4} = \frac{188.5\text{rad}}{\text{s}} \text{ or } 1800\text{rpm},$$

The voltage-to-frequency ratio is:

$$d = V_v/\omega_\text{b} = 132.79/188.50 = 0.7045$$

The supply voltage is:

$$V_s = 132.79\ \text{V}$$

(a) Torque Angle δ\deltaδ:

The synchronous speed is

$$\omega_s = 188.5\text{rad/s}.$$

Since the armature resistance Rs is negligible, the torque angle δ can be determined using:

$$\delta = -\tan^{-1}\frac{(I_a\, X_q cos\theta_m)}{V_a - I_a\, X_q sin\theta_m}$$

Substituting the values:

$$\sin 2\delta = -\frac{12.5 \times 2 \times 188.5 \times 22.5 \times 3.5}{3 \times 132.79^2 \times (22.5 - 3.5)}$$

Solving for δ:

$$\delta = -10.84°$$

(b) **Line Current Ii:**

The output power is:

$$P_0 = T_\text{l}\, \omega_s$$

$$P_0 = 12.5 \times 188.5$$

$$P_0 = 2356\ \text{W}$$

The relationship between torque angle and current is:

$$\tan(10.84°) = \frac{3.5I_a\cos\theta_m}{132.79 - 3.5I_a\sin\theta_m}$$

And from the power equation:

$$P_0 = 2356 = 3 \times 132.79 I_a \cos\theta_m.$$

Using an iterative solution method, we find:

$$I_a = 9.2 \text{ A}$$

$$\theta_m = 49.98°$$

(c) Power Factor PF:

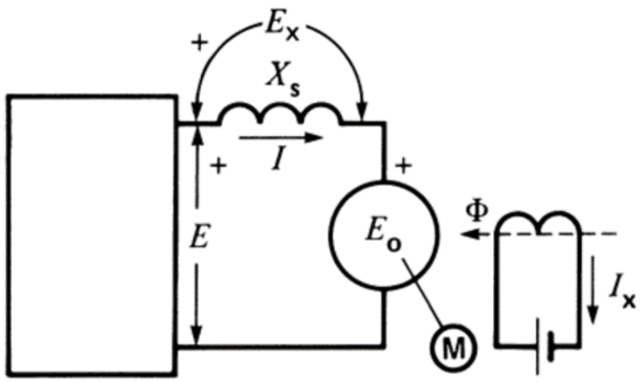
The power factor is the cosine of the phase angle θ_m:

$$\text{PF} = \cos(49.98°) = 0.643$$

Exercise 18:

The synchronous motor in the figure has the following parameters per phase:

- V=2.4 kV
- E_0=3 kV
- Xs=2 Ω
- I=900 A



Plot the phasor diagram and determine:

a. The phase angle

b. Active power per phase

c. Power factor of the motor

d. Reactive power absorbed (or delivered) per phase

Solution of Exercise 18:

a. Phase Angle

Determine the Excitation Voltage:

$$E_x = X_s I$$

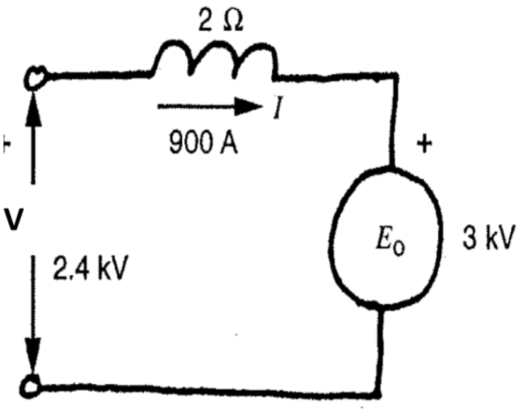
$$E_x = 2 \times 900$$

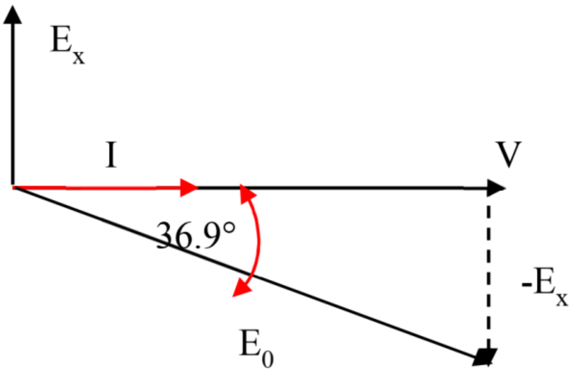
$$E_x = 1800 \text{ V}$$

Using the tangent function for the phase angle:

$$\tan\delta = \frac{E_x}{E}$$

$$\delta = 36.9°$$





b. Active Power per Phase

$$P = \frac{E_0 V}{X_s}\sin\delta$$

$$P = \frac{3000 \times 2400}{2}\sin 36.9°$$

$$P = 2.16 \text{MW}$$

C. Power Factor of the Motor

Since

$$P = VI\cos\varphi = 2.16MW$$

And assuming

$$\cos\varphi = 1$$

So V is in phase with I

D. Reactive Power Absorbed (or Delivered) per Phase

$$Q = 0VAR$$

Exercise 19:

