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Soil Mechanics I

Course Support and Tutorials

2nd year Civil Engineering

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FOREWORD

The current work introduces the fundamentals of soil mechanics to students beginners of the 2nd year of the common core specializing in Civil Engineering. It constitutes a synthesis of the basics of soil mechanics allowing students to grasp the essentials of the subject within the limits of the program.

The content represents a reminder covering the four chapters of the official program, namely:

1- Introduction to soil mechanics
2- Identification and classification of soils
3- Soil compaction
4- Water in the soil

Each chapter presents the course, accompanied by solved exercises at the end of the chapter.

The objective of this course with some applications is to help the student understand soil mechanics, and get used to using it and recognizing its delicate points.

The document is organized into chapters. Each chapter presents the course, accompanied by solved exercises at the end of the chapter, but it should be noted that the availability of this document should not in no case discourage the student from attending the oral course: writing can never replace learning from a master.

I hope that this modest work will be of benefit to our dear students in their training, for this reason I invite them to explore the various concepts and their knowledge through the consultation of the bibliographical references cited at the end of the work, I would like to warmly thank their authors, in particular Mr Mekerta Belkacem , Mr Guettouche Amar, Mr Khaled Meftah and Mr Khelifa Harichane .

AKACEM Mustapha

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<u>CHAPTER I</u>:

Introduction to soil mechanics

Chapter 1. Introduction to soil mechanics

I.1 Purpose of soil mechanics (history and domain)

I.2 Definitions of soils

I.3 Origin and formation of soils

I.4 Structure of soils

1.1 Purpose of soil mechanics

Soil mechanics is the application of mechanical and hydraulic laws to soil material. Compared to the many other materials studied in mechanics, concrete, steel, plastics, wood, etc., soil has two original features. First, it is a discontinuous medium that must therefore be studied both in its entirety and in its elementary composition. Secondly, it is a three-phase material formed of solid grains, water and air. We will see that non-solid phases play a fundamental role.

1.2 Areas of application

The fields of application of soil mechanics are numerous and varied. They concern the profession of public works, as well as that of construction.

1.2.1 Natural environments

The field of application of soil mechanics is not limited to construction; it also includes natural environments such as slopes (landslide problems) and the banks of watercourses or reservoirs.

1.2.2 - Ground works

The works where soil is the basic material are also: - embankments (roads, railways, dams, earth basin dikes, maritime platforms, etc.); - or cuttings (slopes, canals, basins, etc.).

1.2.3 - Mixed works

In mixed structures, the soil is involved in relation to another material, such as concrete or steel. The anchoring conditions in the soil are often essential for structures such as:

- retaining walls (concrete, reinforced earth, geotextile-reinforced soil, etc.);
- sheet piles used in canals, ports, urban construction, etc.;
- diaphragm walls (for waterproofing or retaining purposes).

1.2.4 - Foundations of works or buildings

In the study of foundations, the ground and the structure do not constitute a mixed whole, but two wholes whose interactions must be understood.

Soil mechanics distinguish:

- surface foundations (footings or rafts);
- deep foundations (piles, wells, bars).

All structures such as water towers, sewage treatment plants, silos, earth or concrete dams, retaining walls, must be the subject of a foundation study which will determine the depth of the foundation and the dimensions of the base of the structure. This is too often neglected and many serious disorders have resulted from it.

The structures use the soil as much as an element of the infrastructure that transmits the overall load of the structure to a sufficiently stable and resistant layer of soil. As a result, the success of the structure depends on the success of the foundation project. Depending on the type of structure and its design method, the soil can constitute a support base for the entire structure such as a road, tunnel, gravity dam, retaining wall, airfield, or a support point for only a few elements such as a building, bridge, arch dam, etc. Soil (and rock) mechanics is the science that brings together all the knowledge and techniques that make it possible to carry out the following tasks:

- Identify the characteristics that govern the mechanical behavior of the soil.
- Analysis of soil-structure interaction
- underground works correctly .

