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وزارة التعليم العالي والبحث العلمي

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ET DE LA RECHERCHE SCIENTIFIQUE

Université Ahmed DRAIA – Adrar

Faculté des sciences et de la technologie

Département de Génie Civil

جامعـــة أحمـد دراعيــة –أدرار

كليــة العـلوم والتكنولوجيــا

قسم الهندسة المدنية

جامعة أحمد دراية – أدرار

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مستخرج من محضر اجتماع اللجنة العلمية

نحن رئيس المجلس العلمي لقسم الهندسة المدنية بكلية العلوم والتكنولوجيا

بناءًا على المحضر المؤرخ بتاريخ 25 جوان 2025

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قام بإنتاج مطبوعة بيداغوجية بعنوان: **TOPOGRAPHY 1**

خضعت هذه المطبوعة البيداغوجية لخبرة علمية تحت إشراف اللجنة العلمية للقسم، وأسفرت عن المصادقة على قبول إصدارها ونشرها بمكتبة الجامعة وعلى الموقع الرسمي للجامعة، وذلك بعد الحصول على موافقة إيجابية من طرف الأساتذة الخبراء:

- **د. أقاسم مصطفى**............ جامعة أدرار
- **د. حاج مصطفى عدة**........ جامعة غليزان

أدرار في 26 جوان 2025

رئيس اللجنة العلمية للقسم

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رئيس اللجنة العلمية لقسم الهندسة المدنية

اللجنة العلمية

قسم الهندسة المدنية

People's Democratic Republic of Algeria
Ministry of Higher Education and Scientific Research
University of Ahmed Draia - ADRAR
Faculty of Science and Technology
Department of Civil Engineering

جامعة أحمد دراية ـ أدرار

Handout course

TOPOGRAPHY 1

Field**: Science and Technology**
Level: **2nd Year License - Civil Engineering -**

Author
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Academic Year
2024/2025

Preface

One of the oldest arts practiced by man. Since from the earliest times it has always been necessary to mark boundaries and divide tracts of land. Topography cover a wide range in scope and complexity, from the staking out of simple structures or the surveying of small parcels of land to the extensive and difficult surveys required in the construction of subdivisions, bridges, highways, canals, dams, railroads, wharves, missile and rocket launching sites, drainage and irrigations systems, or the survey of relatively large portions of the earth's surface.

This pedagogical support is designed to teach Topography to second year –Civil Engineering-license students at Ahmed Draia university, Adrar. This handout is produced to enhance and help students' comprehension, assimilation and skills about the relative importance of measurements which i see crucial to equally develop in order to achieve survey proficiency among civil engineering students.

The basics are presented in our handout in five main chapters. In the first chapter, it was a question of seeking to present the generalities on topography, by presenting the importance of the calculation of measurements on land.

In the second chapter, the measurement of distances by direct and indirect methods was treated and well detailed.

The third chapter aims to present the measurements of horizontal and vertical angles.

The fourth chapter is devoted to the determination of surfaces. In this chapter, examples of area measurement of various types are presented.

In the fifth and last chapter, we see how to measure the difference of heights between the points (the rise) via direct and indirect leveling.

L.M.D. ACADEMIC LISENCE
Filière : Génie Civil
2nd Year

Semester: 4
Teaching unit: UED 2.2
Course 2: Topographie 1
VHS: 22H30 (Cours: 1h30)
Credit: 1
Coefficient: 1

Teaching objectives:

The student will be able to know the basics of topography allowing him to carry out and subsequently control the implantation of a construction, leveling, measurement of angles and coordinates, the drawing of topographic plans.

Recommended prior knowledge:

Mathématics ; Physics 1 ; Technical drawing

Content of the course:

Chapter 1. Generalities **(3 weaks)**

Topography in the act of building, The different topographic measuring devices, Scales (plans, maps), Errors and mistakes

Chapter 2. Measurement of distances **(3 weaks)**

Direct distance measurement, Alignment methods and accuracies, Measuring practice, Indirect distance measurements

Chapter 3. Measurement of Angles **(3 weaks)**

Operating principle of a theodolite, Setting up a theodolite (Adjustment, Reading), Reading horizontal angles, Reading vertical angles.

Chapter 4. Determination of surfaces (Areas) **(3_weaks)**

Calculation of the area of a polygon, Determination of the areas of the contours represented on the plan, Planimeter and measurement of areas.

Chapter 5. Direct and Indirect Leveling **(3 weaks)**

Direct Leveling, Indirect Leveling.

Assessment method:

Exam : 100%.

OUTLINE

Chapter II : Measurement of Distances

Chapter III : Measurement of Angles

Chapter IV : Measurement of Surfaces

Chapter V : Direct and Indirect Leveling

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CHAPTER I: Generalities

I.1.Definition

I.1.1. Geodesy: is the basic science necessary for topography, which studies the shape of the earth. It has developed in two fundamental directions, a theoretical direction and a practical direction.

*) **Theoretical**: knowledge of the shape and dimensions of the Earth, its gravitational field, and development of precise measurements in the space domain (satellite tracking and guidance).
*) **Practical**: determination of remarkable points and materialize them in a lasting way allowing the establishment of exact maps and plans and providing the geometric data essential to major civil engineering works.

I.1.2. Topometry: Topometry (the words topo = place and metry = measurement) is a technique which allows obtaining on the ground the data necessary for calculating the metric values of all the elements of a plan on a large or very large scale (case of detailed survey).

It should be noted that topography serves the following areas:

Construction topography: Construction topography consists of providing alignments and altitudes which are used for the construction of buildings, sewer and aqueduct networks (tubes, canals), streets, and the rest.

Road topography: Road surveying is brilliantly linked to highways, railways, pipelines, in general, over large distances.

Cadastral topography: Cadastral surveying, also called boundary marking, consists mainly of determining the delimitation and division of land properties. It is a field of activity exclusively reserved for surveyors.

Underground topography: Operations such as the orientation and dimensions of tunnels and cavity passages, the calculation of volumes, etc., fall under underground topography.

Hydrographic topography: Hydrographic topography, or simply hydrography, aims to represent the coastline (shore), lakes and rivers, seabed, and the rest.

Industrial topography: The layout of industrial installations, using optical instruments, constitutes the main application of industrial topography.

I.1.3. Topography: the word topography comes from the Greek (topo = the place and graphy = to describe), is that which gives the means of graphic or digital representation of a terrestrial surface. The topography subservient (attached) to fundamental sciences such as mathematics and physics is (strictly) linked to geodesy, cartography, photogrammetry and recently to computer science and electronics.

Its purpose is the flat representation at a given scale of a certain area of land including details on a plan or on a map.

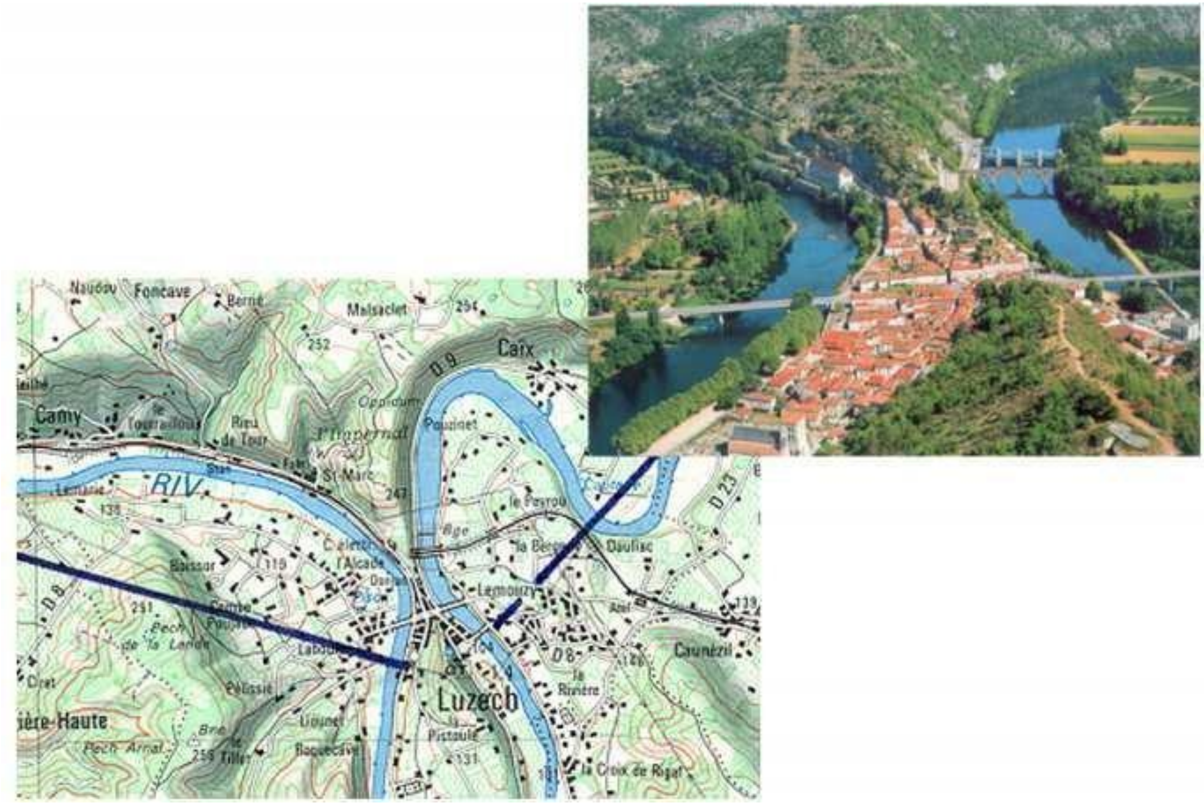


Figure I.1: Representation of an area of land on a map

This science also determines the position and altitude of any point located in a given area, whether it is the size of a continent, a country, a field or a street body. These details can be:

- Natural: Watercourses, rocks, woods, rivers, mountains, fields, etc. Artificial:
- Road, railway, building, embankment, canals, ports, roads, etc. Conventional:
- Municipality, departmental boundary, etc.

I.1.4. Photogrammetry: Photogrammetry is the technique that allows us to measure and represent an object, a construction or a land using aerial photographs (**Figure I.2**) or satellite images (Teledetection) (**Figure I.3**).

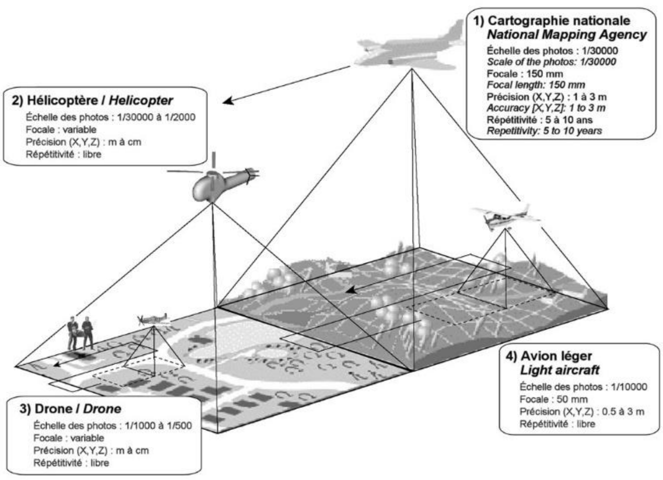


Figure I.2: *Aerial photographs*

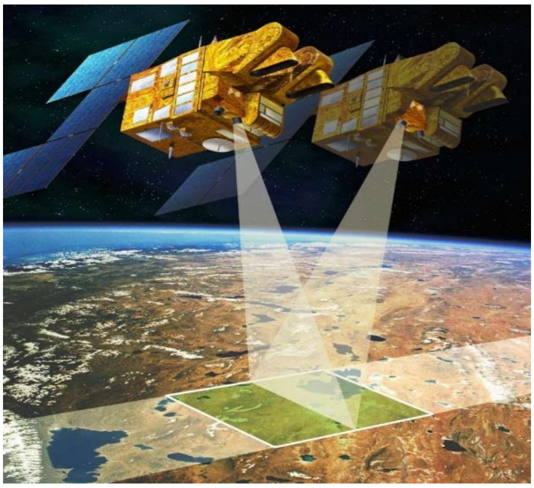


Figure I.3: *Satellite images (Teledetection)*

I.1.5. Mapping: Cartography is the set of scientific, artistic and technical studies and operations based on direct observation or the exploitation of documentation, with a view to developing maps, plans and other modes of expression.

I.1.6. Planimetry: Execution and exploitation of observations and measurements which make it possible to represent on a horizontal plane the details located on the surface of the ground (method for measuring the flat surfaces).

I.1.7. Altimetry: execution and exploitation of observations which lead to the representation ground relief (method for measuring altitudes).

I.1.8. Outline: it is the set of points known in planimetry and altimetry with homogeneous absolute precision.

I.1.9. Scale: The scale of a plan or map is the ratio expressed in the same unit between a length measured on the map and the same length measured on the ground. In topography, it is expressed in the form of **1/*Scale***. The larger the denominator, the smaller the scale. An image given in a map in the form below (**Figure I.4**), allows you to get an idea of the order of magnitude.

Figure I.4: Small extract from a 1/25000 topographic map

Examples:

a) if we measure a distance of 2.5cm on map, the distance on the ground is 25m.

The scale will be: 0.025/25 = 0.001 = **1/1000**

b) if we measure a length of 7.4 cm on a plan at a scale of 1/500, the actual length will be:

7.4 x 500 = 3700 cm = **37 m**

c) Conversely, if a length measured on the ground is 85m, it will be represented on plane at scale of 1/200 with : 85/200 = 0.405m = **42.5cm**

Scale	Purpose (Objective, Aim)
1/1000000, 1/500000	Geographic maps (Small scale maps)
1/250000, 1/100000	Topographic maps (Small scale maps)
1/50000,1/25000,1/20000	Topographic map (Medium scale map)
1/10000	Topographic map (Large scale map)
1/5000	General topographic study plan, Urban planning plans
1/2000	Land use plan (POS) and descriptive plot plans
1/1000,1/500	Land plot plans, Urban cadastral plans
1/200	Road plan, Subdivision implementation documents
1/100	Cleanliness plan, site plan
1/50	Architect's plan, formwork...

The creation of a plan or map encompasses several sciences:

- **Geodesy** which studies the shapes of the earth and makes it possible to determine the geographic or rectangular coordinates of a certain number of points serving as a framework for topographic surveys.

- **The topography** which uses graphic methods of raising or transferring plans.

- **Topometry** which groups together all the measurements and calculations specific to the establishment of plans. Topometry is a part of topography.

- **Topographic surveys** which allow the establishment of plans subsequently used by the State Public Works Engineers. These plans will be presented in the form of a preliminary project, a site plan and a detailed plan.

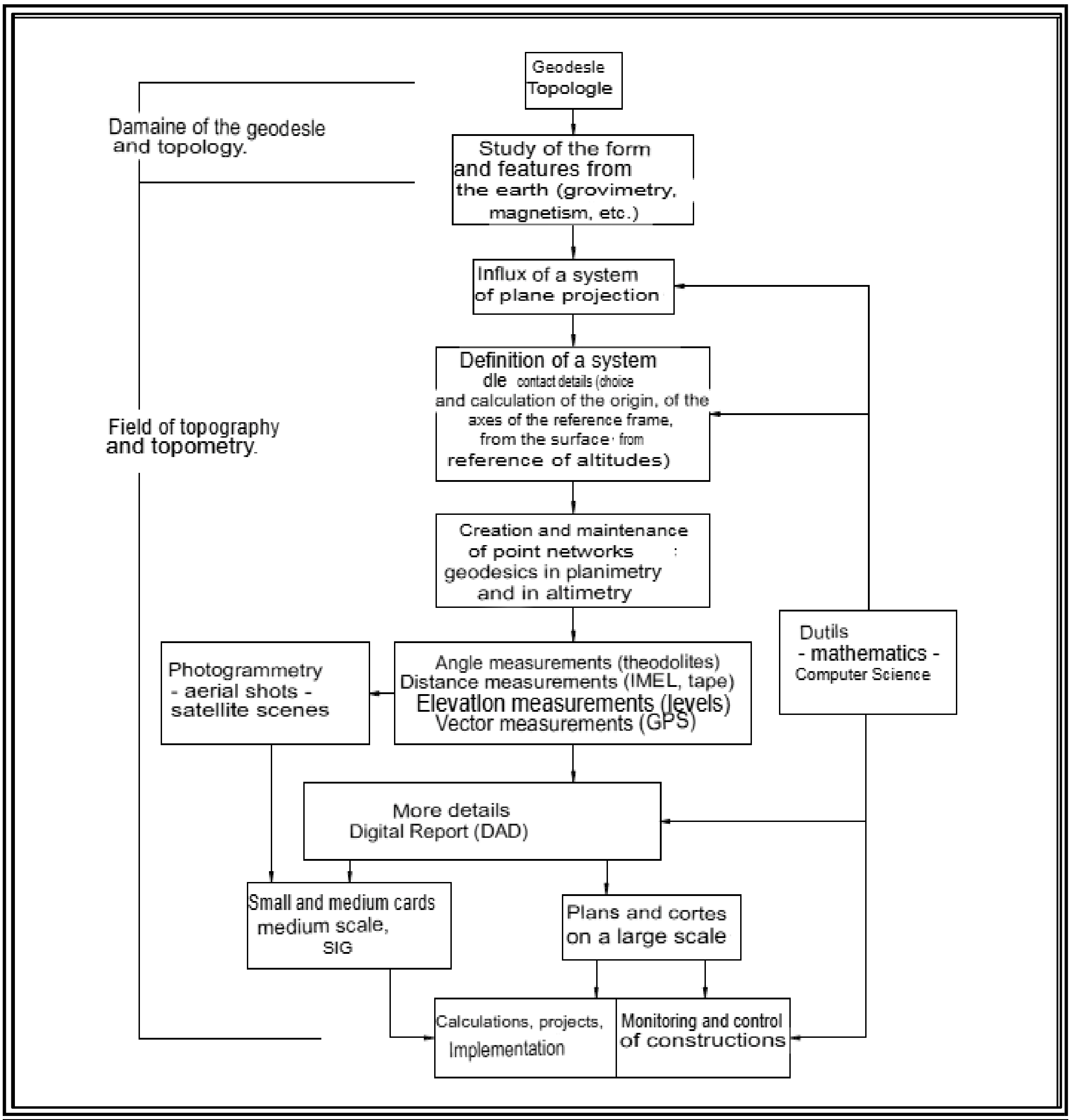


Figure I.5: Flow chart of topographic operations

I.1.10. A plan is a graphic representation of a restricted portion of the earth obtained by orthogonal projection onto a flat surface. The details are represented to scale.

I.1.11. A map is a conventionally reduced representation of a certain portion of land at a small scale. Such as geographic maps, topographic maps and road maps whose scales vary from 1000000 1 to 25000. The map also allows to show the variations and developments of phenomena in time, as well as their factors of movement and displacement in space.

- ***Reading a map***

North, by convention, is always at the top of the map (**Figure I.6a**, **Figure I.6b**). A topographic map represents a certain region. This reproduction is **an oriented drawing** and according to the convention, the **North** is still on top, the **South**, below, the **West** on the left and the **East** on the right (**Figure I.7**). The direction of north is indicated by the meridians (**Figure I.6b**) which are represented by two or three very thin vertical lines running across the map from top to bottom.

Figure I.6a: Topographic map (North at the top of the map)

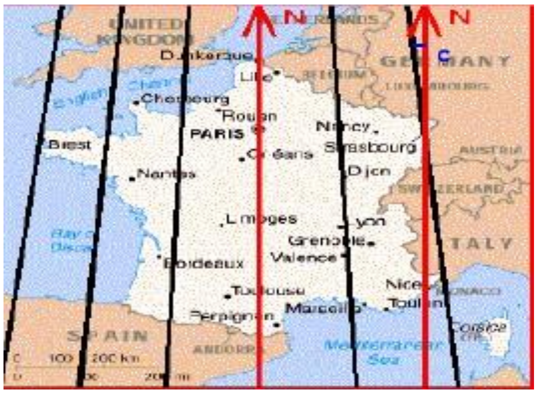


Figure I.6b: Topographic map (direction of north indicated by the meridians)

Magnetic north, indicated by the magnetic needle of a compass, and geographic north, called true north, correspond to the point of convergence of the meridians: the north pole.

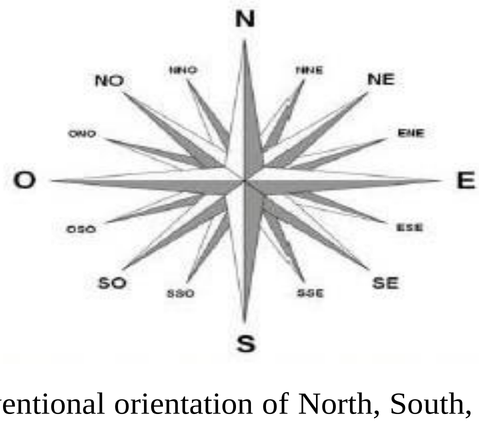


Figure I.7: Conventional orientation of North, South, West and East

I.1.12. The locations

Development projects generally established from topographic data, which must be carried out on the ground. To do this, the topographer ***implant*** in other words ***puts in place on the ground***, the planimetric and altimetric elements necessary for this achievement.