A three-phase 460-V, 60-Hz, 10-pole Y-connected cylindrical rotor synchronous motor has a synchronous reactance of Xs=0.8 per phase, and the armature resistance is negligible. The load torque, which is proportional to the speed squared, is Tl=1250 Nm at 720 rpm. The power factor is maintained at 0.8 lagging by field control, and the voltage-to-frequency ratio is kept constant at the rated value. If the inverter frequency is 45 Hz and the motor speed is 540 rpm, calculate the:

(a) Input voltage, Va

(b) Armature current, Ia

(c) Excitation voltage, V_f

(d) Torque angle, δ

(e) Pull-out torque, Tp

Solution of Exercise 19:

Given Data:

- Rated Voltage (V_{rated}): 460 V (**phase voltage**)
- Frequency (f_{ratedf}): 60 Hz
- Number of Poles (p): 10
- Synchronous Reactance (Xs): 0.8 Ω per phase
- Load Torque (T_l): 1250 N·m at 720 rpm
- Power Factor (PF): 0.8 lagging
- Inverter Frequency (f_{inv}): 45 Hz
- Motor Speed (n): 540 rpm

Calculate Synchronous Speed

The synchronous speed n_s is given by:

$$n_s = \frac{120\,f}{p}$$

For 60 Hz:

$$n_s = \frac{120\ \times 60}{10}$$

$$n_s = 720\ rmp$$

a. Calculate Input Voltage Va

The voltage-to-frequency ratio is maintained constant. Using the ratio at the rated frequency:

$$V_{\text{rated}} = \frac{460}{\sqrt{3}} = 265.6\text{V}$$

$$\frac{V_{rated}}{f_{rated}} = \frac{V_a}{f_{inv}}$$

Substituting the values:

$$\frac{265.6}{60} = \frac{V_a}{45}$$

Solving for Va:

$$V_a = \frac{265.6 \times 45}{60}$$

$$V_a = 199.2\ V$$

So, the phase voltage at 45 Hz is approximately **199.2 V**.

Calculate Armature Current Ia

We can calculate the armature current using the torque equation for a synchronous motor:

$$T_l = \frac{3\ V_a\ I_a\ PF}{\Omega_S}$$

First, calculate $\Omega_S$at the synchronous speed for 45 Hz (inverter frequency):

Using the formula for synchronous speed:

$$n_s = \frac{120\ \times 45}{10}$$

$$n_s = 540\ rmp$$

Now convert the synchronous speed n_s to angular speed Ω_S:

$$\Omega_S = n_s \frac{2\pi}{60}$$

$$\Omega_S = 450 \frac{2\pi}{60}$$

$$\Omega_S = 56.55 \text{rad/s}$$

Now, solve for Ia in the torque equation

$$I_a = \frac{T_l \, \Omega_S}{3 \, V_a . PF}$$

Substitute the known values:

$$I_a = \frac{1250 \times 56.55}{3 \times 199.2 \times 0.8}$$

$$I_a = 147.86\text{A}$$

So, the **armature current** is approximately **147.86 A**.

Calculate Excitation Voltage V$_f$

The excitation voltage V_f can be calculated using the phasor relationship between the terminal voltage V_a, the synchronous reactance Xs , and the armature current Ia:

$$V_f = \sqrt{{V_a}^2 + (I_a \times X_s)^2}$$

$$V_f = \sqrt{199.2^2 + (147.86 \times 0.8)^2}$$

$$V_f = 231.66 \, V$$

So, the **excitation voltage** is approximately **231.66 V**.

Calculate Torque Angle δ

The torque angle δ can be approximated from the power factor angle.

$$\delta = \tan^{-1} \frac{-(IX_s cos\varphi - IR_s sin\varphi)}{V - IX_s sin\varphi - IR_s cos\varphi}$$

Since Rs is negligible, we can simplify the formula by ignoring the terms involving Rs:

$$\delta = \tan^{-1} \frac{-(IX_s cos\varphi)}{V - IX_s sin\varphi}$$

$$\delta = \tan^{-1} \frac{-(147.86 \times 0.8 \times 0.8)}{199.2 - 147.86 \times 0.8 \times sin 36.87°}$$

Finally, calculate δ:

$$\delta = -36.33°$$

The **torque angle δ** is approximately **-36.33°**, indicating that the voltage leads the current in a lagging power factor condition.