1.3 Disciplines of soil mechanics

In order to achieve the objectives mentioned above, several disciplines will be necessary such as:

Terrain geology - Physicochemical characteristics - Mechanical characteristics - Theoretical research and numerical modeling - Design and implementation.

1.4 History of soil mechanics

Soil mechanics is a young science, the first foundations can be attributed to COULOMB (1773), but TERZAGHI (1883-1963) truly initiated modern soil mechanics.

The evolution of soil mechanics can be followed through its emergence as a science in its own right and the development of its major theories (see Table 1.1).

Tab.1.1 **:** *Soil mechanics through its major theories* **.**

Century	Author	Theory
18th	Coulomb	Shear strength
19th	Collin	Breakage in clay slopes
	Darcy	Water flow in sand
	Rankine	Earth pressure on retaining walls
	Gregory	Horizontal drainage. Compact embankment with buttress to stabilize the slope of railway trenches
20th	Atterberg	Consistency limits of clay
	Terzaghi	First modern textbook of soil mechanics
	Casagrande	Essay on the liquidity limit

1.5 Definitions of soils

What is soil? - Soils can be defined as aggregates in which particles are loosely bound and can be separated by light mechanical action - A soil in place is made up of solid grains bathed in water, in air or in a mixture (water + air), the soil is a 3-phase material: solid, liquid (water) and gas (air).
In geotechnical studies, the materials existing on the surface of the earth's crust are classified into two broad categories:

- rocks : agglomerates of mineral grains bound by strong and permanent cohesive forces, even after prolonged immersion in water ⇒ Rock mechanics.

- soils : A soil is a heterogeneous assembly of particles or crystals with very variable properties: dimensions, shapes, physicochemical properties, etc., which can be separated under the effect of relatively weak mechanical actions ⇒ Soil mechanics.

So, we can say that soil is defined in opposition to the word rock, in its geotechnical definition. It is a natural aggregate of mineral grains, separable by light mechanical action. Soil is the result of a natural physical or chemical alteration of rocks. It is therefore understandable that the limit between soil and altered rock is not clearly defined. Soil is a loose material, this characteristic being fundamental. However, it is not enough to define natural soil because some materials produced by man also have this characteristic. For example, mining by-products and crushed aggregates (sand, gravel, ballast, etc.) are also loose materials. The soil mechanic therefore studies both natural soils and materials artificially manufactured from soils or rocks and having a loose character.

1.5.1 Constituent elements of a soil

A soil is a mixture of solid elements constituting the solid skeleton, water that can circulate or not between the particles and air or gas. It is therefore, in general, made up of three phases (Fig 1.1):

Sol = solid phase + liquid phase + gas phase

Between the grains of the skeleton, the voids can be filled by water, by a gas or both at the same time. The gas contained in the voids between the particles is usually air when the soil is dry or a mixture of air and water vapor when the soil is wet (the most common case). When water fills all the voids, the soil is said to be saturated. In temperate regions, most soils in place, a few meters deep, are saturated. When there is no water, the soil is said to be dry.