I.1.13. Development projects

These are projects that modify the planimetry and altimetry of a piece of land: land development, subdivisions, road and railway lines, water management: drainage, irrigation, canals, ditches, etc.

I.1.14. Monitoring and control of works

Once built, civil engineering structures often require monitoring at more or less regular intervals depending on their purpose: dikes, bridges, subsidence, etc. The corresponding topographical work results in measurements of the variations in coordinates (**X, Y, Z**) of rigorously defined points, followed by various digital processing noting a state and possibly predicting an evolution.

I.2. PLACE OF THE CIVIL ENGINEER IN TOPOGRAPHY

The civil engineer not specialized in topography must be able to:

- Understand any document established by a topographer,
- Being able to communicate with a surveyor,
- Know-how of topography operations,
- Monitor the proper execution of a survey,
- Possibly receive the work carried out,
- Handle topographical equipment.

I.3. The Different Topographic Measuring Devices

Theodolite: It is a geodesy instrument used to measure reduced angles to the horizon and zenith distances.

Figure I.8: Optical and electronic Theodolite

Tacheometry: It is an instrument with the functions of a theodolite, plus a distance measuring method (stadimetric or electronic).

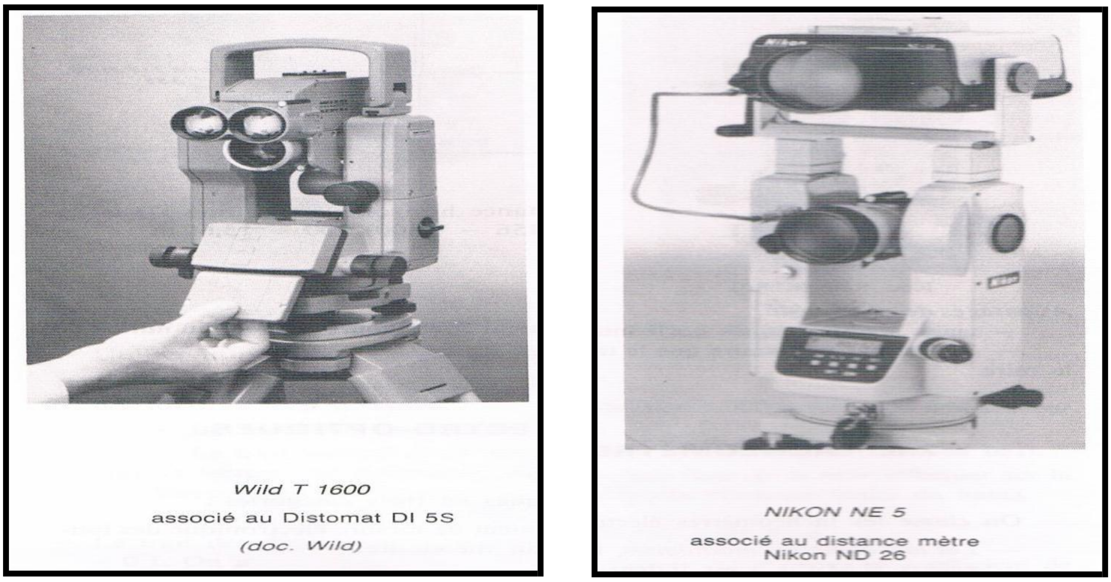


Figure I.9: Modular tachometer

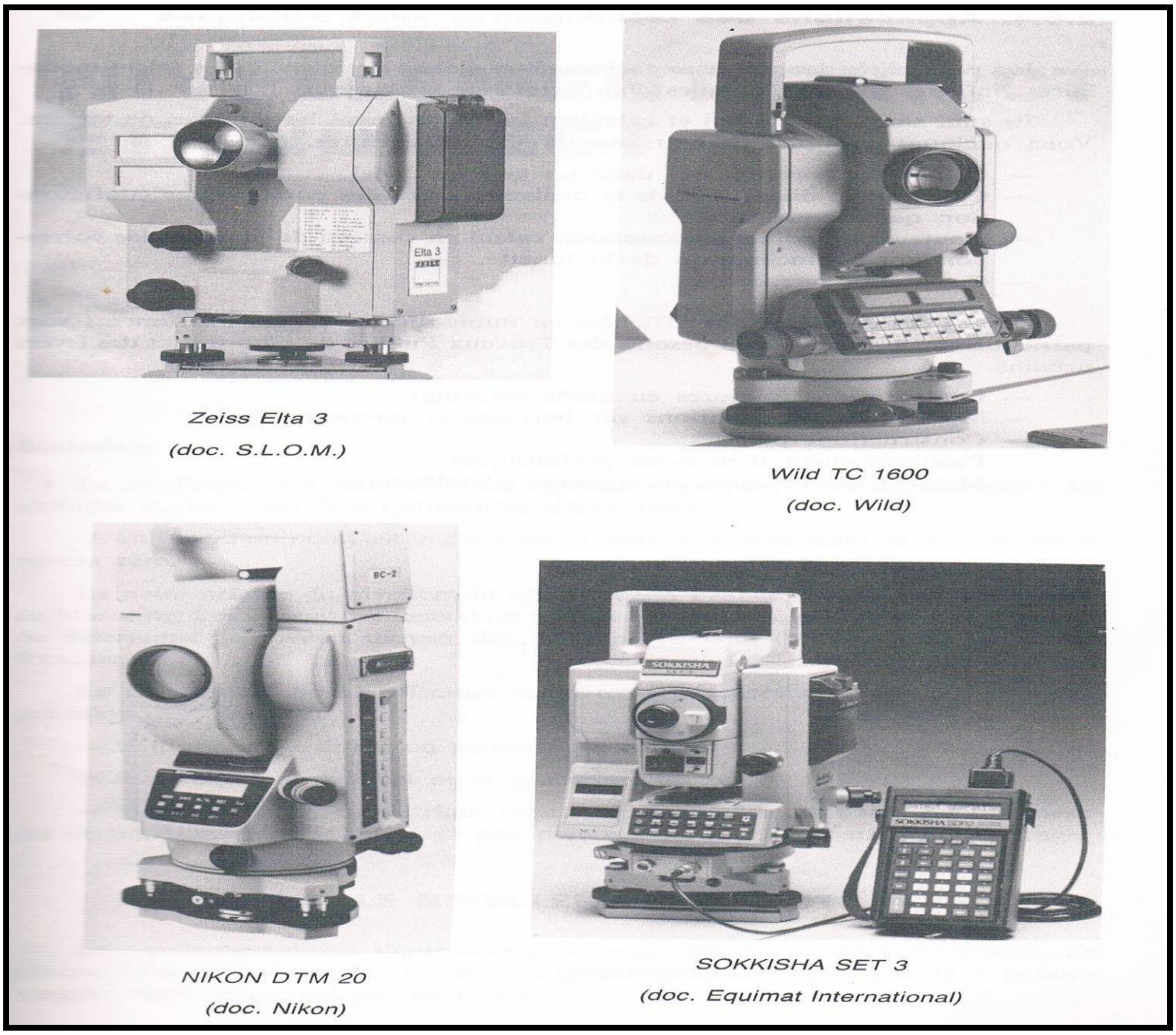


Figure I.10: Compact electronic tachometers

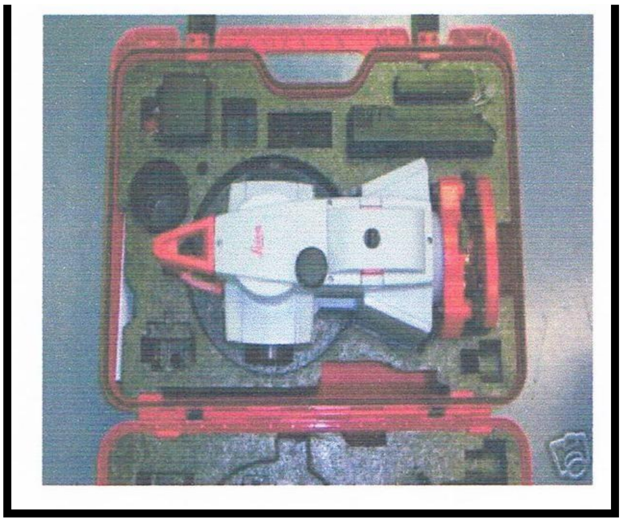
I.3.1. **Theodolite** type (020A) old model**:**

Theodolite type (**020A**)

Theodolite Tripod

Figure I.11: Theodolite type (020A)

I.3.2. **Tacheometer:** type (Leica TCR110,Version101.2401) The new model



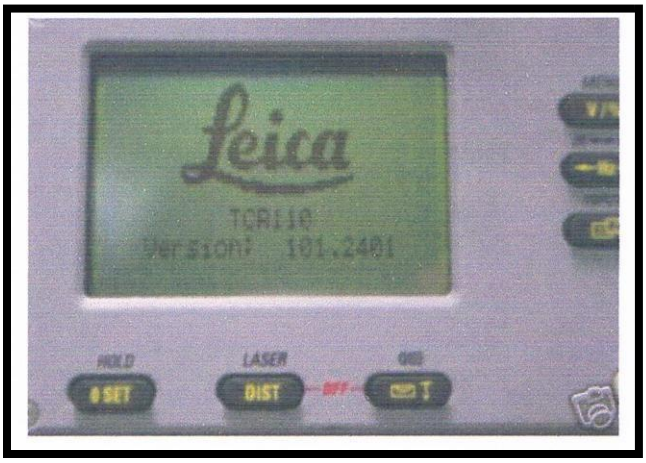


Figure I.12: Typical electronic Tachometer (**Leica** TCR110 Version 101.2401)

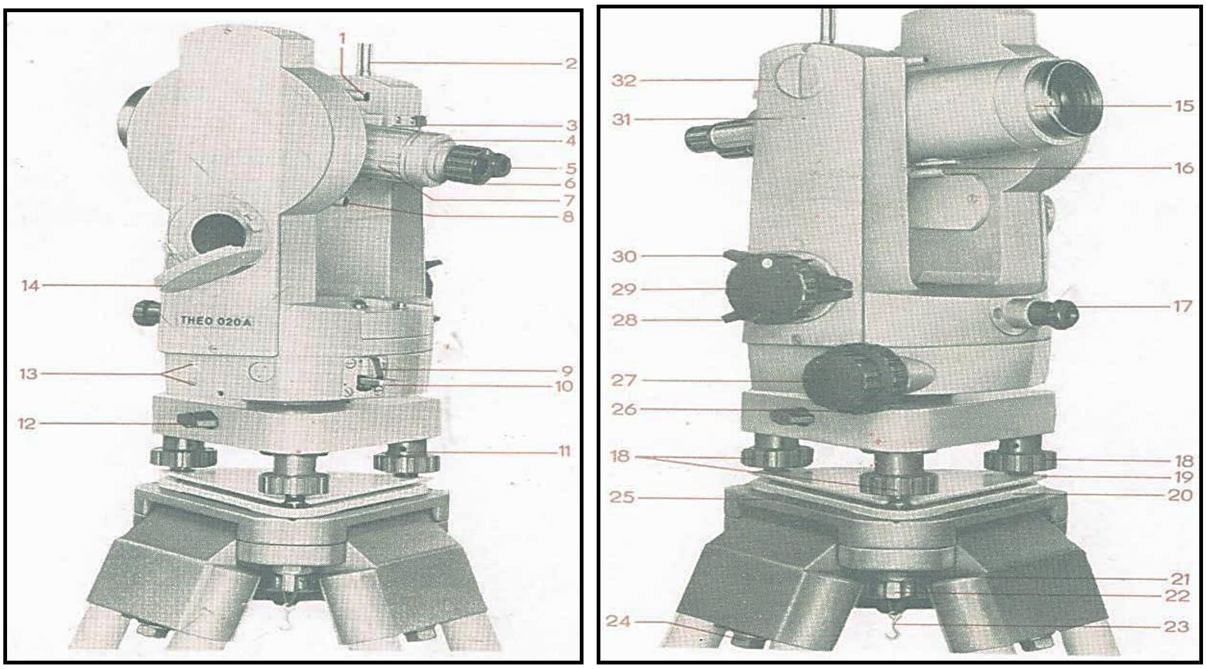


Figure I.13: Description of Theodolite type (020A)

Description:

1. Optical view finder for coarse sighting.
2. Compass receiving pin.
3. Stop for compasses.
4. Protective cover for adjustment screw of the telescope reticle.
5. Microscope eyepiece for reading graduated circles.
6. Telescope eyepiece with graduation in diopters.
7. Focusing ring for the sight image.
8. Optical viewfinder for coarse sighting, with centering point for centering under ceiling marks.
9. Stop lever for the repeater clamp.
10. Locking lever for the repeater clamp.
11. Adjustment hole to make the movement of the leveling screws more flexible or harder.
12. Forced centering pivot clamp for fixing the instrument in the base.
13. Advantage screw for the drive pin of the board.
14. Illumination mirror.
15. Telescope objective.
16. Button for adjusting the illumination of the telescope reticle.
17. Eyepiece of the optical plummet.
18. Leveling screws for the horizontality of the instrument.
19. Elastic plate of the base (with M16 and 5/ inch screw thread).
20. Base plate of the base.
21. Hexagonal screws to make the movement of the tripod legs freer or harder.
22. AS1 fixing screw for fixing the instrument on the tripod.
23. Hook.
24. Hexagonal screws for tightening the wooden legs of the tripod.
25. Tripod head plate.
26. Fine height adjustment.
27. Fine azimuth adjustment.
28. Azimuth movement locking lever.
29. Button for retracting the horizontal or vertical circle image.
30. Vertical movement locking lever.
31. Marking point for the height of the secondary axis.
32. Cover (with, below, adjustment screw for locking in height).

I.4. Field of use of Theodolite

The theodolite (tacheometry) has worked well for all topographic and engineering work requiring an average accuracy of ±10°° for a direction measured once in both positions of the telescope. Its main areas of use are:

- Polygonation
- Complementary triangulation
- Staking work
- Precision Tacheometry (Cadastre)
- Topographic Tacheometry (Board)
- Underground measurements
- Measurement of astronomical connections
- Leveling technique (leveling bubble)

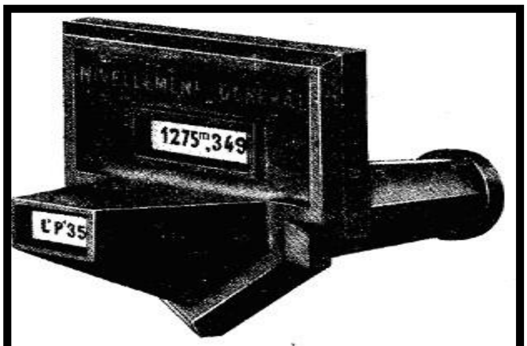
I.5. Boundary plan

It is a plan that consists of specifying the project on the following land of the given terminal. It is carried out by the urban planning department, and it includes all the details raised the indication of the polygon, the captured points and the designation of the curves at the equidistance of 1m.

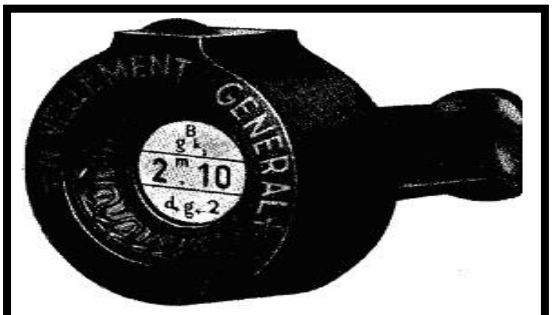
I.6. Topographic survey

It is a designation of the land concerned which indicates the constraints encountered such as housing to be demolished, backfill displacement led.

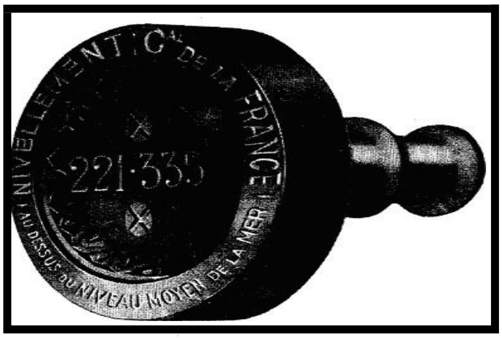
I.7. The Different Types of Stations (Reference)



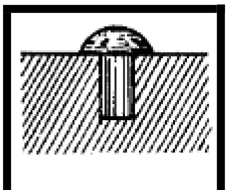
Repère à console (*cl. I.G.N.*)



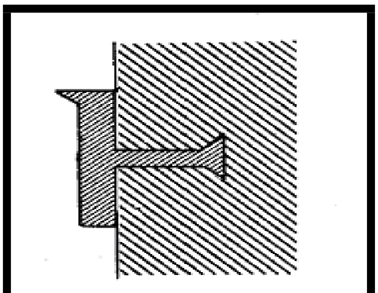
Repère cylindrique (*cl. I.G.N.*)



Repère Bourdaloué (*cl. I.G.N.*)



Rivet.



Repère Ponts et Chaussées.