Calculation of Pull-Out Torque Tp

The pull-out torque can be determined using the relationship with the synchronous speed and the maximum electromagnetic torque

$$T_p = \frac{3\, V_a V_f}{X_s \Omega_s}$$

For the inverter frequency f_{inv}=45 Hz:

$$\Omega_s = 56.55 rad/s$$

Now, Substitute the Values into the Formula:

$$T_p = \frac{3 \times 199.3 \times 231.66}{0.8 \times 56.55}$$

$$T_p = 3062.6 \text{Nm}$$

The **pull-out torque Tp** is approximately **3062.6 N·m**.

Exercise 20:

A 10 kW, 400 volt, 3-phase, star connected synchronous motor has a synchronous impedance of 0.3 + j 2.5. Find the voltage to which the motor must be excited to give a full load output at 0.866 leading pf.

Given:

- Power Output (P_{out}): 10 kW
- Voltage : 400 V
- Synchronous impedance (Zs): 0.3+j2.5
- Power factor (PF=0.866 (leading)
- Armature efficiency: 90%

We need to calculate:

1. The **excitation voltage** required to achieve full-load output at a leading power factor.
2. The **total mechanical power developed**.
3. The **losses in the armature winding**.

Solution of Exercise 20:

Calculate the Input Power

The input power to the motor is given by:

$$P_{in} = \frac{P_{out}}{\eta}$$

$$P_{in} = \frac{10}{0.9}$$

$$P_{in} = 11.11 \text{ kW}$$

So, the **input power** is 11.11 kW.

Calculate the Phase Voltage

Since the motor is **star-connected**, the phase voltage V is:

$$V = \frac{400}{\sqrt{3}} \text{ volts} = 230.94 \text{ Volts}$$

Calculate the Armature Current Ia

We can now calculate the armature current using the formula for 3-phase power:

$$P_{in} = \sqrt{3}\, I_a \times \text{V} \times \text{PF}$$

Rearranging for Ia:

$$I_a = \frac{11.11 \times 10^3}{0.9 \times \sqrt{3} \times 400 \times 0.866}$$

$$I_a = 18.52 \text{ A}$$

Thus, the **armature current** is approximately **18.52 A**.

Calculate the Voltage Drop Across Synchronous Impedance

The voltage drop across the synchronous impedance is given by:

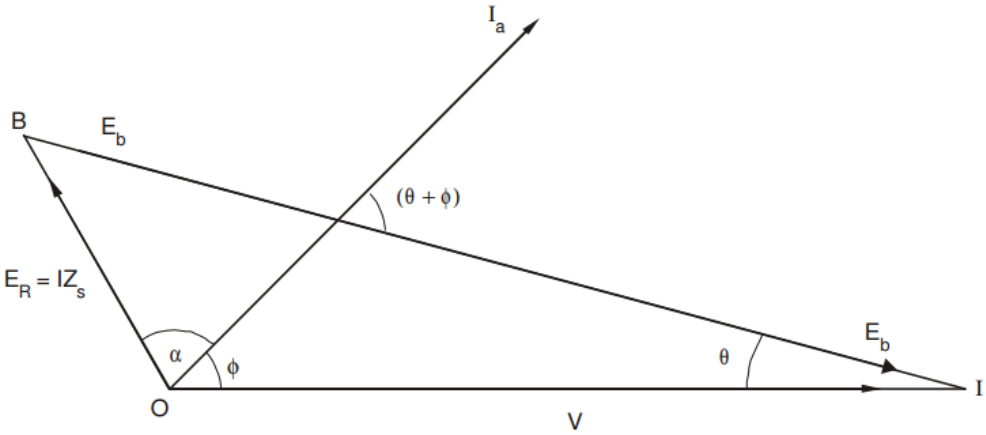
$$Z_s = \sqrt{(0.3)^2 + (2.5)^2} = 2.52\Omega$$

$$I_a\, Z_s = 18.52 \times 2.52 = 46.67 = E_R$$

This is the voltage drop due to the armature current.

Determine the Phase Angles

In the phasor diagram



Impedance angle α:

$$\alpha = \tan^{-1}\frac{X_s}{R_a}$$

$$\alpha = \tan^{-1}\frac{2.5}{0.3} = 83.157$$

Power factor angle ϕ

$$\phi = \cos^{-1} 0.866 = 30°$$

Thus, the total phase angle between the voltage and current α+ϕ is:

$$\alpha + \phi = 113.157°$$

Calculate the Excitation Voltage E$_f$

In a synchronous motor, the excitation voltage E_f is calculated using the phasor relationship:

$$E_f^2 = [IZ_s\cos(\theta + \phi - 90^2)]^2 + \left[V + IZ_s\cos[80° - (\phi + \theta)]\right]^2$$

$$E_f^2 = (ZZ_s)^2 \sin^2(\theta + \phi) + [V + IZ_s \cos(\theta + \phi)]^2$$

$$E_f^2 = (IZ_s)^2\sin^2(\theta + \phi) + V^2 + (IZ_s)^2\cos^2(\theta + \phi)$$
$$- 2V(IZ_s)\cos(\theta + \phi)$$

$$E_f^2 = V^2 + (IZ_s)^2 - 2V(IZ_s) \cos(\alpha + \phi)$$

Substitute the known values:

$$E_f^2 = (230.94)^2 + (46.67)^2 - 2 \times 230.94 \times 46.67\cos 113.157$$

Thus, the **excitation voltage** E_f is approximately **252.96 V**.