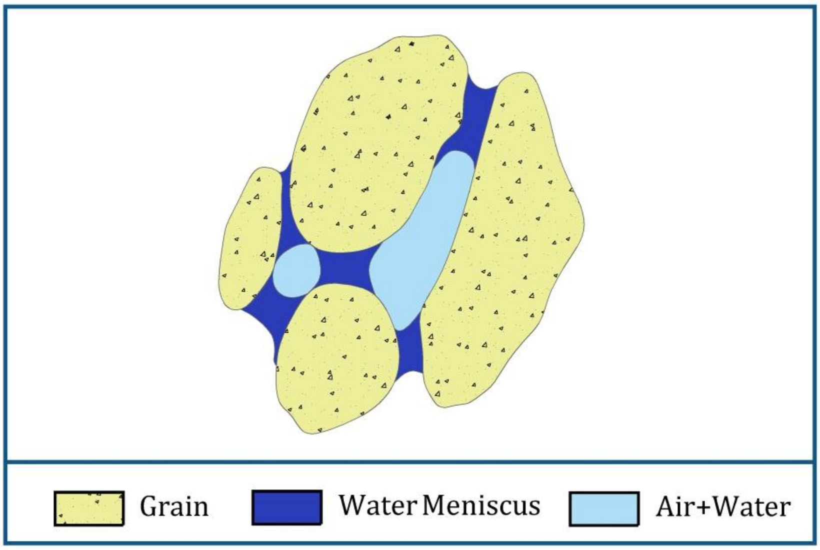

Fig. 1.1: *Constituents of a soil*

1.5.2 Origins and formation of soils

Where do soils come from? Simply rocks, but they can also contain organic matter.

Soils have two main origins: - the disintegration of rocks by mechanical or physicochemical alteration under the effect of natural agents - the decomposition of living organisms: plants (peat) or animals (chalks). Depending on the types of alteration, the resulting soils will have different compositions.

The physical and mechanical disintegration of consolidated rocks gives rock fragments of the same composition as the parent rock: gravel, sand, silt.

The physicochemical decomposition of the rock in place or of rock fragments gives new compounds: clays.

We also distinguish:

- **residual soils** resulting from the on-site alteration of rocks;
- **transported soils** originating from the deposit of alteration products previously taken up by a physical transport agent. It is transported soils which pose the most delicate problems for the designer of works. Finally, depending on their conditions of formation and deposit, soils may contain organic matter in greater or lesser proportions.

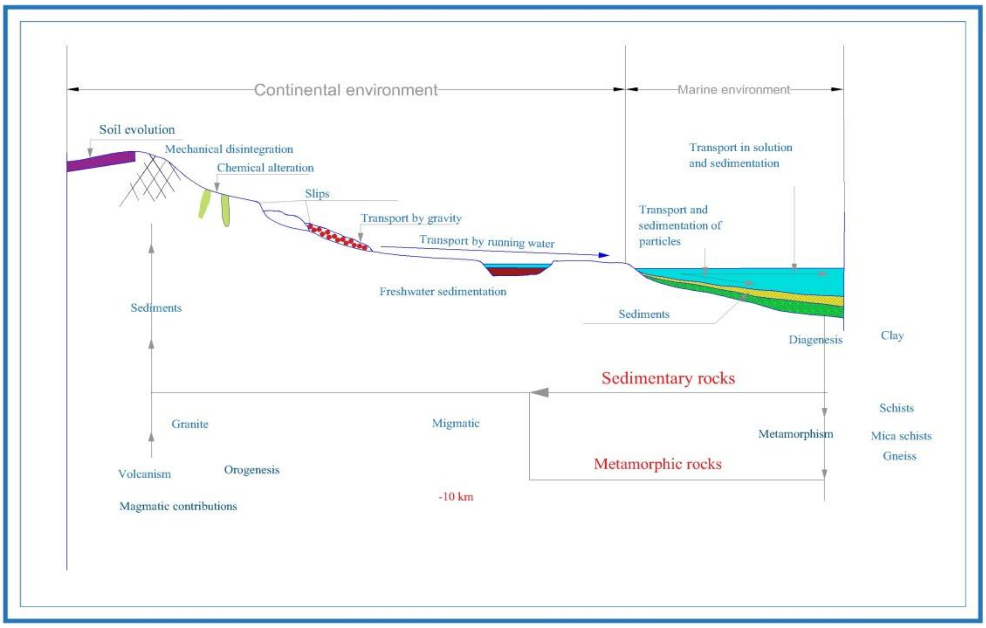

Fig.1.2: *Origins of soils*

1.5.3 Soil structure (grainy soils and fine soils)

Soil is a material made up of particles. The dimensions of these particles can be uniform or varied, ranging from 10 cm pebbles and extending to fine particles of less than a micron. The main physical characteristics of soil particles are:
the size, shape and specific surface area. These characteristics influence the hydraulic and mechanical properties of the soil.