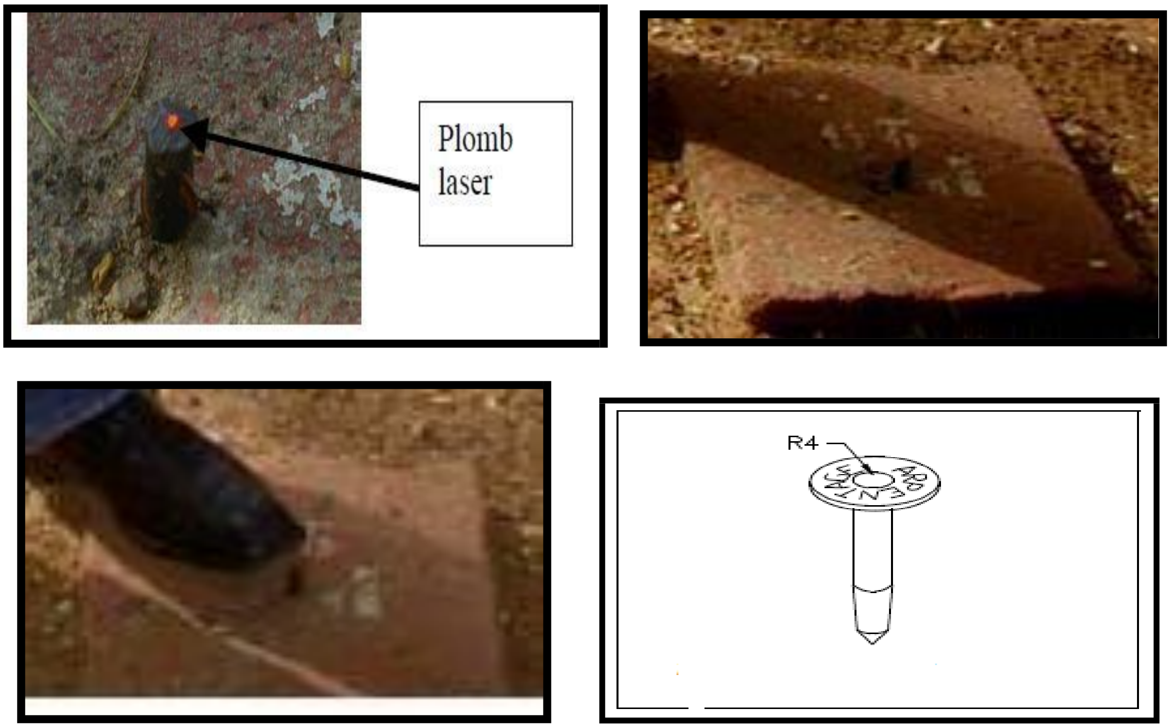


Figure I.14: Different Types of Landmarks

I.8. Typical example of a ground plan at a scale of 1/500

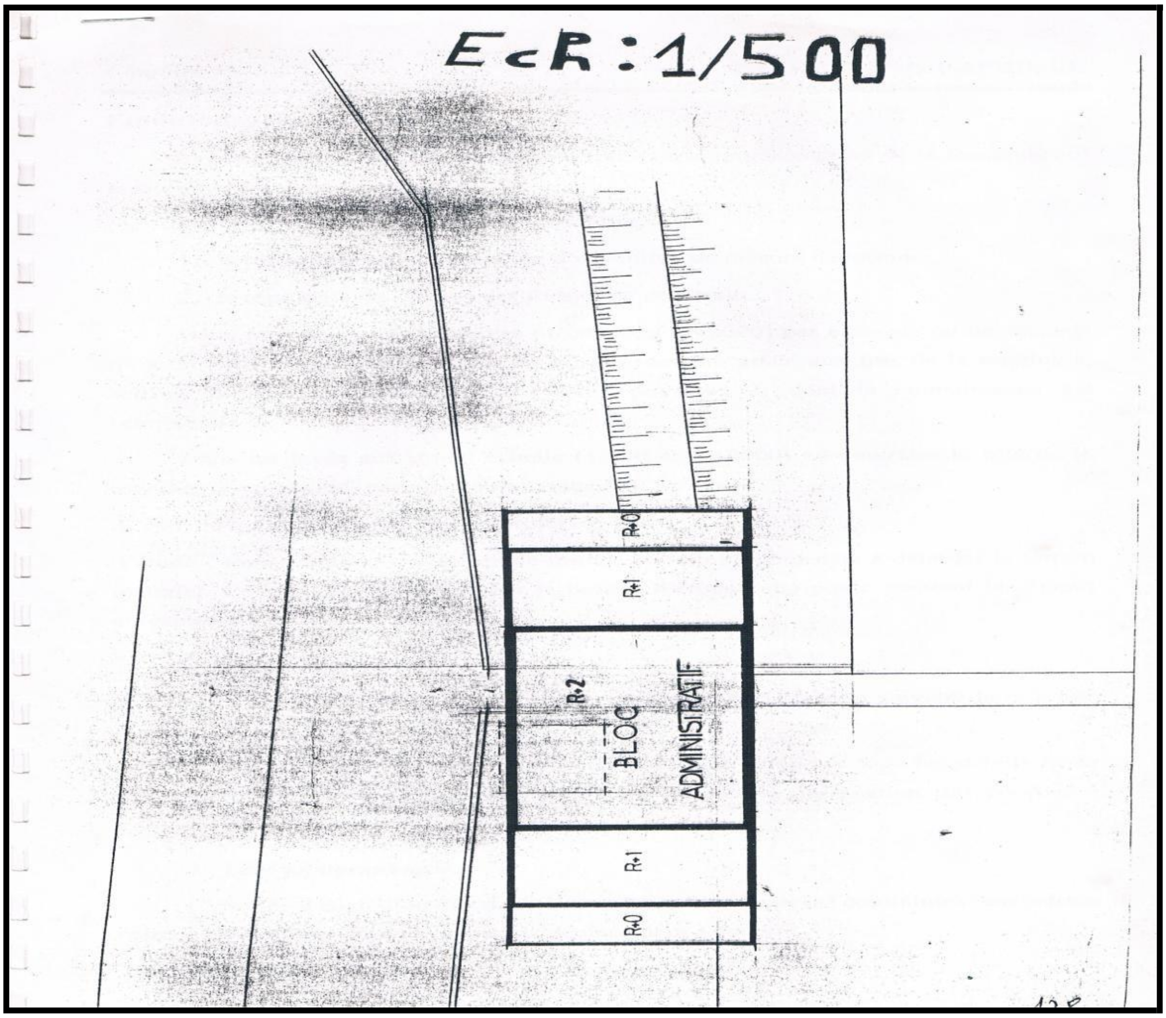


Figure I.15: Example of a ground plan

I.9. Contour lines

A contour line or altitude isopleth is in cartography, a line formed by the points of the relief located at the same altitude. To draw the contour lines, it is necessary to divide the terrain into "slices" to then be projected onto paper. The thickness of the slices is constant, called equidistance of the curves and is indicated in the cartridge of the map.

Every five or ten curves, a master curve is drawn in bold, with the indication of its altitude. The numbers of this curve are always written in the direction of the ascent (**Figure I.16**).

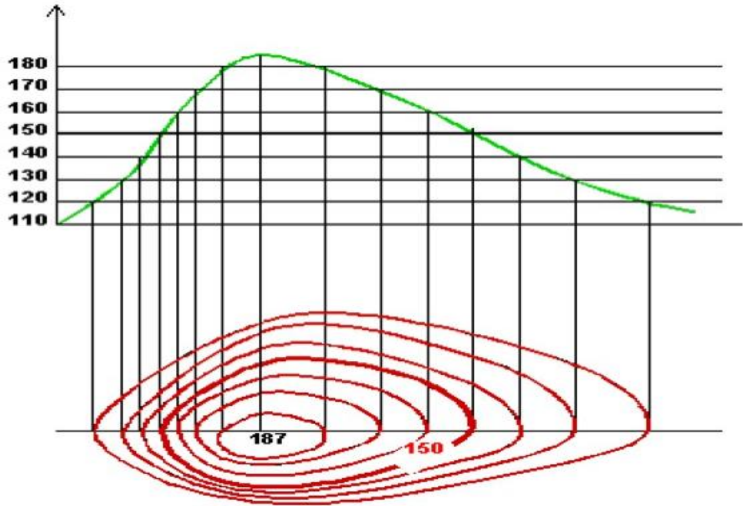
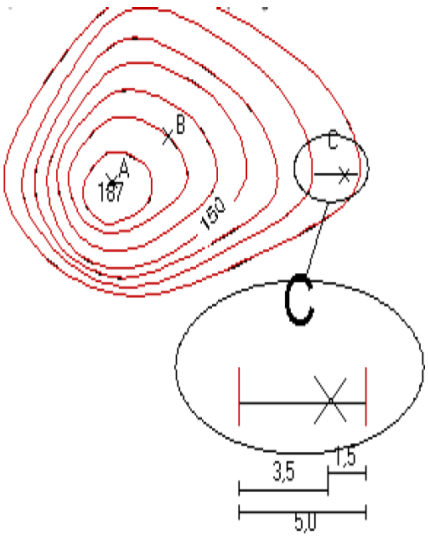


Figure I.16: Principle of determining contour lines

Finally, the points marked in **Figure I.16** (187 m) give the altitude of the particular points which complete the contour lines.

I.10. Calculation of the altitude of a point

To calculate the altitude of a point, you must first study the contour lines and the elevation points. Three points **A**, **B** and **C** have been indicated on the diagram of the **Figure I.17**.



- The point **A** is on a listed point: its altitude is 187 m

- The point **B** is on a contour line: its altitude is 170 m

- The point **C** is located between two contour lines… it's more complicated!

Figure I.17: Principle of calculating the altitude of a point

As **C** is located between two contour lines, you must start by drawing the shortest line between the two curves and passing through the point **C**: This is the line of greatest slope. Then, we must measure the length of this line. Here it is 5m. Then we should measure the distance between the lowest curve (here 120m) and the point, and we will find 1,5m in the example. Finally, a rule of three allows us to calculate the height difference. In the example of the **Figure I.17**, if 5mm represents an elevation of 10m (the difference in altitude between two curves, i.e. the equidistance), then 1,5m will correspond to 1,5.10/5 = 3m. The altitude of the point is therefore 120+3=123 m.

I.11. Calculation of the percentage of a slope

To calculate the slope of a path, simply apply the following formula:

$$slope\ (\%) = \frac{Difference\ in\ height\ (m)}{Length\ (m)} \times 100 = \frac{Rise\ (m)}{Run\ (m)} \times 100$$

So a slope is equal to 100% when the height difference is equal to the length traveled. The height difference is defined as the total height between the arrival point and the starting point.

Example 1:

Let's say two points on a map. **A** is at an altitude of 450 m and **B** at 600m. The distance between **A** and **B** is 4,5 km, i.e. 4500 m. The calculation of the rise is the calculation of the difference in altitude between the two points **B** and **A**. Elevation:
B–**A**=>600m – 450m = 150m.

Slope between point **A** and the point **B**: **slope** = (150/4500).100=3,33%. Therefore, the percentage of the slope should not be confused with the angle of elevation (expressed in degrees) of this same slope:

On the maps we have the flat distance, that is, the horizontal distance; it does not take into account the relief of the ground. We therefore do not know the real distance traveled during the elevation (**Figure I.18**) represented here by the hypotenuse **R**.

On nearly flat ground or for elevation over a long distance, the difference will be minimal. See the difference between line **A** and line **B** in the graph below (**Figure I.18**). If we line them, the **B** would be much larger than **A**.

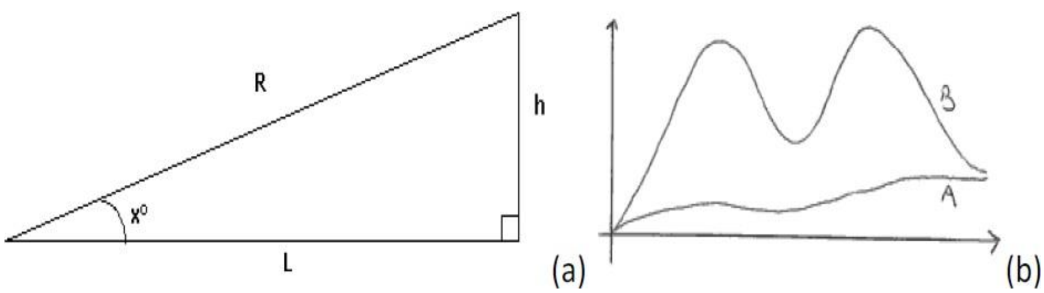
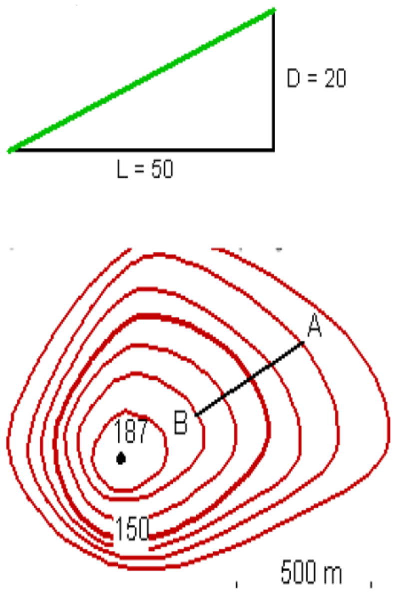


Figure I.18: Calculation of the difference in height between two points **A** and **B**.

To know the actual distance to be covered, we must use the Pythagorean theorem: $L^2 + h^2 = R^2$. After a simple calculation, we obtain: **R = 4502 m**. Concerning the angle of elevation, we have the necessary tool in mathematics: the tangent. $tan(x°) = h/L$. In reverse, to find the angle (x°): $arctan(h/L) = x°$.

Example 2:

The calculation of the slope of the run illustrated by the triangle of the **Figure I.19**, leads to the following result:



Slope: P = (20/50).100 = 40%.

A 40 m rise is the same as travelling 100 m in length (run).

- Going from **A** towards **B**, we go from curve 130 to curve 170: we have therefore climbed 40 m. The length of the path as the crow flies is 500 m. Therefore, the slope is:

P = (40/500).100 = 8%.

- Going from **A** towards **B**, (a climb): the slope is **+8%**
- Going from **B** towards **A**, (a descent): the slope is **–8%**

Figure I.19: Principle of calculating the percentage of a slope.

Table I.1: Measurement units

Lengths			
Submultiples	Decimeter (dm)	0.1m	10^{-1}m
	Centimeter (cm)	0.01m	10^{-2}m
	Millimeter (mm)	0.001m	10-3m
	Micron (µ)	0.0000001m	10^{-6}m
multiple	Decameter (dam)	10m	10^{1}m
	Hectometer (hm)	100m	10^{2}m
	Kilometer (km)	1000m	10^{3}m
Area, surface areas or surfaces			
Submultiples	Square decimeter (dm^2)		
	Square centimeter (cm^2)		
multiple	Square decameter (dam^2)	100 m^2	$10^{2}m^2$
	Square hectometer (hm^2)	10000 m^2	$10^{4}m^2$
	Square kilometer (km^2)	100 ha	
Angles			
Submultiples	Decigrade (dgr)	0.1gr	10^{-1}gr
	Centigrade (cgr)	0.01gr	10^{-2}gr
	Milligrade (mgr)	0.001gr	10^{-3}gr
	Decimilligrade (dmgr)	0.0001gr	10^{-4}gr

Table I.2: Correspondence between the different units of measurement of some angles **features**

400 gr	**360°**	**6.28 rad**	2π rad	**Circumference (Circular)**
200 gr	**180°**	**3.14 rad**	π rad	**Flat angle**
100 gr	**90°**	**1.57 rad**	$\pi/2$ rad	**Right angle**
63.66 gr	**57.30°**	**1 rad**		
1.111 gr	**1°**			
1 gr	**0.9°**	**0.0157 rad**		

I.12. Conclusion

The determination of coordinates and various characteristics of points in space occupies an important place in most studies for environmental purposes. In this chapter, the objective of these determinations was generally the study of the geographical aspect of the interrelationships between the various parameters or indicators recorded.

The purpose of this chapter was to cover all the sciences and techniques available to design offices to acquire both geometric and thematic information on three- dimensional objects that make up our urban and natural landscapes. It is not a question of training seasoned topographers, but rather of providing a basic technical culture to allow, on the one hand, a dialogue with professionals and, on the other hand, when necessary, the implementation of a simple measurement protocol.

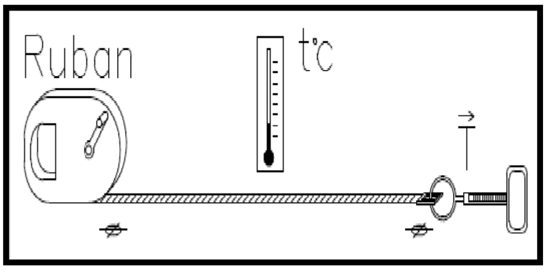
In this chapter, we have reviewed the basic geodesic concepts necessary for understanding this course. Calculation of altitudes and slope percentage have been discussed with examples and applications.

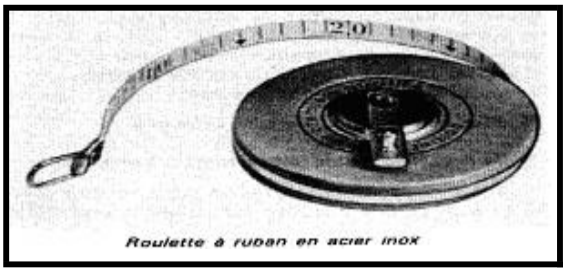
Chapter II: Measurement of Distances

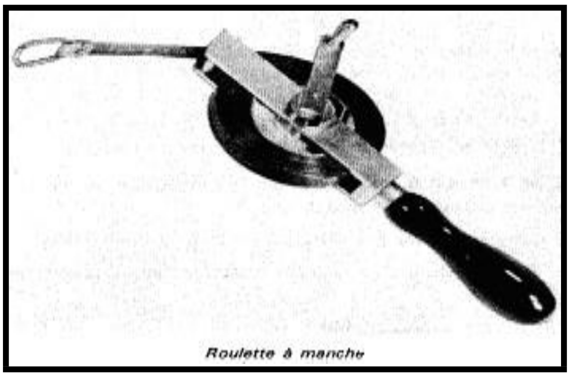
II.1. Introduction

Measuring distances has always been a problem for the topometer. There are several methods (or techniques) for calculating a distance. Depending on the equipment available, the chosen precision and the regularity of the terrain, we distinguish:

- Tape (string) measurements; Direct measurement
- Measure with a stadia; (Indirect measurement) Measurement
- With an auxiliary base; (Direct measurement) Stadiametric
- Measurement; (Indirect measurement)
- Measurement by an IMEL (Electronic Length Measuring Instrument); (Indirect measurement)
- Measurement by GPS (Global Positioning System, satellite positioning). (Indirect measurement)







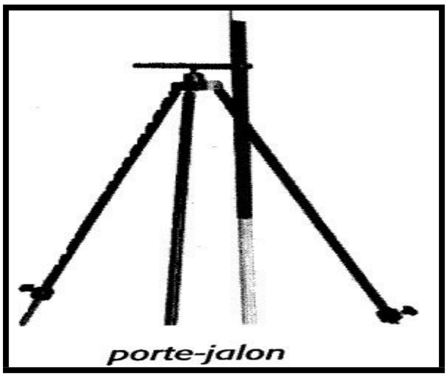


Figure II.1: Tape and chain measurements

II.2. Measure by tape or string

Chain measurement is the most classic and used way to determine distances. Its main disadvantages are:

- They are dependent on the terrain (uneven or not, steep or not, etc.).
- They are limited in range (the tapes commonly used are limited to 100 m).
- The accuracy of the measurement is also limited and depends heavily on the operators.
- The measured reading must be corrected to have an accurate reading (calibration correction, temperature correction, voltage correction, chain correction).

II.3. Measurements on regular ground

II.3.1. Regular and horizontal terrain

If the ground is even and has a slight slope (less than 2%), it is possible to place the tape on the ground and consider that the horizontal distance is read directly. The precision that can be obtained on a measurement is at best of the order of ± 5 mm at 50 m.

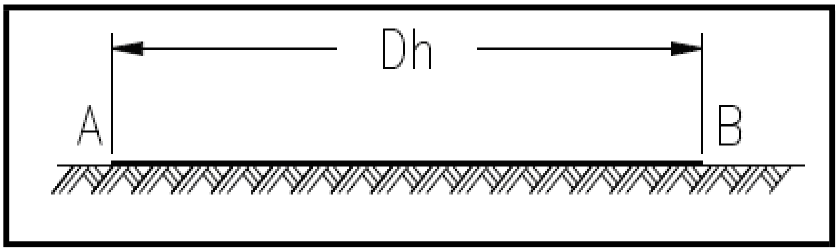


Figure II.2: Tape measure on horizontal distance

II.3.2. Regular sloping terrain

If the terrain is not perfectly horizontal, it must be considered that the distance is measured along the slope. To know the horizontal distance precisely, it is therefore necessary to measure the difference in height **ΔH** between **A** and **B** or the slope **p** of **AB**, the horizontal distance **Dh** will therefore be:

Next by the difference in level **ΔH**: **Dh** $= \sqrt{Dp^2 - \Delta H^2}$

Next by the slope **p**: **Dh** $= Dp*\cos(i) = Dp*\sqrt{\frac{1}{1+tg^2 i}} = \frac{Dp}{\sqrt{1+p^2}}$

With: $p = \tan(i)$ and Dp is the distance calculated by the tape on the slope.

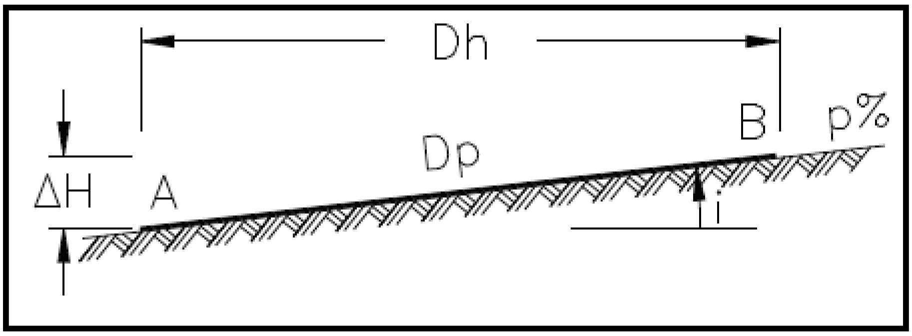


Figure II.3: Tape measure on regular sloping distance

Example:

The distance measured between points **A** and **B** along the slope is 37.25 m, and the slope of line **AB** measured with the Clisimeter (Clinometer) is equal to 2.3%. Calculate the horizontal distance **Dh** and the difference in height **ΔH**.

Answer:

$$Dh = 37.25 \times \frac{1}{\sqrt{1+0.023^2}} = 37.24m \quad \text{and} \quad \Delta H = \sqrt{37.25^2 - 37.24^2} = 0.86m$$

II.4. Measurements on uneven or steep terrain

The tape cannot be stretched on the ground because of its undulations. In addition, the slope (or the distance to be chained) is such that the distance **Dh** cannot be directly measured.

II.4.1. Measurement by horizontal jumps

In the case where the tape cannot be stretched on the ground because of its undulations, the horizontal spring method is used. It requires the use of a spirit level and two plumb lines in addition to the chain and the surveying cards (or markers). Its implementation is long and the process is not very precise. **$Dh = Dh_1 + Dh_2 + Dh_3$**.

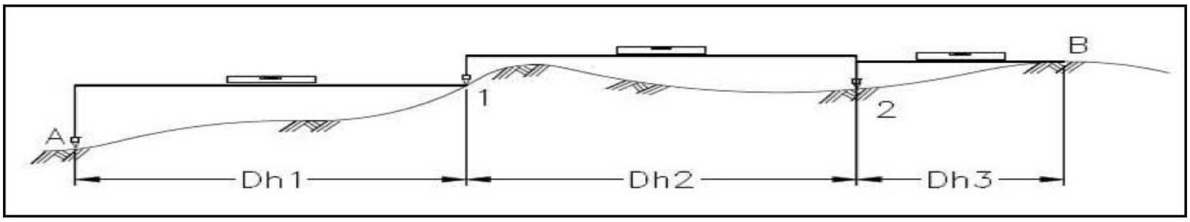


Figure II.4: Tape measurement by horizontal steps

II.4.2. Measurement in suspended mode

It consists of using a wire made of stable material (Invar) which is stretched above the ground by constant tensions supported by weights. The operator must measure the difference in level **ΔH** between the vertices **A'** and **B'** of the wire suspension tripods in order to calculate the length **Dh** in function of the measured inclined distance **Di**: $Dh = \sqrt{Di^2 - \Delta H^2}$

Di: the inclined distance measured between **A'** and **B'**.