Calculate the Line Value of the Induced EMF

The line value of the induced EMF is:

$$E_{lin} = \sqrt{3} \times E_f$$

$$E_{lin} = \sqrt{3} \times 252.96$$

$$E_{lin} = 438.14 \text{ Volts}$$

Also

$$\frac{E_R}{\sin\alpha} = \frac{E_f}{\sin(\theta+\phi)}$$

$$\sin\alpha = \frac{E_R}{E_f}\sin(\theta+\phi)$$

$$\sin\alpha = \frac{46.67}{252.96}\sin(113.157)$$

$$\alpha = 9.766$$

Calculate the Mechanical Power Developed

The mechanical power developed by the motor can be calculated using the excitation voltage and phase voltage:

$$P_{mec} = \frac{E_f \times V}{|Z_s|}\cos(\alpha - \emptyset)$$

Substitute the known values:

$$P_{mec} = \frac{252.96 \times 230.94}{2.52}\cos(83.157^\circ - 9.766^\circ)$$

Thus:

$$P_{mec} = \frac{-252.962 \times \cos 3.157^\circ}{2.52} = 3.6 \text{ kW}$$

So, the **mechanical power developed** is approximately **3.6 kW**.

Calculate the Armature Winding Losses

The armature winding losses are calculated as:

$$p_{ru} = 3I^2R_\text{a}$$

$$p_{ru} = 3 \times 1852^2 \times 0.3$$

$$p_{ru} = 0.31 \text{ kW}$$

Thus, the **armature winding losses** are approximately 0.31 kW.

Iron and Excitation Losses

The **iron and excitation losses** are the difference between the total mechanical power developed P_m and the output power P_0:

$$\text{Iron and excitation losses} = P_m - P_0$$

$$\text{Iron and excitation losses} = 10.8 - 10 = 0.8KW$$

Also, mechanical power developed

$$P_m = P_{out} - p_{su} = 11.11 - 0.31 = 10.8KW$$

Exercise 21:

Given a voltage inverter U_{dc} using IGBT that powers a self-controlled synchronous machine connected in a star configuration with 2 pole pairs and a constant V/f control. Knowing that for:

- N_1 =500 RPM, U_{dc1}=120 V
- N_2 =1000 RPM, U_{dc2}=180 V
- Determine the equation that relates the voltage Udc to the frequency f, and calculate the voltage corresponding to a speed of 1500 RPM.
- Plot the characteristic curve U_{DC}=f(f) .

Solution of Exercise 21:

Given data:

- P=2 (number of pole pairs),
- U_{dc1} =120 V , N_1 =500 RPM ,
- U_{dc2}=180 V , N_2 =1000 RPM,
- V/f control is constant.

The relationship $U_{dc} = a \times f + b$ is linear.

The equation for speed is:

$$N = \frac{60\, f}{p}$$

For N_1 =500 RPM:

$$f_1 = \frac{P \times N_1}{60}$$

$$f_1 = \frac{2 \times 500}{60}$$

$$f_1 = 16.76\ Hz$$

For N_2 =1000 RPM :

$$f_2 = \frac{P \times N_2}{60}$$

$$f_2 = \frac{2 \times 1000}{60}$$

$$f_2 = 33.33\ Hz$$

Now, calculate the slope **a** of the linear relationship:

$$a = \frac{U_{dc2} - U_{dc1}}{f_2 - f_1}$$

$$a = \frac{180 - 120}{33.33 - 16.76}$$

$a \approx 3.6\, V/Hz$

Now, calculate **b**:

$b = U_{dc1} - a \times f_1$

$b = 120 - 3.6 \times 16.76$

$b = 60$

Thus, the equation relating U_{dc} to *f* is:

$U_{dc} = 3.6 \times f + 60$

For N=1500 RPM, the frequency is

$$f_3 = \frac{P \times N_3}{60}$$

$$f_3 = \frac{2 \times 1500}{60}$$

$f_3 = 50\ Hz$

Substituting $f_3 = 50\ Hz$ into the equation:

$U_{dc3} = 3.6 \times 50 + 60$

$U_{dc3} = 240V$

So, the voltage corresponding to a speed of 1500 RPM is $U_{dc3} = 240V$

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