1.5.4 Particle size

Particle size is measured by a diameter called equivalent diameter. The equivalent diameter of a particle is equal to the minimum square opening through which the particle can pass. The equivalent diameter was determined using square mesh sieves used in sieving granulometric analysis.

1.5.5 Particle shape

Although there are an infinite variety of shapes, two types are usually recognized: the bulky shape and the leaf shape.

a- ***Bulky shape:*** The bulky shape characterizes generally gravel, sand and silt particles.

The equivalent diameter of large particles is generally greater than 0.001 mm.

Most large particles are roughly spherical and have more or less rounded or more or less angular edges. (Fig.1.3)

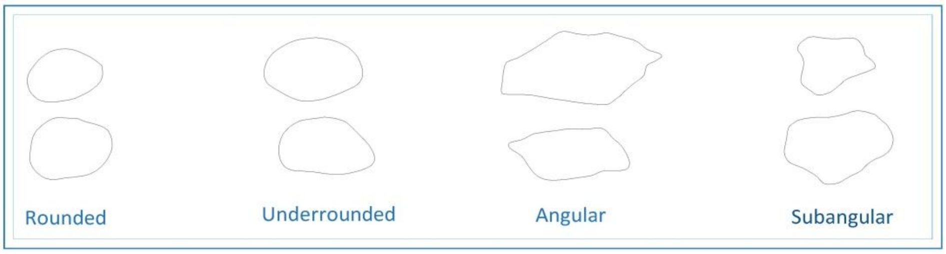

Fig.1.3: *Some typical shapes of coarse grains*

b- ***Sheet shape:*** When the ratio of its length to its thickness is greater than 10, a particle is considered to have a sheet shape.

This shape especially characterizes clay particles.

c- ***Specific surface area***

The specific surface area is the ratio of its total surface area to its mass (in m²/kg). Table 1.2 shows the average value of the specific surface area of particles of different types of soils.

1.5.6 Soil types

Different types of soils are usually identified according to the size of their particles.

In soil mechanics, the simplest division is to group soils into two broad classes: coarse-grained soils and fine-grained soils.

Tab.1.2: *Specific surface area of particles of different soil types.*

Soil type	Equivalent diameter (mm)	Typical thickness (ηm)*	Average specific surface area (m^2/kg)
Sand	1 to 2		1.5
Fine sand	0.25 to 0.5		6
Silt	0.002 to 0.05		82.5
Clay :			
- Kaolinite	0.0003 to 0.002	50 to 100	15000
- Illite	0.0001 to 0.002	30	90000
- Montmorillonite	0.0001 to 0.001	3	800000

*** 1 ηm = 10^{-9} m**

a- Coarse-grained soils:

Pebbles and blocks, or rockfill, have an equivalent diameter greater than 80mm. They are characterized by very high permeability. Gravel and sand are made up of rock particles whose equivalent diameter varies from 0.08mm to 80mm. Generally speaking, they have good permeability.

b- Fine-grained soils:

Silt is composed of fine rock particles whose equivalent diameter varies from 0.002mm to 0.08mm, and whose shape can be observed with a magnifying glass or optical microscope. Clay is made up of crystalline particles that come from the chemical decomposition of rock constituents. These are, for the most part, aluminum, magnesium or iron silicates whose atoms are arranged to form very regular geometric figures. Their equivalent diameter varies approximately from 1ηm to 0.002mm; more sophisticated techniques (such as SEM) must be used to observe these particles.

Each clay mineral is formed by the stacking of microscopic crystals (**sheets)** . These sheets are themselves made up of crystalline units called **fundamental structures** . These are juxtaposed in a single plane, and this is why the sheets have a very large surface area compared to their thickness. The thickness of the sheets and fundamental structures is estimated at about 0.5ηm (5×10^{-7} mm).