ΔH: the difference in height between point **A'** and **B'**.

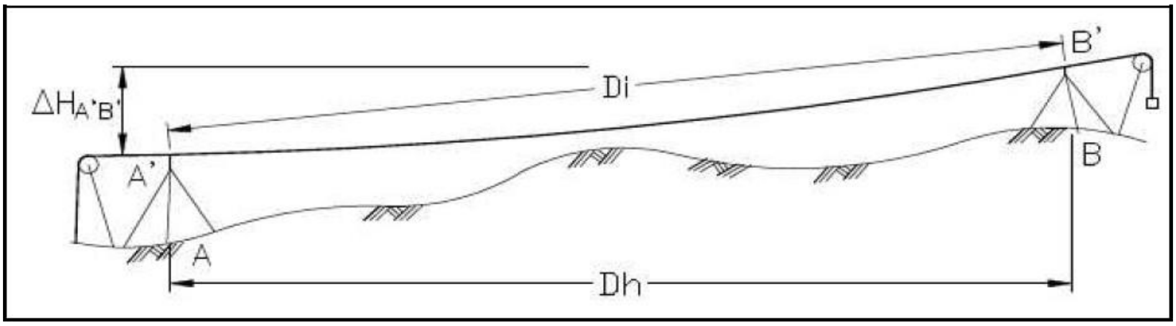


Figure II.5: Measurement in suspended mode

This method gives satisfactory results in precision measurement but it takes a long time to implement.

II.5. Measure with a stadia

This type of parallactic measurement requires the use of a theodolite and a stadia. A stadia is a ruler with two sights (triangular or circular) whose spacing is known (usually 2 m). There are Invar stadia for high-precision measurements. The stadia is equipped with a spherical level and a sight to adjust its perpendicularity with respect to the line of sight A' B'. The operator has a theodolite at A (or an alignment circle) and in B a horizontal stadia perpendicular to the distance to be measured AB. Height adjustment is unnecessary:

The measured angle is the angle projected onto the horizontal plane.

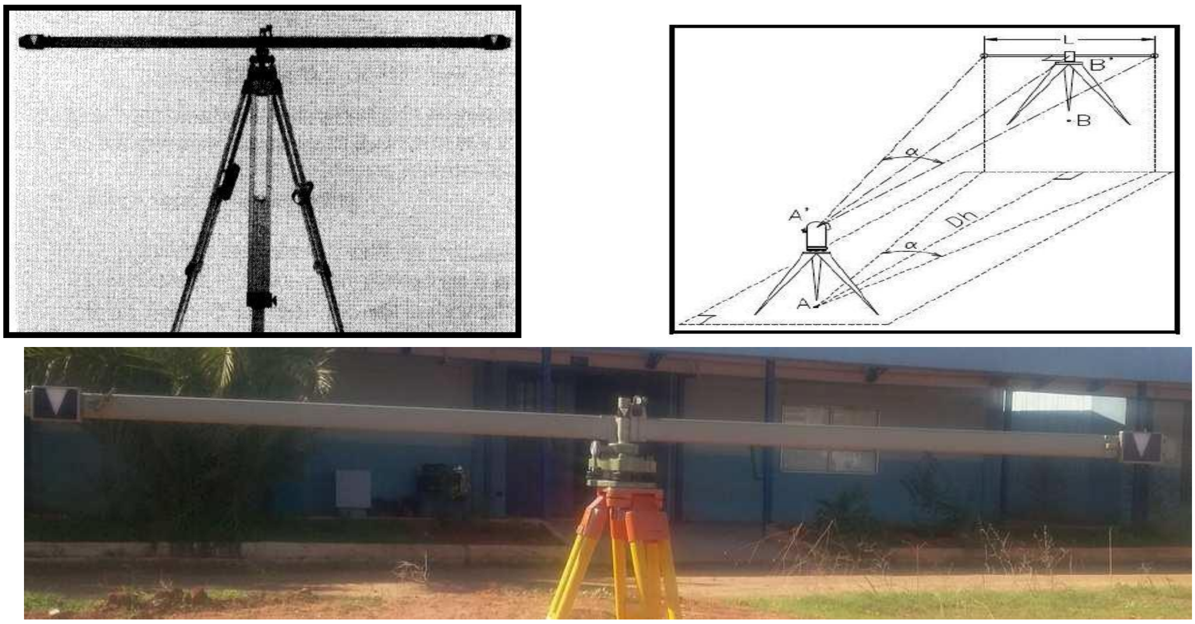


Figure II.6: Measurement with a stadia

Projected onto the horizontal plane passing for example through point **A**, we obtain:

$$tg\frac{\alpha}{2} = \frac{L}{2Dh} \Rightarrow Dh = \frac{1}{tg\frac{\alpha}{2}} = \cot g\frac{\alpha}{2} \qquad \text{L = 2m (cas général)}$$

$$\frac{1}{\sin\frac{\alpha}{2}} = \frac{Dh}{\cos\frac{\alpha}{2}} \quad d'où \quad Dh = \frac{\cos\frac{\alpha}{2}}{\sin\frac{\alpha}{2}} = \cot g\frac{\alpha}{2} \Rightarrow Dh = \frac{1}{tg\frac{\alpha}{2}} \cot g\frac{\alpha}{2}$$

<u>**Example**</u>**:** calculate the horizontal distance between the stadia **A** and the points aimed at according to the observation of the horizontal angles between the two lights of a horizontal stadia.

Station	Trageted Points	Origin of the limbus	Position	Seer	Reading	Angle (α)
A	1	0	CG	G	0.0842	3,2402
				D	3,3244	
A		100	CD	G	100,0114	3,2401
				D	103,2515	
A	2	50	CG	G	50,0428	3,2399
				D	53,2827	
A		150	CD	G	150,0111	3,2404
				D	153,2515	
						12,9606

α=12.9606/4 = 3.2402 gr , SM = cot*g*(*α* / 2) = 39.287 , Dh=SM= 39.287 m

II.6. Measurement with an auxiliary base

The basic principle of this method is also used in the measurement of altitude (or coordinates) of an inaccessible point. This method requires the use of a tape and a classic theodolite. It involves transforming the measurement of a long distance into a measurement of a short distance associated with angular measurements that are all the more precise the further away you aim. We therefore create a base **AB** whose length is perfectly known.

by placing a theodolite at A then at B, we measure the angles (PAB) and (PBA).

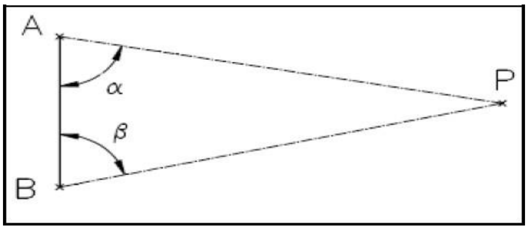


Figure II.7: Use of an auxiliary base

Solving the triangle PAB allows us to obtain:

$$D_{AP} = D_{AB}\frac{\sin\beta}{\sin(\alpha+\beta)}$$

II.7. Stadiametric measurement

Stadiametry is a less precise method than the previous ones. It allows the indirect measurement of a horizontal distance by reading the length intercepted on a staff by the stadiametric wires of the aiming reticle. Point **A**, the optical center of a theodolite, is located vertically from the point stationed at **S**; the operator aims at a staff placed at **P** and takes the reading intercepted by each wire on the staff, i.e. $\mathbf{m_1}$ and $\mathbf{m_2}$.

The horizontal distance can be expressed by the following formula:

$$Dh = \frac{m_2 - m_1}{2tg\,^{\alpha}/_{2}} \times \sin^2 V$$

If the aim is horizontal (V=100 gr) we obtain:

$$Dh = \frac{m_2 - m_1}{2tg\,^{\alpha}/_{2}}$$

With: α stadiametric angle.

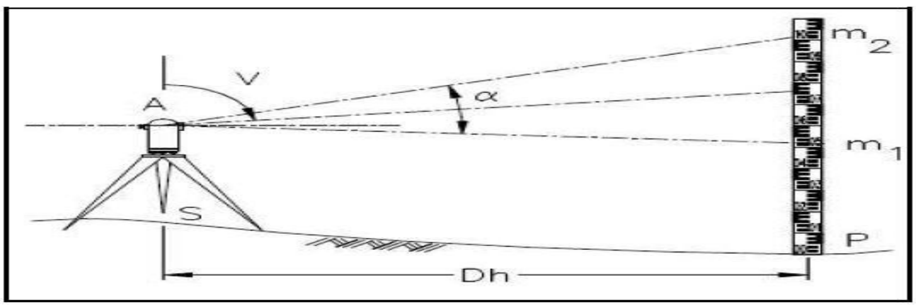


Figure II.8: Stadiametry measurement

II.7.1. Stadiametry constant angle

If the angle α is constant in the device used, we have: $Dh = k.(m_2 - m_1).sin^2V$

With $k=1/2tg(\alpha/2)$: is called the stadiametric constant. It is generally equal to 100; that is why the expression of **Dh** becomes: $Dh = 100(m_2 - m_1).sin^2V$. In the case of a level (device used for calculating differences in level)

V = 100 grade $\quad$ $sin^2 100 = 1$ $\quad$ $Dh = 100(m_2 - m_1)$

- **Precision obtained on the measurement of Dh with a level**

For a level, V=100 gr, hence: $Dh = 100(m_2 - m_1)$ Calculation of the standard deviation on Dh gives allows to obtain: $\sigma_{Dh}^2 = 2\ (k.\sigma_m)^2$

σ_m: is the standard deviation on knowledge of readings ***m***: it can be estimated at best at 1 mm up to 35 m for a classic level or theodolite. Which implies that the precision on **Dh** is of the order of ±14 cm at 35m.

<u>**Reading example**</u> from a construction site level perspective: the operator reads:

m_2**= 7.60 dm** and m_1**= 5.69 dm** and we obtain

Dh =19.1 m±14 cm.

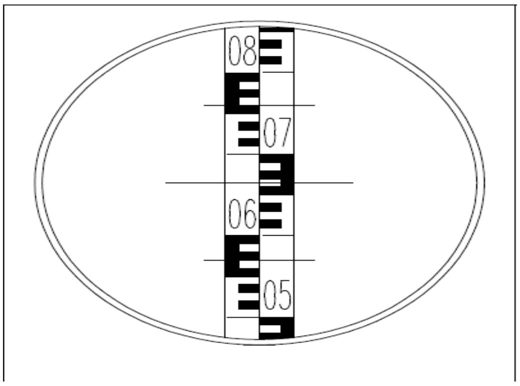


Figure II.8a: Reading on stadiametric lines (marks)

- **Accuracy of measuring Dh with a theodolite**

The standard deviation on **Dh** is calculated from the general expression (which is an approximate expression): $Dh=100\ (m_2 - m_1)\ sin^2V$.

If we use a T16 theodolite, we have σ_V=±2.5 mgr. Each sight reading is estimated at σ_m= ±1mm near up to 35m range.

- For a range of 35 m and a zenith angle V of 100 gr, we find the precision of the level ± 14 cm at 35 m.
- For a range of 35 m and for V=80 gr, we obtain an accuracy of ± 13 cm.
- For a range of 35 m and for V=50 gr, we obtain an accuracy of ± 7 cm.

II.7.2. Variable stadiametry angle

The angle a is variable on some so-called "self-reducing" devices, for example the Wild RDS tacheometer, the term "self-reducing" meaning that the calculation of the reduction of the inclined distance to the horizontal is automated. The stadiametric angle varies continuously according to the inclination of the sight so as to intercept a constant length L = (m_2- m_1) on the target. ***Dh = 100(m2– m1)***.

Note : The accuracy is better in variable angle stadiametry than in constant angle.

II.8. Electronic measurements

Length measuring instruments (IMEL) or also called electronic distance measuring instruments (IMED) work like stopwatches. They use electromagnetic waves that propagate in a straight line, at a constant and known speed.

The intensity of the carrier wave (lime, centimeter and electromagnetic) is modulated at emission by a lower frequency. The carrier wave is emitted by a transmitter-receiver station and returned by it (**Figure II.9**), either by a reflector or by a second (radio waves). The IMEL actually measure travel times.

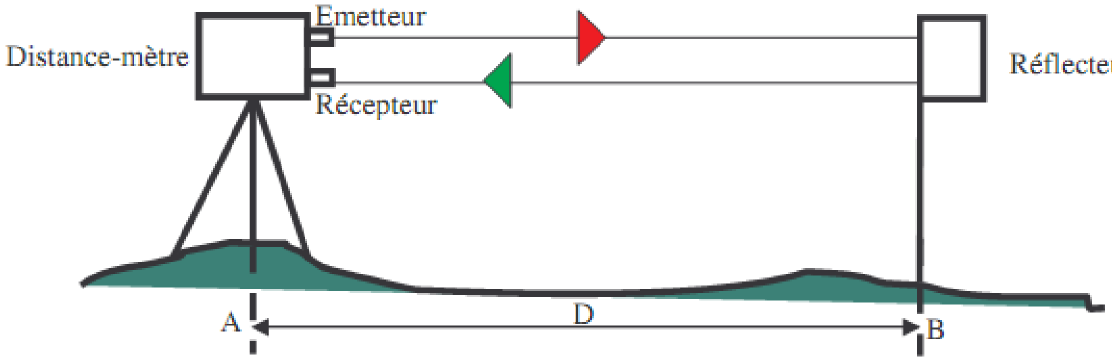


Figure II.9: An IMEL with transmitter and receiver

These measurements are carried out using **Electronic Length Measuring Instruments (IMEL)** who use **the electromagnetic waves** propagating in a straight line at a constant and known speed.

The general formula: $L = V.T / 2$

L: Inclined length,

V: Wave propagation speed,

T: Wave propagation time (The carrier wave making the round trip).

Some instruments that only perform distance measurements are listed:

1. Tellurometers (centimeter radio waves)
2. Geodimeters (light waves);
3. Distances meters (infrared);
4. Electronic rangefinders.

The device located at point **A** emits a train of electromagnetic waves towards a reflector located at **B** (**Figure II.10**). After reflection, the wave returns to the point of emission with a delay which is a function of the distance traveled. The device analyzes this delay and converts it into distance according to the slope (L). Measuring the tilt of the sight (angle α) then allows the horizontal distance to be determined:

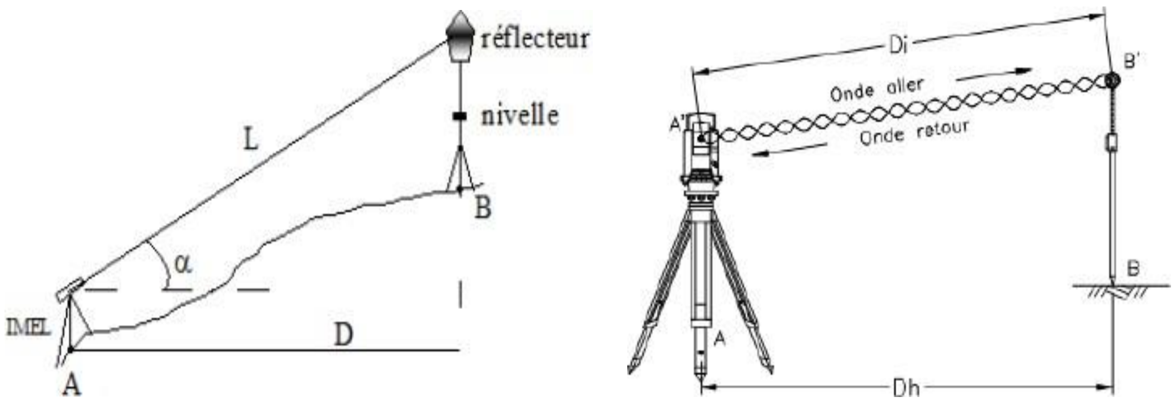


Figure II.10: Distance measurement with an IMEL.

II.9. Measurement assuming the IMEL attached to a theodolite.

The emitted wave (infrared, visible, microwave, etc.) is called a carrier wave.

Its intensity is modulated at emission by a lower frequency. An IMEL is therefore:

an energy source (battery) with its modulation device;

an emitting-receiving optic (reflector prism (**Figure II.11.a, Figure II.11.b**));

- A phase meter;
- An indicating device (which displays the results in 6 or 7 digits);
- A calculator.

Figure II.11.a: Prisms for rangefinder

Figure II.11.b: Reflector for rangefinder

<u>Systematic errors</u>

1. Calibration
2. Dilation
3. Elasticity
4. Horizontality

<u>Accidental errors:</u>

1. Sealing error
2. Non-vertical sheet
3. Errors in putting things together, bad layout.

Chapter III: Measurement of Angels

III.1 Operating Principle of a Theodolite

III.1.1 Definition: A theodolite is a device for measuring horizontal angles (angles projected onto a horizontal plane) and vertical angles (angles projected onto a vertical plane).

III.1.2 Operating principle: Figure III.1 shows the basic diagram of how a theodolite works.

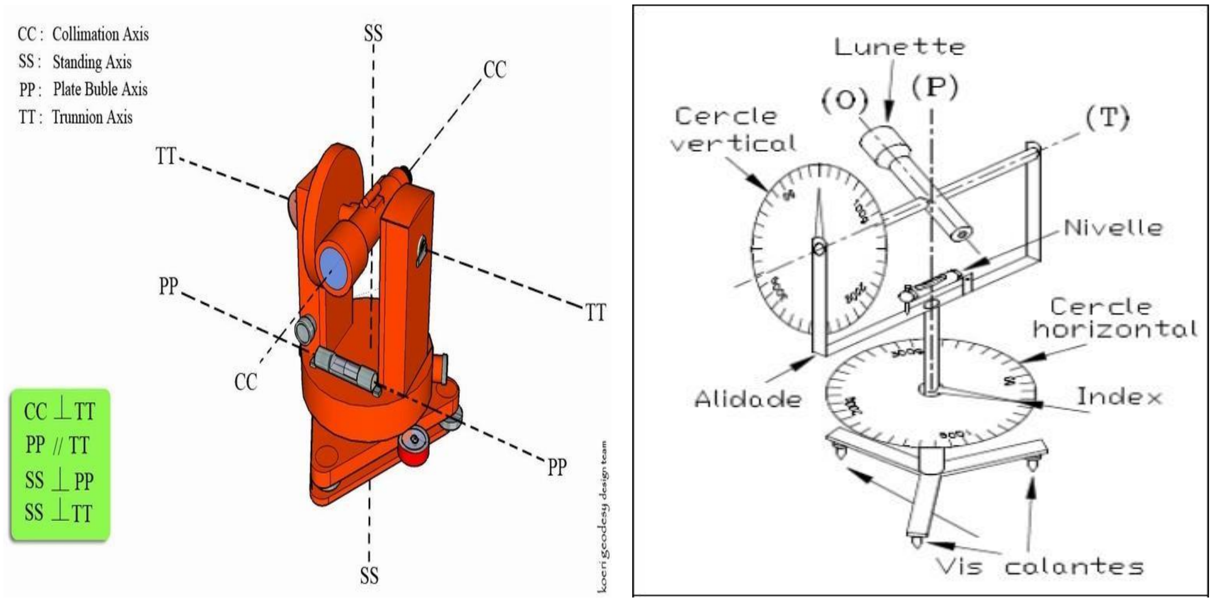


Figure III.1: Schematic diagram of a theodolite

- **(P): main axis** (Standing axis), it must be vertical after the theodolite is stationed and must pass through the center of the horizontal graduation (and the stationed point).
- **(T): secondary axis** (Trunnion axis), it is perpendicular to (P) and must pass through the center of the vertical graduation.
- **(O): optical axis** (Collimation axis), it must always be perpendicular to (T), the three axes (P), (T) and (O) must be concurrent.
- **L'alidade:** It is a mobile assembly around the main axis (P) comprising the vertical circle, the telescope, the alidade toric level and the reading devices (symbolized here by indexes).
- **The vertical circle** (vertical graduation): It is attached to the bezel and pivots around the axis of the trunnions (T). Authorized measurement of vertical angles.
- **The horizontal circle** (horizontal graduation): It is most often fixed relative to the base but it can be secured to the alidade by a clutch system. Allows the measurement of horizontal angles.

III.1.3 Angular readings

III.1.3.1 Aiming reticles: There are four main types of pointing (aiming) (**Figure III.2**):

- Ordinary or contact pointing.
- Pointing by bisection: the vertical thread of the reticle passes through the axis of the object being pointed at.
- Pointing by framing: the object being pointed at is framed by two parallel threads of the reticle.
- Pointing by coincidence: the vertical thread of the reticle tends to merge with the object being pointed at.

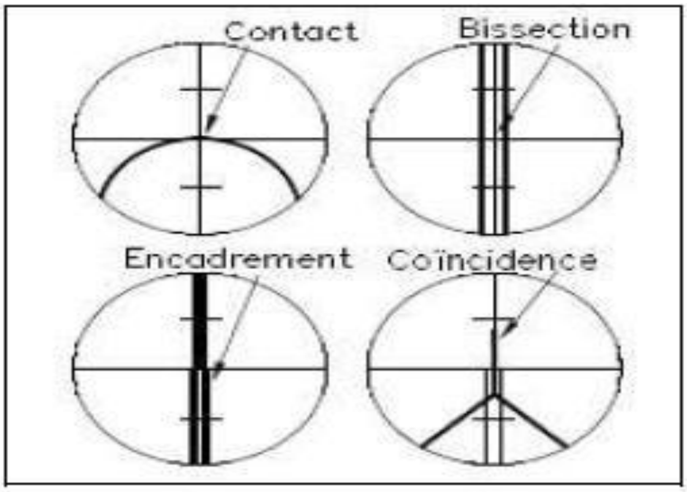


Figure III.2: Different types of pointing

III.1.3.2 Readings on Vernier caliper (Reading microscope)

The reading is taken on a graduated Vernier as in **Figure III.3**:

The theodolite gives the horizontal angles **Hz** and vertical **V**, the reading is done as follows: the Figures before the decimal point pass in front of the fixed graduation of the Vernier, the Figures after the decimal point are read at the place where a movable graduation captures the graduated sector. For example, in the 020A theodolite, the two circles are readable at the same time; we can read: vertical reader: **V = 95.985 grade** and horizontal reader **Hz = 17.965 grade**.

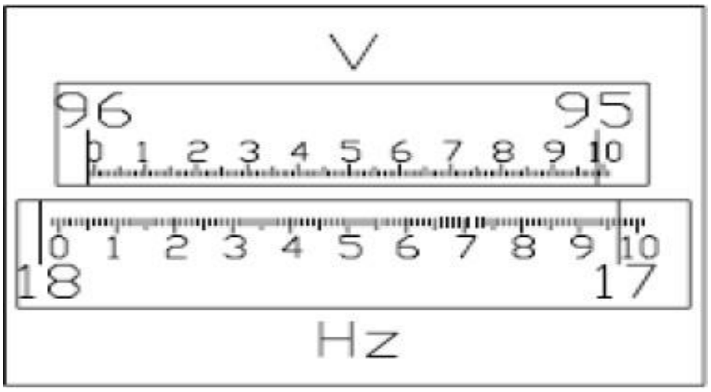


Figure III.3: Reading on graduated Vernier of the Theodolite 020A

III.2 Horizontal circle (horizontal angles)

III.2.1 Definition:

The horizontal circle is the graduation of the theodolite on which the operator reads the horizontal angles. It is linked to the base of the device but can also pivot on itself so as to adjust the zero of the graduations on a given direction.

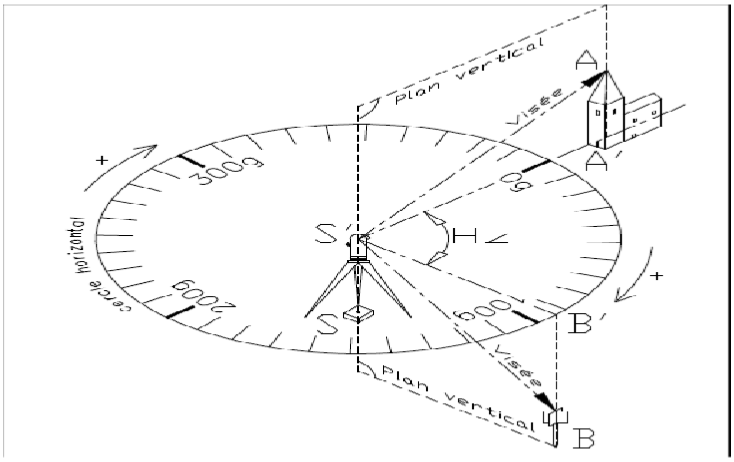


Figure III.4: Measuring horizontal angles

The graduations are **increasing from 0 to 400 grade clockwise**

After the theodolite is positioned, this circle is horizontal, which explains why the angles read are angles projected onto the horizontal plane and called horizontal (or azimuthal) angles, noted **Hz**.

In **Figure III.4**, the apparatus is stationed at point **S**. The operator aims at point **A** (top of the building) and sets the zero of the graduations at this point. Aiming at point **B**, he reads in the theodolite the horizontal angle **A'** – **S'- B'** (**A'**, **B'**, **S'** are the projections of **A**, **B** and **S** on the horizontal plane passing through the trunnion axis (secondary axis (T)) of the device.

III.2.2 The double reversal

This is a manipulation consisting of a simultaneous half-turn of the telescope and the alidade. This measurement technique makes it possible to eliminate certain systematic errors and to limit reading errors. When measuring a horizontal angle, this allows:

- To double the readings and therefore reduce the risk of reading errors;
- Not to always read on the same area of the limb, thus limiting the error due to graduation defects of the limb;
- To eliminate horizontal collimation and swirl defects. In practice, the following steps are performed:

1. A left circle reading (vertical circle of the device to the left of the operator, more generally in the reference position).

2.A double reversal of the bezel and the alidade.

3.A new reading of the same angle in a right circle (vertical circle on the right).

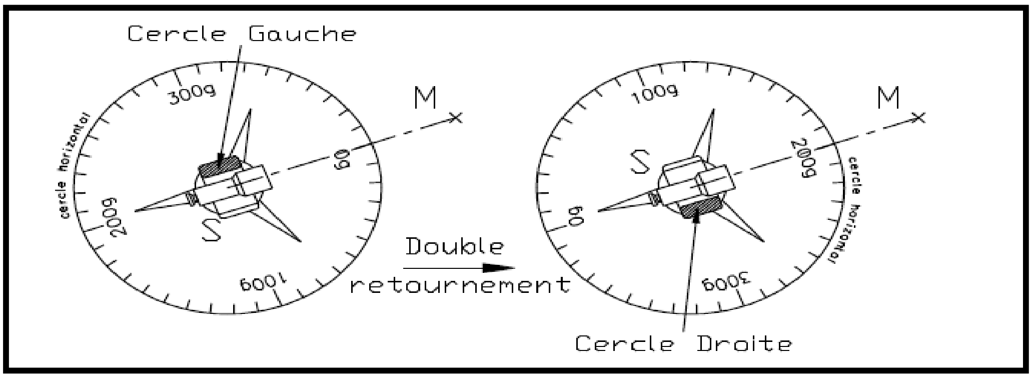


Figure III.5: Double reversal

If we call Hz_{CG} the value read in the left circle, and Hz_{CD} the one read in a right circle, we must observe: $Hz_{CG} \approx Hz_{CD} + 200$

In fact, the double reversal shifts the zero of the graduation by 200 gr; this allows simple and immediate control of the readings in the field.

The difference between the values Hz_{CG} And (Hz_{CD}– 200) represents the combination of collimation, alignment, reading errors, etc.

The measured horizontal angle Hz is then:

$$Hz = \frac{Hz_{CG} + (Hz_{CD} - 200)}{2} \quad Si \quad Hz_{CD} > 200 \text{ grade}$$

$$Hz = \frac{Hz_{CG} + (Hz_{CD} + 200)}{2} \quad Si \quad Hz_{CD} < 200 \text{ grade}$$

III.2.3 Simple reading

With the device in its reference position, and the zero of the horizontal graduation not being modified after setting up, the operator takes an azimuthal reading L_A on point **A** then a reading L_B on **B** and deduces the angle **ASB**: $Hz_{AB} = L_B - L_A$

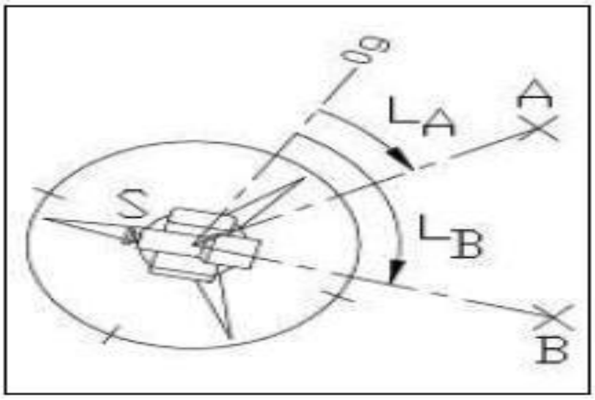


Figure III.6: Reading a horizontal angle

III.2.4 Sequence

A sequence is a set of (**n+1**) readings taken from the same station in **n** different directions with the same position of the horizontal and vertical circles. For example, in **Figure III.7**, the reference is the point **R** on which the operator performs the first reading "L_{R1}", we take a reading on each point by turning clockwise and a final closing reading on the point **R** "L_{R2}". By calculation, the readings are then reduced to the reference **R** by diverting the average of the two readings on the reference to the other readings. To do this, we calculate:

- Closing the sequence: $Fs = | L_{R1} - L_{R2} |$
- The average on the reference: $L_R = (L_{R1} + L_{R2})/2$
- Reading on each point: $L'_j = L_j - L_R$
- The reading on the reference therefore becomes $L_R = 0$

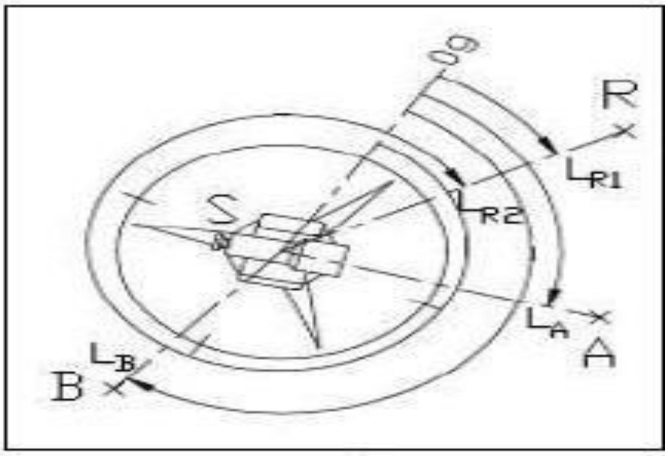


Figure III.7: Example representing a Sequence

The angular closure of each sequence is subject to regulatory tolerances, the values of which set by the decree of January 1980 correspond to:

- **Sequence closure gaps (e.f.s)**: **1,5 mgr in precision canvas.**
- **Sequence closure gaps (e.f.s)**: **2,8 mgr in ordinary canvas.**

III.2.5 Sequence Pair

A sequence pair is the association of two successive sequences with an offset of the origin of the limb of **100 grade**, turning the telescope and reversing the direction of observation.

This method allows to minimize some systematic errors. Generally, the operator performs a sequence in **CG** in the clockwise direction of rotation of the device then performs a double return and shifts the origin of the limb of **100 grade**, and finally he performs the sequence in **CD** in the trigonometric direction (counter-clockwise).

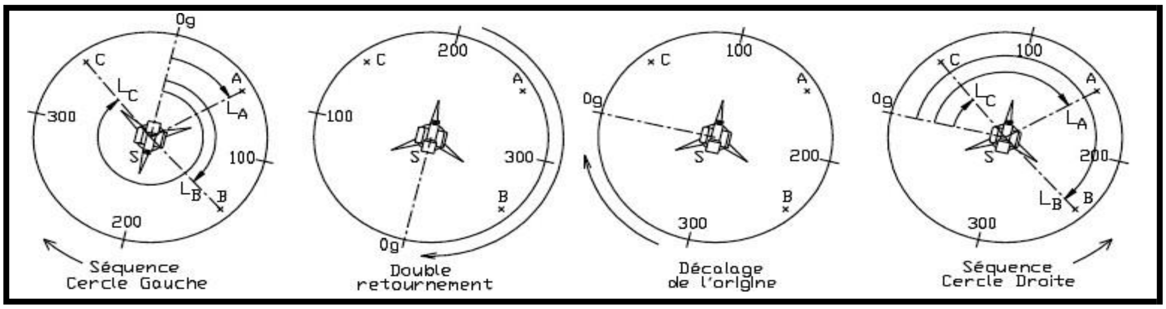


Figure III.8: Pair of sequences

III.3 Horizon closure

III.3.1 Definition: The Horizon closure is the final result of the combination of angular observations (sequences) at the same station and reported to the same reference (in our examples the point **R**).

When calculating, the average value of is determined **the deviation from the reference**: it is the algebraic sum of all the reading differences of the same pair divided by **(n+1)**, **n** being the number of directions targeted including the reference. This deviation is subject to regulatory tolerances:

- **T=0.8 mgr in ordinary canvas** for two pairs (0.9 mgr for four pairs);
- **T=0.7 mgr in precision canvas** for four pairs (0.8 mgr for eight pairs).

III.3.2 Reduced Sequence Pair

It is a pair of sequences without closure and without shift of the limb. It is used in detail surveying or for the measurement of single angles, for example in ordinary polygonation.

Station	Points	Lecture CG (gon)	Lecture CD (gon)	Moyenne
1	A	114,75	314,71	114,73
	B	207,23	7,28	207,23
	C	373,64	173,60	373,62
	D	86,19	286,14	86,16

- When you reach **D**, you perform a double turn and reverse the direction of rotation.
- The difference between **CG + 200** and **CD** must remain constant (± 1 graduation).
- The average of the two readings based on **CG** is taken.
- Deviations of the averages of the readings by pairs compared to the average: **T=1.3mgr** (2 pairs of sequence and for an ordinary canvas).
- Deviations of the averages of the readings by pairs compared to the average: **T=1.2mgr** (4 pairs of sequence and for a precision canvas).

III.4 Vertical circle (Vertical angles)

III.4.1 Definition

The vertical circle (**Zenith angle**) is an angle projected into the vertical plane of the station point. The reading index is fixed and positioned vertically (**Zenith**) of the axis of the trunnions (T). When the line of sight passes through a point **M**, the index then gives the reading of the angle **Z** (Or **V**) merged on the vertical circle.

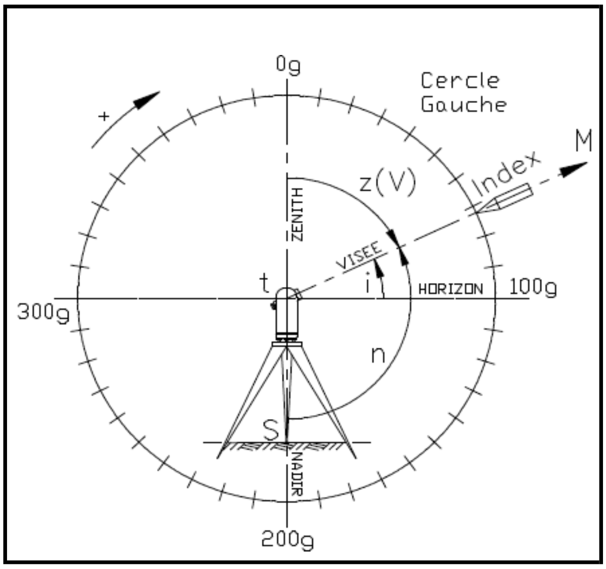


Figure III.9: Reading the zenith angle **z(V)**

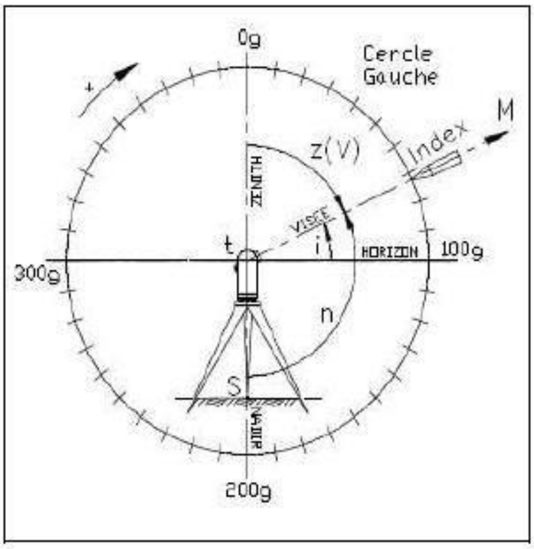


Figure III.10: Vertical angles **z**, **i** and **n**

To simplify the reading diagram of a zenith angle, we consider that the zero of the graduation is at the zenith when the device is stationary. We then consider that everything happens as if the vertical circle were fixed and that the reading index moved with the sight.

- **V** or (**Z**): vertical (zenith) angle measured in a vertical plane.
- **i**: the elevation angle measured between the horizon and the sight.
- **n**: the nadiral angle measured between the nadir and the sight.
- The relationships between these angles are: **n =200 – V**; **i = 100 – V**; **n–i = 100**

III.4.2 Average value of a vertical angle by double reversal

The reference position of our mechanical device is the left circle (CG),

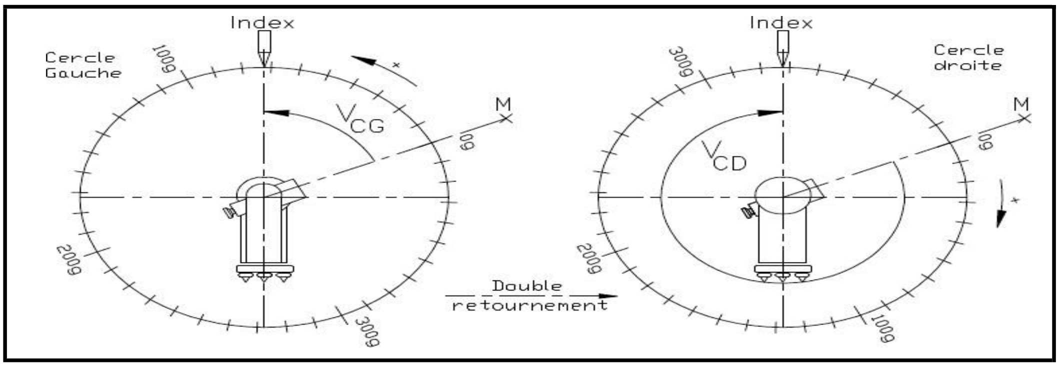


Figure III.11: Effect of double reversal on the measurement of the vertical angle

In **Figure III.11**, we see that after a double reversal the direction of evolution of the graduation of the vertical circle is reversed. The angle read in a right circle $\mathbf{V_{CD}}$ is therefore not "directly comparable" with the angle read in a left circle $\mathbf{V_{CG}}$, as was the case with horizontal angles. The relationship between the two readings $\mathbf{V_{CG}}$ and $\mathbf{V_{CD}}$ after a double flip is:

$V_{CG} = 400 - V_{CD}$

And the average final angle is: $V = \frac{V_{CG} + (400 - V_{CD})}{2}$

III.4.3 Vertical index error (V_0 or Z_0)

The interest of **the double flip** is, as for horizontal angles, to limit reading errors and to eliminate certain systematic or accidental errors. In the case of measuring vertical angles, the double reversal makes it possible to eliminate:

-**The eccentricity error** of the collimation axis relative to the secondary axis;

-**The vertical circle index error**: in fact, whether manual (index level) or automatic (compensator), the device of modern devices does not exactly set the zero (reading index) vertically to the center of the circle but in two neighboring positions symmetrical with respect to this vertical;

-**The trunnion defect** (non-perpendicularity of secondary axis and the principal axis).

In **Figure III.12**, the presence of an angular error is assumed $\mathbf{V_0}$ or ($\mathbf{Z_0}$) position of the index of the vertical circle relative to the vertical of the center of the circle.

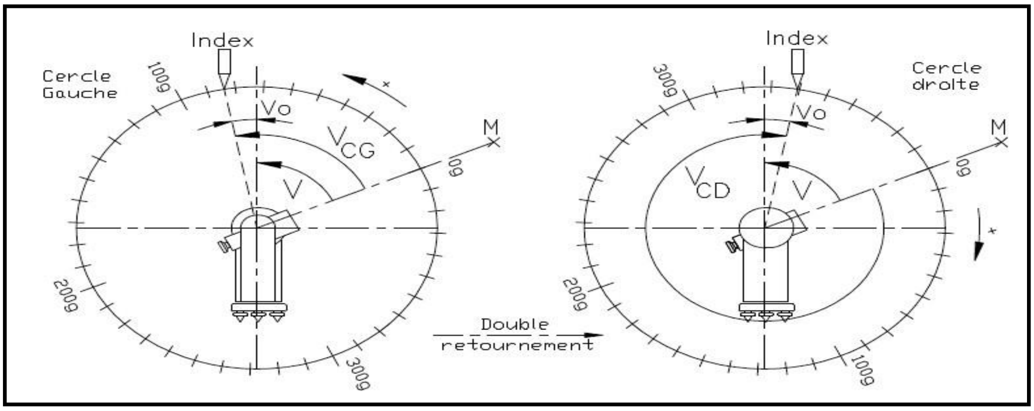


Figure III.12: Vertical circle index error

This defect is a device constant which can vary. It can suddenly increase if the vertical index level is out of adjustment or if the compensator is faulty. It is therefore advisable to evaluate it regularly and check that it is approximately constant (apart from reading errors). If it is found to vary greatly from one station to another, it is because the vertical index setting system is out of adjustment.