There are two fundamental structures:

- the fundamental tetrahedral structure, figure 1.4

- the fundamental octahedral structure, figure 1.4

There are three main families of clay minerals:

- kaolinite: this clay is dangerous for the engineer (see its schematic representation in figure 1.5);

- Montmorillonite: susceptible to significant swelling or shrinkage depending on variations in water content (see its representation in figure 1.6);
- illite , (see its schematic representation in figure 1.7).

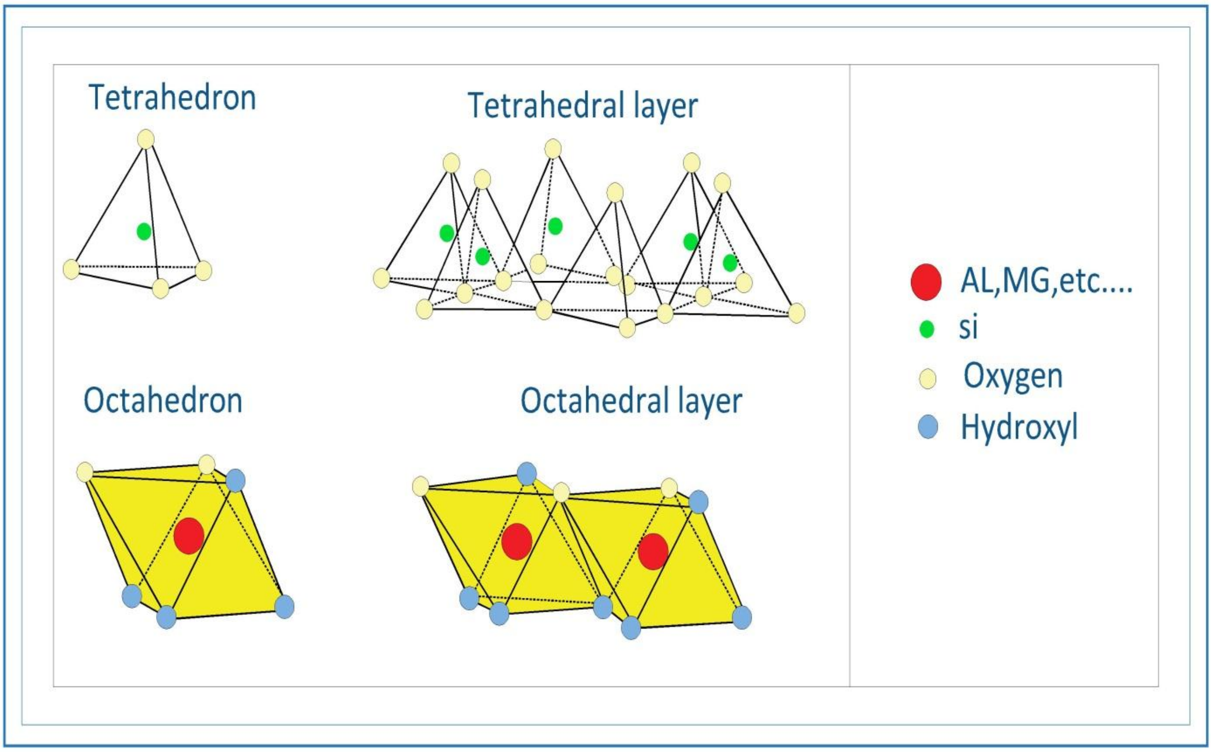

Fig.1.4. Schematic diagram of tetrahedral and octahedral structure

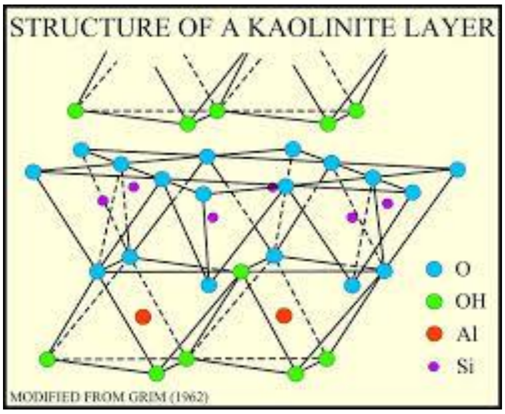

Fig.1.5 : Schematic representation of kaolinite

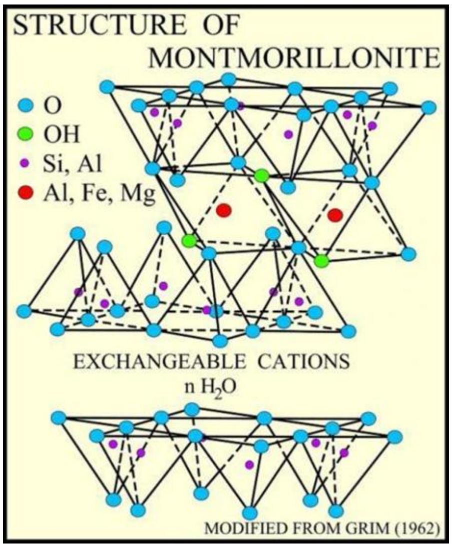

Fig.1.6 : Schematic representation of montmorillonite