- In the left circle, the operator reads $\mathbf{V_{CG}}$, the angle **V** searched is worth: $\mathbf{V=V_{CG}-V_0}$.
- In a right circle, the operator reads $\mathbf{V_{CD}}$, the angle **V** searched is worth:

$\mathbf{V=400 - V_{CD}+V_0}$.

- If we take the average of the two values, we find: **$V = (V_{CG} + 400 - V_{CD})/2$**.
- If we subtract the two equations, we isolate **V_0** and we get: **$V_0 = (V_{CG} + V_{CD} - 400)/2$**

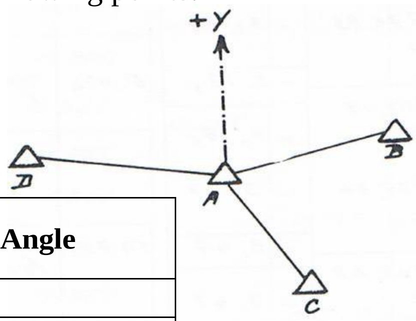
The value of **V_0** is also tainted by errors of reading, pointing, etc., so that it is impossible to say whether it is indeed the value of **V_0** only one that is measured in this way (except in the case of a gross timing error or in the case of a malfunction of the manual or automatic timing system of the vertical index).

Thanks to the double reversal, we can therefore eliminate certain errors and in particular the vertical index error. On low-end devices that do not have a precise vertical index setting device (manual or automatic), this average of the readings **CG** and **CD** helps to improve the measurement accuracy of **V**.

III.5 Exercises

Exercise Nº. 01: Observations made with theodolite 020A of the following points:

1. calculate the average readings of each target point
2. calculate horizontal angles
3. Determine the angles on paper using a protractor



Station	Targted Points	Readings		Readings averages	Angle
		CG	CD		
ST	A	114,750	314,710	**114,730**	
	B	207,230	7,180	**207,205**	92,475
	C	373,640	173,600	**373,620**	166,415
	D	86,190	286,140	**86,165**	- 287,455 (112,545)
					28,565

Exercise Nº. 02: Observations made with theodolite 020A of the following points:

1. Complete the following table: vertical circle

Station	Targted Points	V_{CG}	V_{CD}	*V_0 (Z_0)*	*V (Z)*	i	n
ST	A	98.242	301,770	**0.006**	**98.236**	**1,764**	**101,764**
	B	105.188	294,822	**0.005**	**105.183**	**- 5.183**	**94,817**
	C	98,790	301.220	**0.005**	**98,785**	**+ 1.215**	**101.215**

Exercise Nº. 03:

Calculate the value of horizontal (azimuthal) angles from observations of a horizon survey carried out at a station. The work is part of a precision survey. The tolerances to be respected are as follows:

- Sequence closure gaps (e.f.s) T=1.5 mgr;
- Deviation of the means of the paired readings from the mean T=1.3 mgr;
- Deviation from the reference T=0.8 mgr.

St	Pt	Reading CG=0	**Reduction Origin-0**	Reading CD=100	**Reduction Origin-100**	Average on two seq Pair	Reading CG=50	**Reduction Origin- 50**	Reading CD=150	**Reduction Origin-150**	Average on two sequences, Pair	Average reading on two Pairs
	1	0.3891		100,4691			50,3440		150,1188			
	2	23,7434		223,8270			173,7024		273,4750			
	3	56,3036		356,3855			306,2592		6,0347			
St												
	4	31,6436		31,7247			381,6010		81,3750			
	5	62,1364		62,2172			12,0940		111,8678			
	1	0.3893		100,4700			50,3440		150,1179			
Avg/ Ref												
e,f,s T=2.8 mgr												

Value of Azimuthal Angles: α

$\alpha_1=$ grade

$\alpha_2=$ grade

$\alpha_3=$ grade

$\alpha_4=$ grade

$\alpha_5=$ grade

The deviation of the means of the paired readings from the mean

Points	**Pair 1**	**Pair 2**	**Sum**
2			
3			
4			
5			
sum			

The gap on the reference

Pair 1	**Pair 2**

Chapter IV: Determination of Surfaces

IV.1 Shape of the Earth

The Earth is an ellipsoid of revolution rotating around its minor axis, called the Earth's axis. The Equator is the imaginary great circle traced around the Earth at an equal distance from the two poles. The Meridian is the imaginary semi-great circle of the Earth's surface limited to the poles (**Figure IV.1**). It is appropriate to distinguish the ellipsoid of revolution which is a surface generated by an ellipse with half axis **a** and **b** (~6400 km) rotating around the minor axis **b**, parallel to the axis of **poles**.

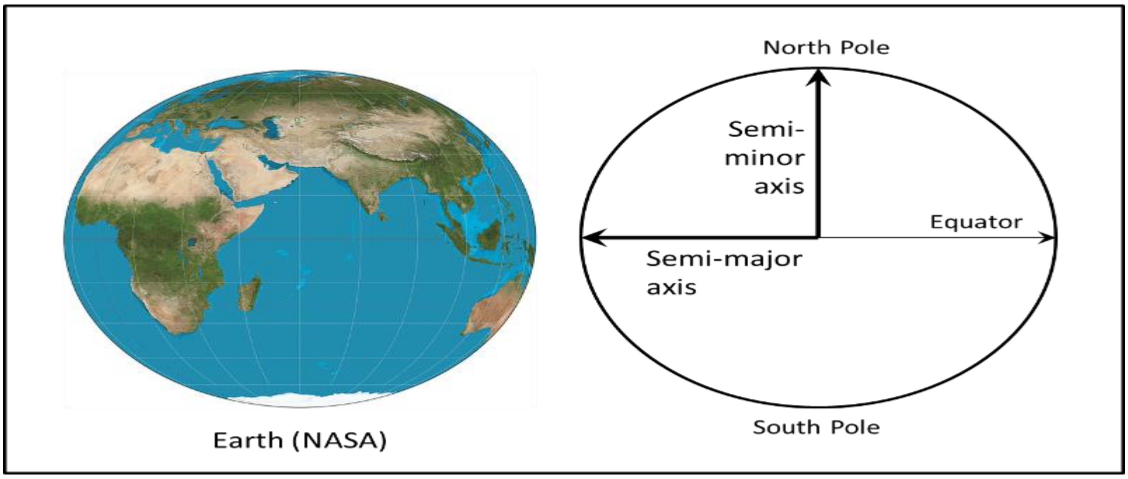


Figure IV.1: Shape of the earth.

IV.1.1 Geoid

The geoid is a body that does not have a regular geometric shape because of this we change the surface of geoid to the surface of ellipsoid of revolution.

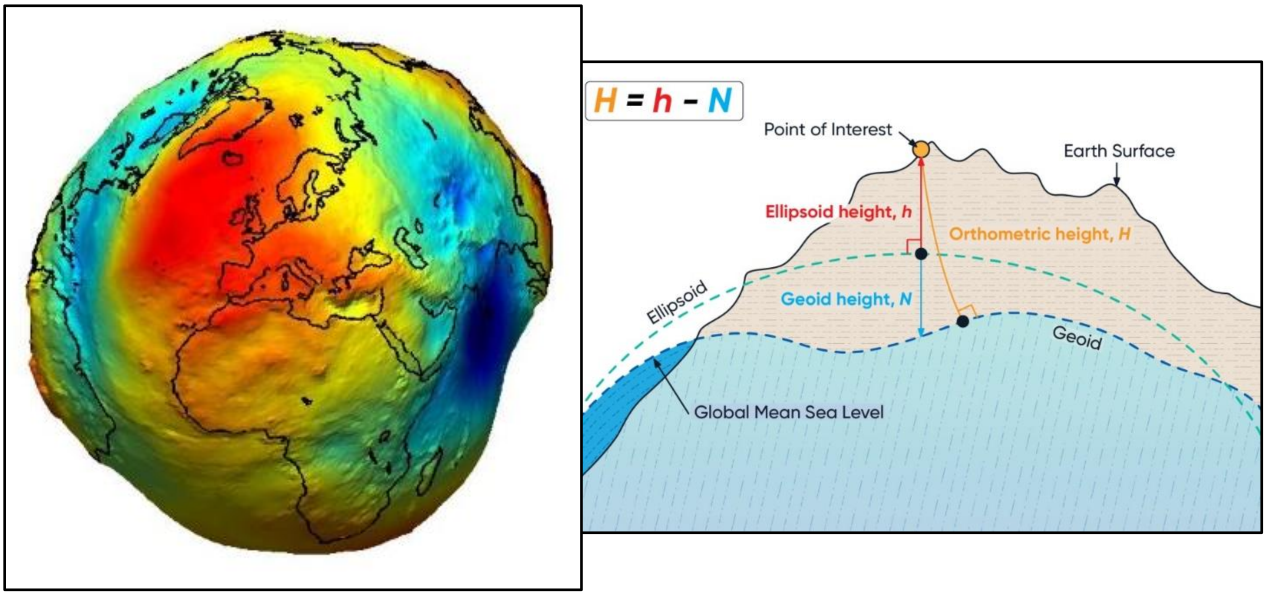


Figure IV.2: Earth's Geoid

IV.1.2 Reference ellipsoid (revolution):

The ellipsoid of revolution (sphere flattened at the poles) is a mathematical model used for calculation, defined so that it is as close as possible to the geoid. There are many models of ellipsoids. Each geodesic reference is associated with an ellipsoid on which a meridian has been fixed as the origin of the longitudes and which is perfectly defined by the semi-major axis a and one of the different values:

- Radius **a= 6378.1370 KM** = semi-major axis
- Polar radius **b = 6356.7523 KM** = semi-minor axis.

$$\text{Flattening}: \ \alpha = \frac{(a - b)}{b} = \frac{(6378.137 - 6356.7523)}{6378.137} = 0.0033528$$

We can imagine that the ellipsoid is a sphere of radius R=6378KM

Approximately =6,4*10^3KM. Allow $2\pi R = 2*3,14*6,4*10^3 = 40*10^3$

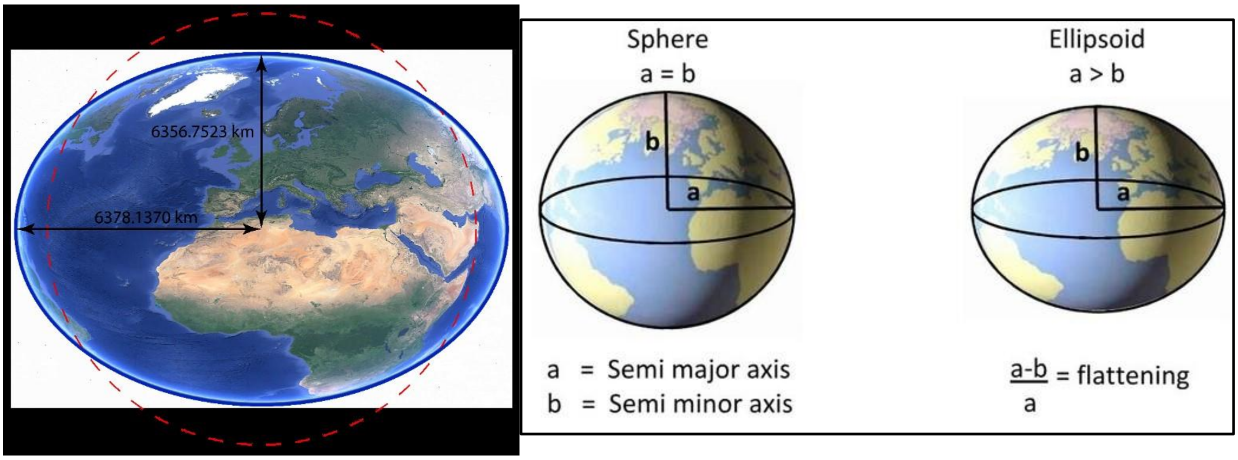


Figure IV.3: Earth's dimensions and flattening

Or what amounts to the same thing, the square of his eccentric $\mathbf{e^2}$ given by the relationship $e^2 = (a^2 - b^2)/b^2 \neq 2a$

First eccentricity: $e = \sqrt{(a^2 - b^2)/b^2}$

Inverse of flattening: $1/a = b/(a-b)$

IV.1.3 Coordinate systems

Coordinates can be expressed in the form of coordinates:

Geocentric Cartesians (X, Y, Z) relating to the three (3) axis of a reference frame having its origin at the center of the Earth's masses (**Figure IV.4**). These coordinates can be used, for example, as an intermediary when calculating changes in geodetic reference systems.

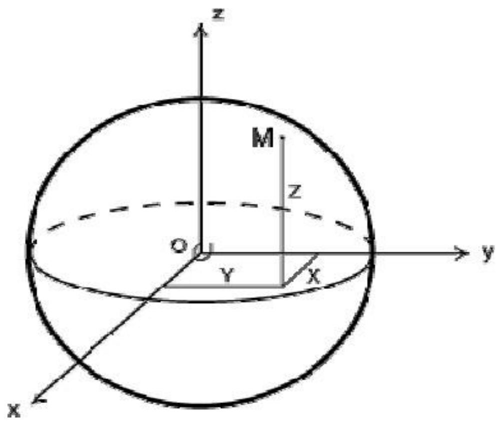


Figure IV.4: Cartesian coordinates of the Earth

Geographical (λ, φ, h), A point on the earth is located exactly using its geographic coordinates, **LONGITUDE** and **LATITUDE**.

- **Longitude (λ)**: it is the angle formed by the meridian plane containing this point with a meridian plane arbitrarily chosen as the origin, the longitude is counted positive towards the west and negative towards the east (clockwise).
- **Latitude (φ)**: It is the angle formed by the vertical of the point and the plane of the equator. Latitude is measured north or south from the equator.
- The letter **h** corresponds to the ellipsoidal height (not to be confused with the **altitude**). It is defined in a geodetic reference system and can differ from the altitude by several tens of meters.

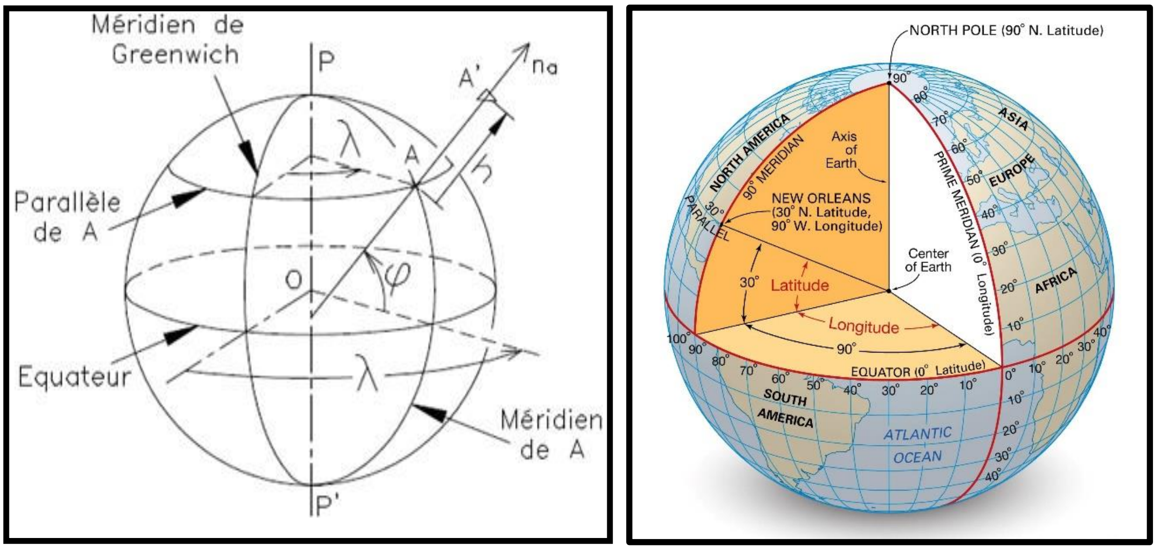


Figure IV.5: Geographic coordinates of the Earth

In projection or plane representation

Using coordinates on a reference surface such as an ellipsoid is not easy and does not allow direct measurements of distance, angle or area. It is more practical to have a graphic image of the world on a plane.

Contrary to custom, it is preferable to use the convention **E**, **N**(**E**st, **N**ord) to designate coordinates in projection (**Table IV.1**) to avoid confusion with cartesian coordinates **(X, Y, Z)**.

Table IV.1: Elements required to describe a coordinate type

Coordiantes type	X, Y, Z	λ, φ, h	E, N
Angular unit		*	
Linear unit	*	*	*
Projection			*
Meridian origin		*	*
Ellipsoid		*	*
Geodic System	*	*	*

Some notations of angular units for latitudes and longitudes are given in **Table IV.2**.

Table IV.2: Angular unit notations for latitudes and longitudes

Degrees, Minutes, sexagesimal Seconds	**° ‘ “**
Degrees, Decimal Minutes	**° ‘**
Decimal Degrees	**°**
Grades (or Gon)	**gr**
Radians	**rad**

Some numerical approaches are given by the **Table IV.3**.

Table IV.3: Numerical approaches

1°	= 60 ’	= 3600 “
180°	= 200 gr	= 3.141592654 rd= π (flat angle)
360°	= 400 gr = -6,28 rd = 2 π rd	Circumference
48.61°	= 48° 36.6 ’	= 48° 36 ’ 36 “
48.60°	= 54 gr	

Example of transformation

57,30° = 63,66 gr = 1rad. 1° = 1,111 gr. 0.9° = 1 gr = 0,0157 rd.

IV.2 The orientations

IV.2.1 Direction of a point

To define the direction of a point we use **the zenith** the angle from 0 to 400 grade then a reference grade position in the direction of a clockwise, this reference is made according to three axis.

IV.2.2 The axis

- **North of the Map (Lambert)**: It is the direction of the positive "Y" at a point and is also called grid North.
- **Geographic North**: is defined as the direction of the North Pole. At a given point the direction of the Lambert grid (or positive Y axis) coincides with geographic north only along the original meridian. The angle between Lambert north and geographic north is called "meridian convergence".
- **Magnetic North**: the direction taken by a dismantled needle, it varies with time, it is influenced by several factors, it varies according to the years, the centuries and it is influenced by all the surrounding magnetic bodies (grids, pipes).

IV.3 The deposit (G)

IV.3.1 Definition

The deposit of a direction **AB** is the horizontal angle (North) measured positively in the clockwise direction between the ordinate axis of the projection system used and this direction **AB**. We denote $\mathbf{G_{AB}}$ Mathematically, it is the positive clockwise angle between the ordinate axis of the reference frame and the line (**AB**).

A deposit is always between 0 and 400 grade. $\mathbf{G_{AB}}$ is the angle between North (ordinate) and the **AB** direction.

$\mathbf{G_{BA}}$ is the angle between the North and the **BA** direction.