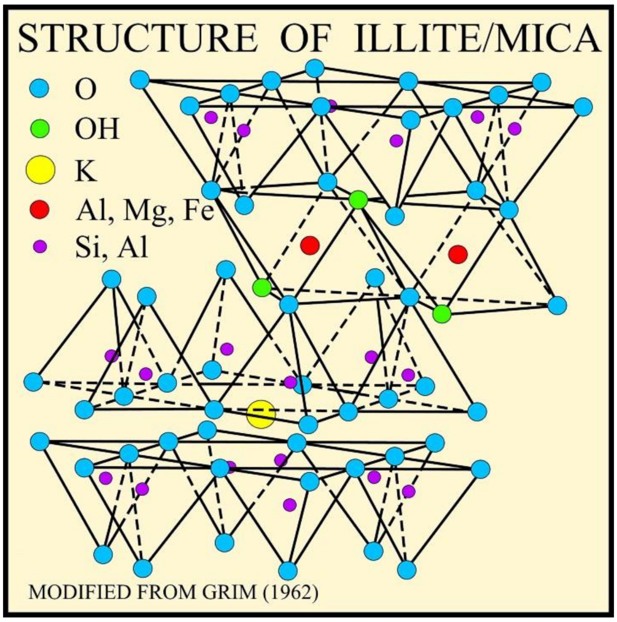

Fig.1.7: Schematic representation of illite

c- Organic soils:

They contain a high percentage of organic matter.

MO < 3%: inorganic soil

3% < MO < 10%: weakly organic soil

10% < MO < 30%: moderately organic soil.

CHAPTER II:

Identification and classification of soils

Chapter 2. Identification and classification of soils

2.1 Physical characteristics
2.2 Granulometric characteristics
2.3 Consistency of fine soils (Atterberg limits)
2.4 Geotechnical classification of soils

2.1 Physical characteristics

2. 1. 1 Elementary model of a soil

Since a soil is composed of solid grains, water and air, each phase can be grouped into a single partial volume of unit section. The following notations are used (Fig.2.1):