The relationship that binds $\mathbf{G_{AB}}$ and $\mathbf{G_{BA}}$ is: $\mathbf{G_{BA}=G_{AB}+200}$

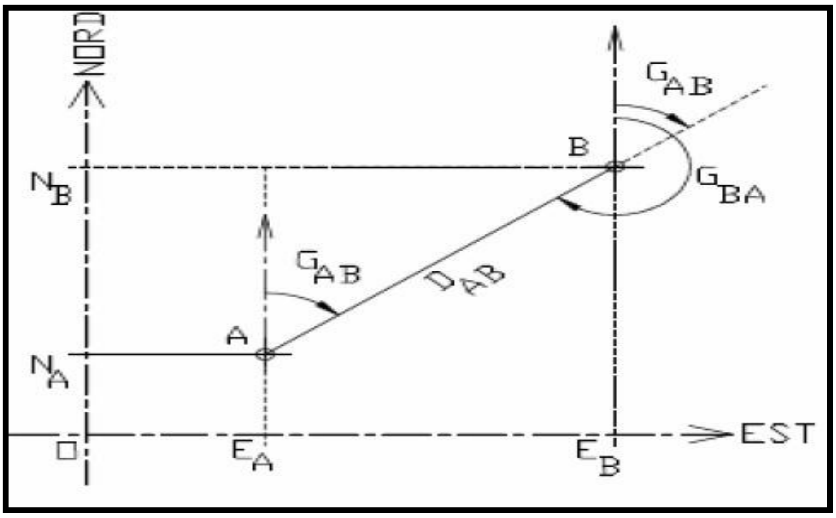


Figure IV.6: Rectangular coordinates and the deposit

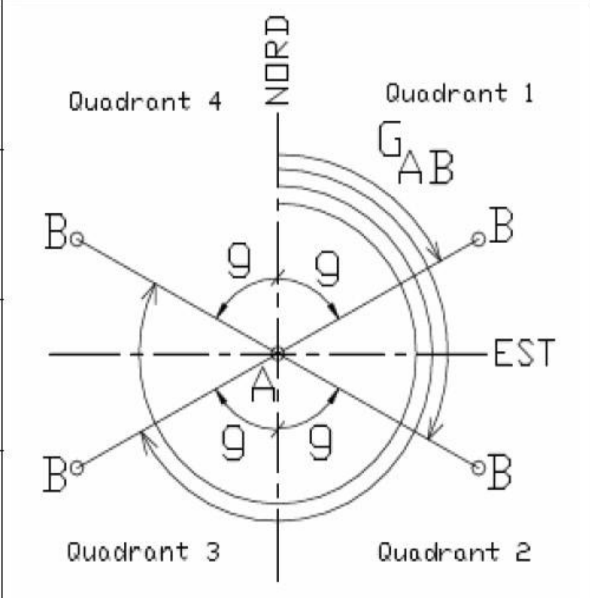
IV.3.2 Uses of deposits

- ***Calculation of a deposit from Cartesian coordinates***

Consider the coordinates of two points **A** (E_A,N_A) and **B** (E_B,N_B)

The following relationship allows us to calculate $\mathbf{G_{AB}}$:

$$\tan G_{AB} = \frac{E_B - E_{HAS}}{N_B - N_{HAS}}$$

Quadrant 1 : B est à l'est et au nord de A (ΔX > 0 et ΔY > 0). **G**$G_{AB} = g$	
Quadrant 2 : B est à l'est et au sud de A (ΔX > 0 et ΔY < 0). $G_{AB} = 200 + g$ (avec $g < 0$)	
Quadrant 3 : B est à l'ouest et au sud de A (ΔX < 0 et ΔY < 0). $G_{AB} = 200 + g$ (avec $g > 0$)	
Quadrant 4 : B à l'ouest et au nord de A (ΔX < 0 et ΔY > 0). $G_{AB} = 400 + g$ (avec $g < 0$)	

In topography, it is very common to know a point **S** (E_S, N_S) and to look for the coordinates of a point **P** visible from **S**. We say that **P** is radiated from **S** if we can measure the horizontal distance $\mathbf{D_{SP}}$ and the deposit $\mathbf{G_{SP}}$. Whatever the quadrant, we can then calculate the coordinates of point **P** by the following formulas:

$$E_P = E_S + D_{SP} * \sin G_{sp}$$
$$N_P = N_S + D_{SP} * \cos G_{sp}$$

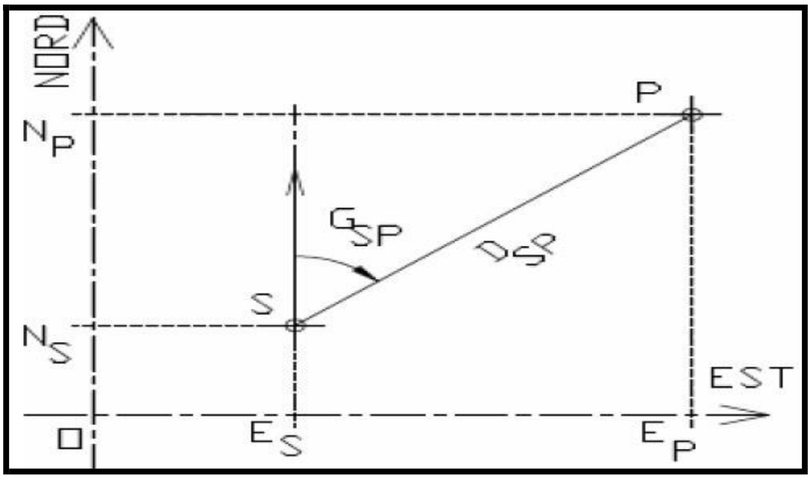


Figure IV.7: Cartesian coordinates

IV.4 Calculation and compensation of polygons

IV.4.1 Definition

A polygon is a set of connected survey lines of which the length of each is known and the angles defined between them. A polygon is necessarily closed on itself and can have any number of vertices. The angles at each of the vertices can be interior or exterior angles.

IV.4.2 Polygons defined by polar coordinates of the vertices

- Choose a direction of rotation.
- Each summit (**n**) is attached to a point **O** by length L_n and the deposit G_n.

Let us designate byαthe central angles such as:

$$\alpha_{(n-1)} = G_{(n)} - G_{(n-1)}$$
$$\alpha_{(n)} = G_{(n+1)} - G_{(n)}$$

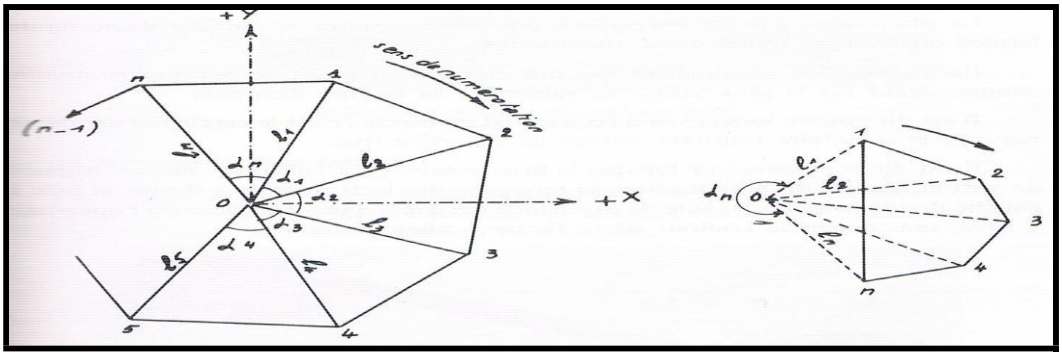


Figure IV.8: Polygon calculation

The area of the polygon is the algebraic sum of the areas of all the triangles.

$$2S = l_1 l_2 \sin\alpha_1 + l_2 l_3 \sin\alpha_2 + l_3 l_4 \sin\alpha_3 + \ldots\ldots\ldots\ldots + l_n l_{n+1} \sin\alpha_n$$

If the station **O** is outside the polygon the surface of the last triangle $l_n l_{n+1} \sin\alpha$ to a negative sign.

IV.4.3 Polygons defined by rectangular coordinates of the vertices

The area of a polygon whose rectangular coordinates of the vertices are known can be calculated by decomposing the Figure into triangles, rectangles or trapezoids. This time-consuming and difficult method is rarely applied.

After generalization and transformation, the previous formula becomes:

$$2S = \sum \left[X_{(n)} \times (Y_{(n-1)} - Y_{(n+1)}) \right]$$
$$2S' = \sum \left[Y_{(n)} \times (X_{(n-1)} - X_{(n+1)}) \right] = -2S$$

Both formulas are generally used simultaneously.

IV.5 Planimeters

The planimeter is an instrument designed to measure the surface of any closed figure (plan, map, diagram). Among all the planimeters that have been developed, the polar compensating planimeter is the most used because of its ease of use. It is called polar when its direction is a circle: it is the polar circle described by its polar arm rotating around its fixed pole. It is called compensating when the polar arm can be placed successively on either side of the tracing arm, the average of the readings pole to the right, and pole to the left gives the exact value of the measured surface, to the extent that the operator has followed exactly the outline of the planimetric.

- ***Rolling disc planimeters:***

They have the advantage of being able to work on surfaces extending over a great length.

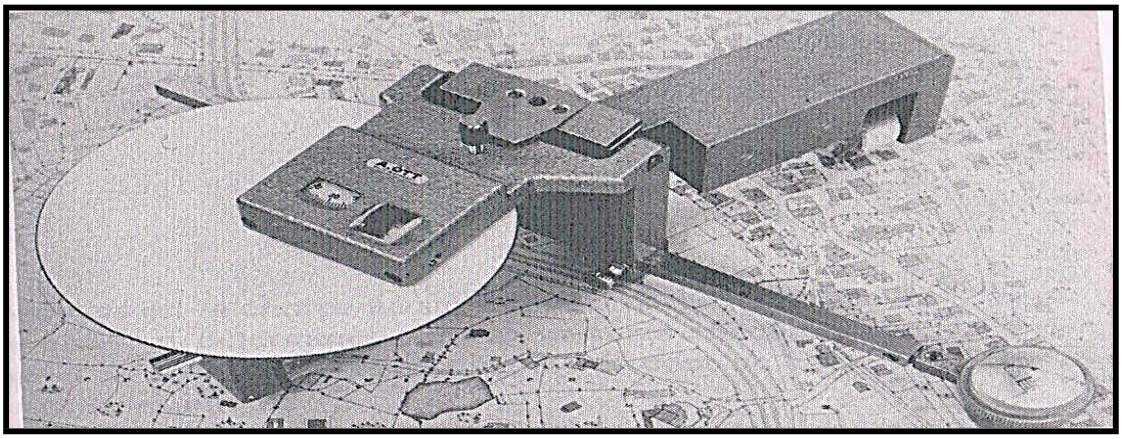


Figure IV.9 : Rolling disk planimeter

- ***Digital planimeters:***

Equipped with a digitizer, calculator, the determination of the surfaces is carried out either as with a classic planimeter by the continuous mode, or point by point. Very efficient, they are increasingly used.

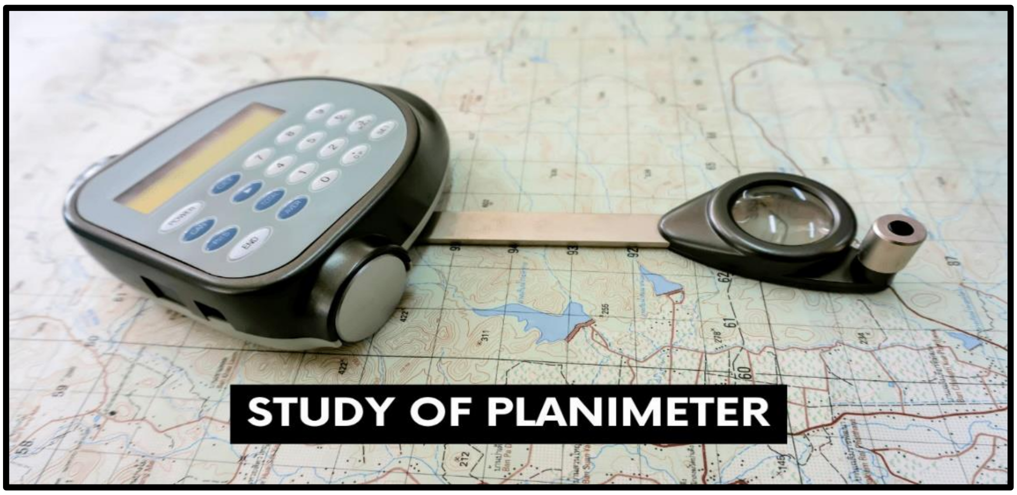
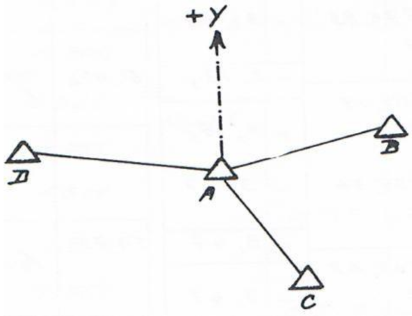


Figure IV.10 Digital planimeter

IV.6 Exercises

Exercise #1:



Knowing the coordinates of points A, B, C and D

1. calculate the deposits AB, AC, AD.
2. Calculate the distance between two points (AD)
   a. by the Pythagorean theorem.
   b. by the deposit.

Data: XA=875.17m XB=975.73m XC=924.17m XD=753.04m
YA=275.30m YB=309.14m YC=175.49m YD=295.79m

Exercise #2:

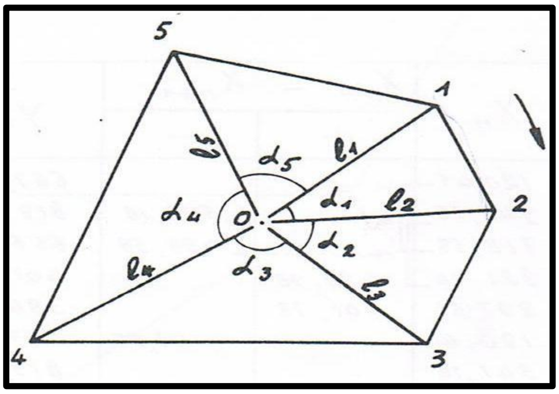
We have point A with coordinate (680379; 210257), knowing that the distance between the point A relative to the point B: D_{AB}= 45,53 m and the bearing of the line AB is G_{AB}=172,622 grade, calculate the coordinates of the point B.

Exercise #3:

Calculate the area of polygon defined by polar coordinates shown in the Figure below.

Data:

L_1=22.50m α_1=87.825gr
L_2=30.50m α_2=84,000gr
L_3=50.50m α_3=76.235gr
L_4=27.50m α_4=114.200gr
L_5=29.20m α_5=37.740gr



Exercise #4:

Calculate the area of polygon defined by rectangular coordinates.

Data :

1. X=1000.00m Y=1000.00m
2. X=1025.82m Y=991.60m
3. X=1047.61m Y=947.07m
4. X=1000.20m Y=953.76m
5. X=984.31m Y=995.13m

Chapter V: Direct and Indirect Leveling

V.1 Direct Leveling

V.1.1. Definition

Direct leveling, also called geometric leveling, consists of determining the difference in level **ΔH_{AB}** between two points **A** and **B** using a device (the level) and a grading rod (staff).

- The level consists of a sighting optic rotating around a vertical axis, thus defining a horizontal sighting plane (**Figure V.1**).
- The sight is a vertical scale placed successively on the two points **A** and **B**. The operator reads the value **m_a** on the sight placed in **A** and the value **m_b** on the sight placed in **B**. The difference in readings on the sight is equal to the height difference between **A** and **B**.

The difference in height from **A** to **B** is: $\Delta H_{AB} = m_a - m_b$

The difference in height from **B** to **A** is: $\Delta H_{BA} = m_b - m_a$

The altitude **H_A** from a point **A** is the distance calculated along the vertical which separates it from the geoid. If the altitude of point **A** is known, we can deduce that of point **B** by: $H_B = H_A - \Delta H_{AB}$

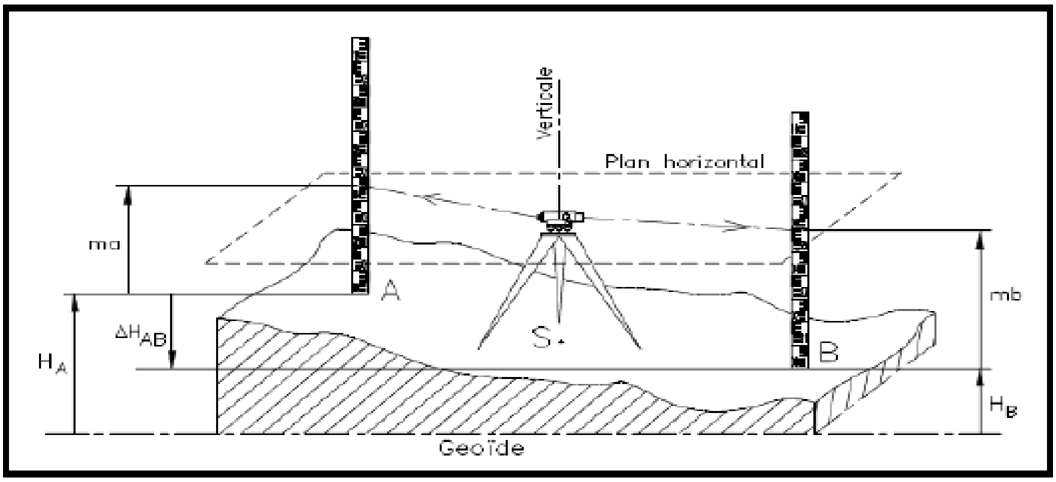


Figure V.1: Basic principle of direct leveling

The span is the distance from the level to the sight; it varies depending on the equipment and the precision sought, and Ordinary leveling: must be at most **60m**. Precision leveling: and **35m**.

Where possible, the operator places the level approximately equal in distance from **A** and **B** (**Figure V.2**), so as to achieve equality of spans.

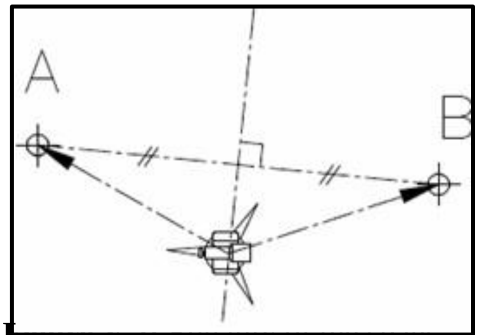


Figure V.2: Equality of the spans

V.1.2. The level

V.1.2.1. Operating principle

The level is schematically made up of a sighting optic (**collimation axis (O)**) rotating around a vertical axis (called **main axis (P)**) which is perpendicular to it (**Figure V.3**). The adjustment of the verticality of the main axis is done by means of **a spherical level**. The optical axis **(O)** rotating around the principal axis therefore describes a horizontal plane passing through **the optical center** of the level which is the intersection of the axes **(P)** and **(O)**.

The main axis **(P)** can be stationed vertically above a point by means of a plumb line, but generally the level is placed at any location between points **A** and **B**, if possible on the intermediate line of **AB** (**Figure V.3**). A level is therefore not equipped with an optical plumb line like a theodolite.

Some devices have a graduation (or **horizontal circle**) which allows horizontal angles to be read with average precision, of the order of ± 0.25 grade: they are only used for implantations or rough surveys.

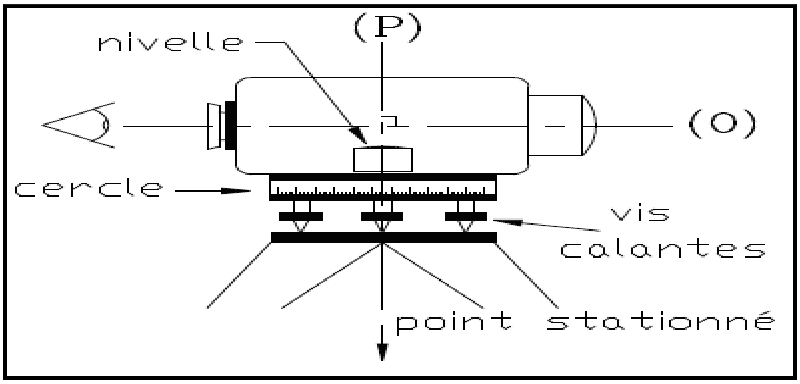


Figure V.3: Sketch of a level

The constituent elements of a level are as follows:

1. Base
2. Shim screws (3 screws)
3. Slow rotation
4. Focus on the object
5. Objective
6. Fast approach sight
7. Eye-piece
8. Removable ring
9. Control of automation
10. Pendulum compensator
11. Horizontal circle (option on NA2)
12. Spherical level (invisible here)

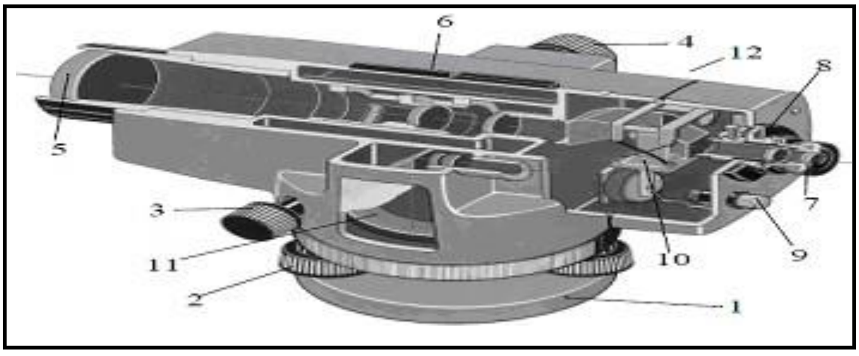


Figure V.4: Sectional view of the Leica NAK2 Doc -type level-

To accurately determine differences in level, the device must check:

- The perpendicularity of (O) and (P);
- That the horizontal line of the aiming reticle is located in a plane perpendicular to the main axis (P);
- That the optical axis (O) is parallel to the bubble tube, if it is a toric bubble, or that the plane described by the optical axis (O) rotating around the principal axis (P) is parallel to the plane in which the centering circle of the bubble is inscribed, if the bubble is spherical.

V.1.3. Readings on grade rod

The rod is a linear scale that must be held vertically (it includes a spherical bubble) on the point involved in the difference in level to be measured. The precision of its graduation and its maintenance in a vertical position strongly influence the precision of the difference in level measured.

The classic rod is generally graduated in centimeters. **The encryption** is often in decimeters (**Figure V.5**).

There are reverse graduation sights for optics that do not straighten the image (old models). The reticle of a level is generally made up of four lines (hairs):

- **The upper stadia hair (s')**: which gives a reading $\mathbf{m_1}$ on the sight;
- **The lower stadia hair (s)**: which gives the reading $\mathbf{m_2}$ on the sight;
- **The middle crosshair (n):** which gives the reading **m** on the sight;
- **The vertical crosshair (v)**: which allows the aiming of the sight or an object.

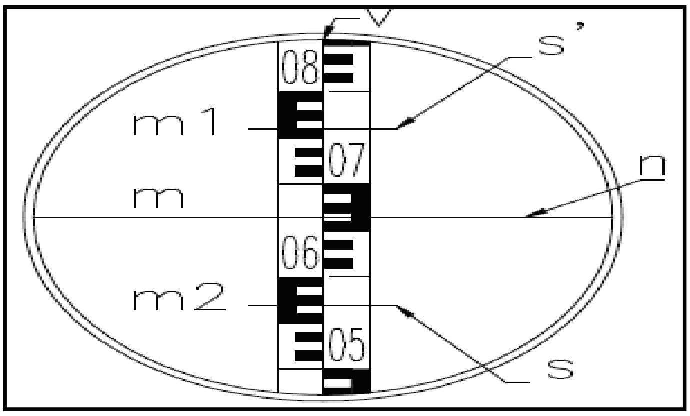


Figure V.5: Aiming reticle

The reading on each wire is visually estimated to the nearest millimeter (6.64 dm in **Figure V.5**, leveling line). The stadiametric lines make it possible to obtain an approximate value of the range. For each reading, it is advisable to read the three horizontal line in order to avoid reading errors: we check, directly on the ground, that:

$m = \frac{(m_1 + m_2)}{2}$; For example, Figure 04: $m = \frac{(5.69 + 7.60)}{2} = 6.64 dm$.

V.1.4. Accuracy and tolerance of readings

The various possible faults and sources of error are calculated below.

V.1.4.1. Errors

We distinguish the following faults:

- **Adjustment:** forgetting to adjust the bubble, blocked compensator;
- **Reading:** confusion of the leveling line with a stadiametric line; confusion of graduation or unit;
- **Transcription on paper:** bad copy of the value read.

V.1.4.2. Systematic errors

Systematic errors are:

- The calibration error of the rod;
- The lack of verticality of the rod: bubble out of adjustment;
- Optical axis tilt error: optical axis not perpendicular to the principal axis;
- The malfunction of the compensator.

V.1.4.3. Accidental errors

Accidental errors are:

- A bad adjustment of the bubble;
- Reading error on the grade rod due to the millimeter estimate;
- A bad choice of an intermediate point: unstable point;
- The air flare: avoid aiming rod foot near the ground when it is hot;
- Object pointing (aiming) error: It is due to the shape of the reticle.

V.1.5. Regulatory tolerances

Tolerances applicable to leveling (**Table V.1**).

L is the total length of the route in kilometers. **N** is the number of elevation differences. **n** is the number of elevation differences per kilometer (**n=N/L (km)**). The limit value **n=16** corresponds to a path whose average distance between points is **62.50 m** or an average range of approximately **30 m**. This value is the upper limit allowed in high precision leveling.

Table V.1: Tolerances applicable to leveling

Tolerances $T_{\Delta H}$ in (mm)	**n≤16**	**n>16**
Ordinary	$4\sqrt{36L + L^2}$	$\sqrt{36N + \frac{N^2}{16}}$
Precision	$4\sqrt{9L + L^2}$	$\sqrt{9N + \frac{N^2}{16}}$
High precision	$8\sqrt{L}$	$2\sqrt{N}$

V.1.6. Practice of leveling by path

Leveling by path is carried out by the following operations:

- The rod being on the origin point **A**, the operator parks the level in $\mathbf{S_1}$ whose distance he determines by counting the number of steps separating **A** of $\mathbf{S_1}$, so as not to exceed the maximum range of 60 m. The operator makes a **back sight reading**, that is to say in the chosen direction of travel, on point **A**, noted $\mathbf{m_{bsA}}$;
- The rod holder moves to come to the first intermediate point $\mathbf{I_1}$ as stable as possible (stone, metal base called a "toad", stake, etc.) and the distance of which he determines by counting himself the number of steps separating **A** from $\mathbf{S_1}$ in order to be able to reproduce this number of steps of $\mathbf{S_1}$ to $\mathbf{I_1}$;
- Still parked in **S1**, the operator reads on the rod a **front sight reading**, on $\mathbf{I_1}$ noted $\mathbf{m_{fsI1}}$; it is then possible to calculate the difference in height from **A** to $\mathbf{I_1}$ in the following manner:

$$DH_1 = m_{bsA} - m_{fsI1} = \text{back sight reading on A} - \text{front sight reading on } I_1$$

- The operator must read the stadia wires and verify that: $m = \frac{(m_1 + m_2)}{2}$

; the operator move to choose a station S_2 and so on;

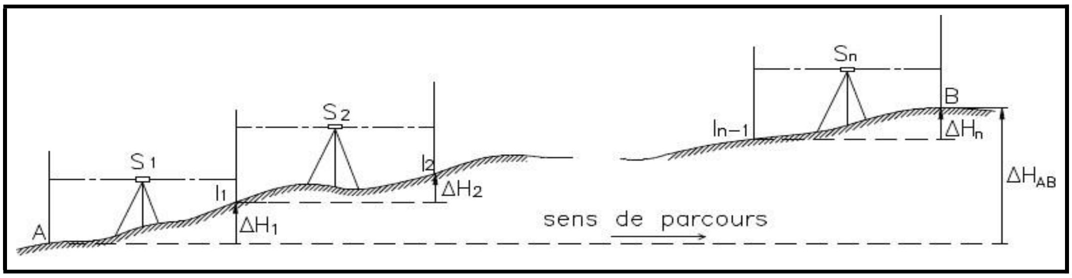


Figure V.6: Leveling path

The partial differences in level are as follows:

$m_{bsA} - m_{fsI1} = \Delta H_1$	Elevation difference from A to I1
$m_{bsI1} - m_{fsI2} = \Delta H_2$	Elevation difference from I1 to I2
$m_{bsI(i-1)} - m_{fsI(i)} = \Delta H_i$	Elevation difference from I(i–1) to I(i)
$m_{bsI(n-1)} - m_{fsB} = \Delta H_n$	Elevation difference from I(n–1) to B

$$\sum_{i=1}^{i=n} m_{bs} - \sum_{i=1}^{i=n} m_{fs} = \sum_{i=1}^{i=n} \Delta H_i = \Delta H_{AB}$$

The total elevation difference ΔH_{AB} from A to B is equal to the sum of the backward readings minus the sum of the forward readings.

Note: If the path is closed, the total height difference must be zero.

V.1.7. Closing the path

Knowing the altitude of **A**, we can calculate again from the ground measurements, the altitude of **B**: we call this value $\mathbf{H_B}$ the observed value, noted $\mathbf{H_{Bobs}}$. It is defined by:

$$H_{B(obs)} = H_A + \sum_{i=1}^{n} \Delta H_i$$

If the measurements were error-free, we would find exactly the known altitude $\mathbf{H_B}$. In reality, there is a gap called **path closing error** (or more simply **closing**) which is subject to tolerance. This closure noted f_H is worth:

$$f_H = H_{B(obs)} - H_B$$

A mnemonic to help remember the meaning of this subtraction is to remember that the sign of the closure error f_H must be positive if the observed altitude is higher than the actual altitude, i.e.: $f_H > 0$ implies that $H_{Bobs} > H_B$.

If we call $T_{\Delta H}$ **tolerance** regulatory closure of the path, we must therefore check: $|f_H| < T_{\Delta H}$. If this is not the case, the measurements must be redone.

V.1.8. Path compensation

Compensation is the operation of distributing the closure over all the measurements. Compensation, noted C_H, is therefore the opposite of closure, that is to say:

$$C_H = -f_H$$

This adjustment consists of modifying the partial differences in height by distributing the total compensation C_H on each of them. This distribution can be done in several ways:

V.1.8.1. Proportionally to the number *N* of differences in height: This type of compensation will be chosen in the case where the closure is very weak, that is to say less than the standard deviation, $\sigma_H = T_H/2{,}7$. So the compensation on each difference in heigth is:

$$C_{Hi} = \frac{C_H}{N}$$

In the case where the closure is between standard deviation and tolerance, one can choose between the following two distribution methods:

V.1.8.2. Proportionally to the span: it is considered that the greater the span, the more the height difference can be affected by error. This requires knowing an order of magnitude of the span, which is obtained by stadiametry. The compensation on each height difference is then:

$$C_{Hi} = C_H \frac{L_i}{\sum L_i}$$

V.1.8.3. Proportionally to the absolute value of the difference in height: the compensation to be applied to each partial difference in height of the path is therefore:

$$C_{Hi} = C_H \frac{|\Delta H_i|}{\sum |\Delta H_i|}$$

V.2 Indirect Leveling

V.2.1.Definition

Indirect trigonometric leveling is used to determine the difference in height **ΔH** between the station **T** of a theodolite and a point **P** aimed (**Figure V.7**). This is done by measuring the inclined distance along the line of sight **Di** and the **zenith** angle, from the sketch, we can write that:

$$\Delta H_{TP} = ht + D_i * \cos V - hv$$

ΔH_{TP}: is the height difference of **T** towards **P**.

ht: is the station height (or trunnion height).

hv: is the height of the indicator or more generally the height aimed at above the point sought (we can also place a rod in **P**).

Di: is the sloping distance.

We deduce the horizontal distance $\mathbf{Dh_{TP}}$: $Dh_{TP} = D_i * \sin V$

We deduce the distance following the slope **Dp**: $Dp = \sqrt{(\Delta H)^2 + (Dh)^2}$

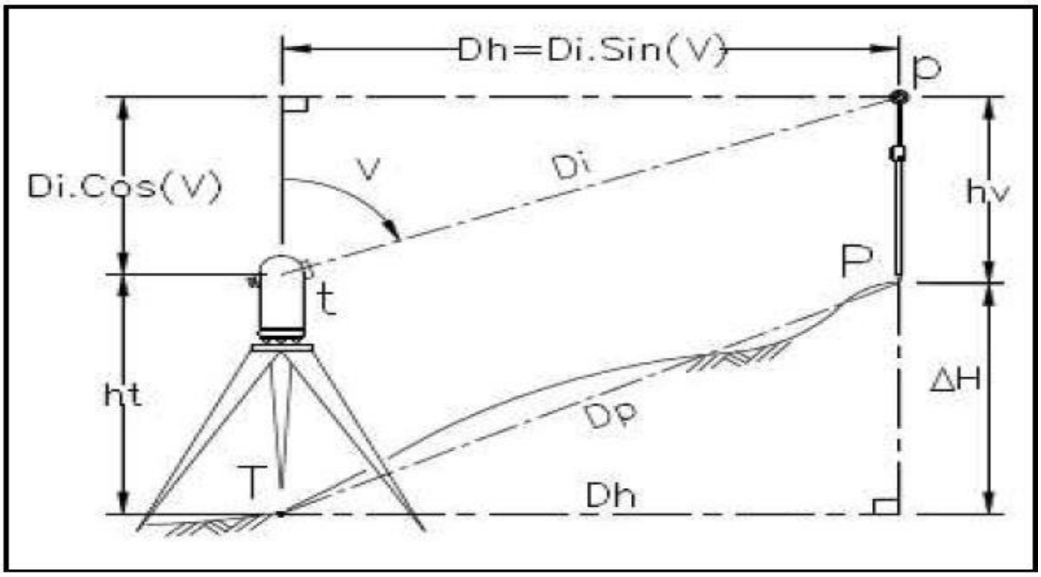


Figure V.7: Indirect trigonometric leveling

V.2.2. Comparison with direct leveling

- ***Advantages of indirect leveling over direct leveling:***

1) Indirect leveling can be done on steep terrain without increasing the number of stations, unlike direct leveling;

2) The measurement of the difference in altitude is made at the station on the known point, which can save time during a route or during a detailed survey by radiation since we directly obtain the altitude of the targeted points in addition to their coordinates in planimetry;

3) if we use an electronic theodolite, we can make long sightings, of several kilometers, which is not possible with direct leveling, a reading on a staff at 100m being already difficult.

- ***Disadvantages of indirect leveling compared to direct leveling:***

1) on a construction site, to obtain a simple difference in level for the purpose of verification or installation, the level remains simpler and quicker to set up and above all easier to control by non-specialists;

2) long spans require taking into account errors due to terrestrial spherical, atmospheric refraction, and ellipsoid reduction corrections. But computer science overcomes this drawback by directly providing data corrected for these errors. Only direct precision leveling makes it possible to obtain millimetric accuracies on the differences in height. the precision of modern station electronic length measuring instruments (IMEL) makes it possible to approach centimeter precision on the difference in height over spans of the order of a kilometer.

V.2.3. Indirect leveling with a theodolite

On very short ranges (**Dh < 100 m**), indirect levelling can be carried out with an optical-mechanical theodolite, a chain and a staff. The precision obtained is poor but may be sufficient in certain cases, for example, for calculating approximate altitudes for a preliminary earthworks project.

V.2.3.1. Case where the distance following the slope *Dp* is measurable

This is the case where the land has a regular slope between **S** and **A**. We can then measure the distance directly. **Dp** to the chain with correct precision

The method is as follows: from the theodolite stationed in **S**, the operator aims the rod by intercepting the corresponding graduation at the height of the trunnions **ht** so that the sight is parallel to the line **SA** whose length the operator measured **Dp**. He reads the angle **V** corresponding, it measures **Dp** and deduces that:

$$Dh = D_p * \sin V$$

$$\Delta H = D_P * \cos V$$

$$H_{HAS} = H_S + \Delta H$$

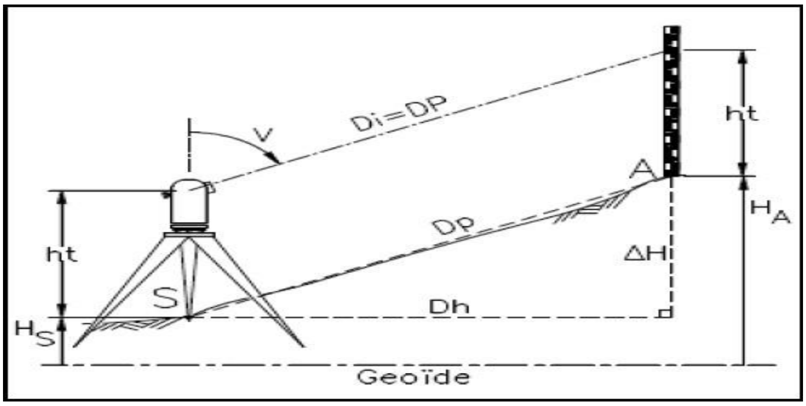


Figure V.8: Distance along the measurable slope

V.2.3.2. Case where if the distance following the slope Dp is not measurable:

This is the case when the slope is irregular, on very bumpy terrain or if there are obstacles.

The method consists of calculating the horizontal distance **Dh** from station **S** from readings on a target placed in **A**. We determine **Dh** by stadiametry from the readings m_1, m_2 and **V**. and the difference in height **ΔH** is then determined by the following formula:

$$\Delta H_{TP} = h_t + D_h \times cotgV - L_m$$

Lm: the height read on the leveling line rod in (m)

And the horizontal distance equals: $Dh = 100 * (m_2 - m_1) * sin^2 V$

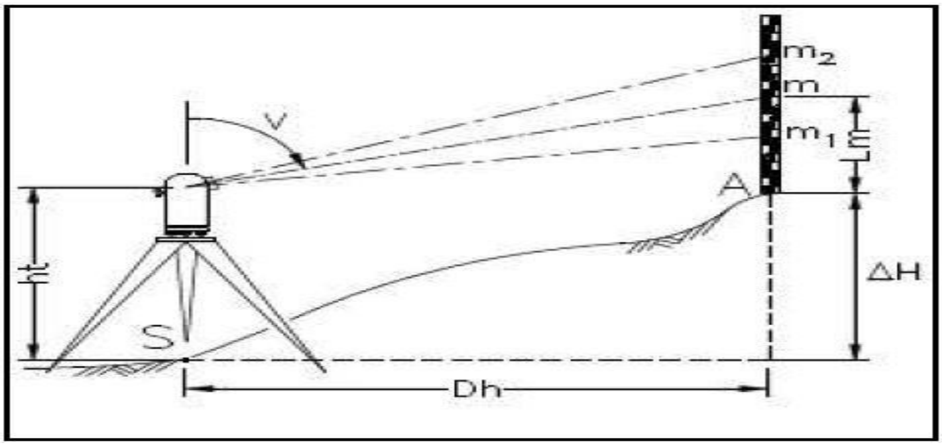


Figure V.9: Distance along the slope not measurable

V.2.4. Regulatory tolerances in indirect leveling

These tolerances applicable to large-scale surveys undertaken by public services, can be used as an indication for routine work. Each of the tolerances ***Ti*** following applies to the difference between two independent determinations of the same height difference by reciprocal sightings. The tolerance on the complete path will be as follows:

$$T = \sqrt{\sum (T_i)^2}$$

V.2.4.1 Elevation difference calculated from the horizontal distance

The tolerance on the measurement of a difference in level is:

$$T_{\Delta H} = \sqrt{A^2 + T_{DH}{}^2 \times tg^2 i + (1 + tg^2 i)^2 \times Dh^2 \times T_i^2 + Dh^4 \times Tq^2}$$

A: is a constant term: knowledge of the station point, of ***ht***, ***hv*** and holding the mirror (or the rod);

i: the elevation angle (i=100-V);

Ti: tolerance on angle measurement **i**;

Dh: horizontal distance of the aim calculated from the coordinates of the points at the average altitude between station and aiming point;

T_{Dh}: tolerance on knowledge of **Dh=4cm** in precision canvas, **20cm** in ordinary canvas;

Tq: tolerance on knowledge of apparent level correction.

V.2.4.2 Elevation difference calculated from an inclined distance measurement

The tolerance on the measurement of a difference in level is:

$$T_{\Delta H} = \sqrt{A^2 + T_{Di}{}^2 \times \sin^2 i + \cos^2 i \times Di^2 \times T_i^2 + Dh^4 \times Tq^2}$$

T_{Di}: (cm) is 3+Di if using a distance meter;

The other terms are the same as in the previous paragraph;

V.1.4.3. Summary tables

These tables provide formulas adapted to the different measurement cases: unilateral, simultaneous or non-simultaneous reciprocal sights. They are established with the following values (the tolerances are given in centimeters):

A=2cm (holding the indicator or rod and knowledge of ***ht*** and ***hv***) ;

T_{Dh}=4 cm or 20 cm (3+Dh (KM)) depending on the case;

Tq=1cm ;

Ti=5.6 mgrade for one-sided aiming.

Dh and **Di** are expressed in kilometers.

Note :

- For reciprocal sights, the vertical angle is measured twice; therefore, the tolerance T_{Dh} is divided by √2.
- For non-simultaneous reciprocal sights, the error due to knowledge of the coefficient of apparent level correction is divided by √2.
- For simultaneous reciprocal sights, it is divided by two.

V.1.4.3.1. Elevation difference deduced from the horizontal distance from the coordinates

The tolerances in centimeters are as follows:

Aims	Tolerances in centimeters
Unilateral (reciprocal)	$\sqrt{4+16\times tg^2 i+40\times Dh^2\times(1+tg^2 i)^2+Dh^4}$
Non-simultaneous reciprocals	$\sqrt{4+16\times tg^2 i+40\times Dh^2\times(1+tg^2 i)^2+\frac{Dh^4}{2}}$
Simultaneous reciprocals	$\sqrt{4+16\times tg^2 i+40\times Dh^2\times(1+tg^2 i)^2+\frac{Dh^4}{4}}$

V.1.4.3.2. Elevation difference deduced from the measured inclined distance

The tolerances in centimeters are as follows:

Aims	Tolerances in centimeters
Unilateral (reciprocal)	$\sqrt{4+(3+D_i)^2\times\sin^2 i+\cos^2 i\times 80Di^2+Dh^4}$
Non-simultaneous reciprocals	$\sqrt{4+(3+D_i)^2\times\sin^2 i+\cos^2 i\times 40Di^2+\frac{Dh^4}{2}}$
Simultaneous reciprocals	$\sqrt{4+(3+D_i)^2\times\sin^2 i+\cos^2 i\times 40Di^2+\frac{Dh^4}{4}}$

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