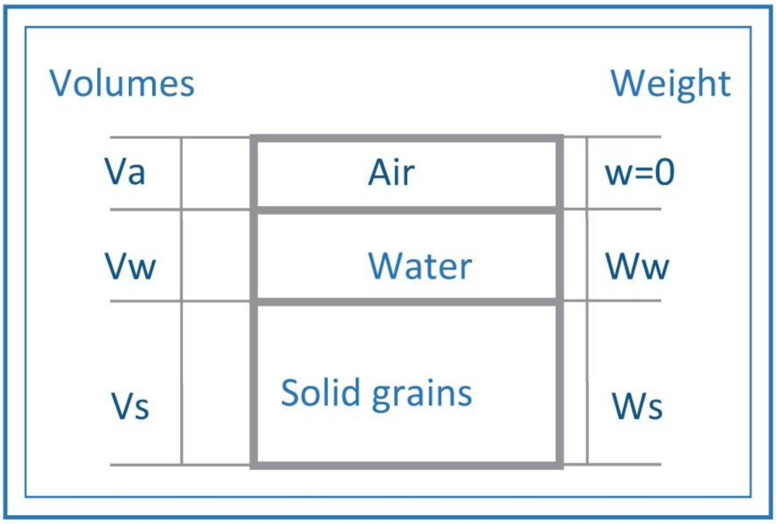

Fig.2.1: *Conventional representation of a volume of soil (Weight and volumes of the different phases)*

Conventional notations :

W : total weight of soil, Ws : weight of solid particles, **Ww** : weight of water
V : total (apparent) volume, **Vs** : volume of solid particles
Vv :volume of voids between particles, **Vw** : volume of water, **Va** : volume of air

with the relations:

W = Ws + Ww ; Vv = Vw + Va; V = Vs + Vv = Vs + Vw + Va

We also define the volumetric weights which, together with the weights and volumes, constitute:

a- Dimensional parameters (volume weights) :

• THE **volumetric weight of solid particles** (of the material constituting the solid grains), noted **γs** : $\gamma s = Ws/ Vs$; sand and clay = 26 to 27 kN/m^3

• THE **volumetric weight of water** , denoted **γw** : $\gamma w = Ww/ Vw = 9.81\ kN/m^3$. We often take $\gamma w = 10\ kN/m^3$. Which immediately results in a 2% relative error.

• THE **soil unit weight** (or apparent unit weight or wet unit weight), denoted **γ** . It is the sum of the weights of the solid particles and the water in a unit volume of soil. $\gamma = W/V$; sand = 17 to 20 kN/m^3, clay = 16 to 22 kN/m^3

• THE **volumetric weight of dry soil** , denoted **γd** :
$\gamma d = Ws /V$; sand = 14 to 18k N/m^3 , clay = 10 to 20 kN/m^3 .
If the soil is dry: $\gamma = \gamma d$.

• THE **volumetric weight of saturated soil** , denoted **γsat** : when all voids are filled with water. $\gamma_{sat} = W/V= (Ws + \gamma w . Vv)/V$

•THE **volumetric weight of the soil when it is leveled** , noted **γ'** : It is taken into account when the soil is completely submerged. It takes into account the presence of water which fills all the voids and the Archimedes' thrust: $\gamma' = \gamma sat - \gamma w$; sand and clay = 9 to 12 kN/m^3 .

We also introduce the notion **of density** , noted **ρi** , and more rarely that of **density** relative to water, noted **Di** :

Density: $Di = \gamma i / \gamma w \Rightarrow$ dry density: $Dd = \gamma d / \gamma w$

b- Dimensionless parameters (state parameters) : They indicate in what proportions are the different phases of a soil. They are very important and essentially variable. We define:

• **Porosity,** noted **n** , which allows us to know the importance of the voids , that is to say to know if the soil is in a loose or tight state. It is defined as being the ratio of the volume of voids to the total volume. $n = Vv /V$; sand: n = 0.25 to 0.5; clay: n = 0.20 to 0.80; porosity is always less than 1. It can also be expressed in percent.

• **The void ratio** , denoted **e** , whose meaning is similar to that of porosity. It is defined by the relation:

e = Vv /Vs ; sand: e = 0.5 to 1, clay: n = 0.3 to 1.

The void ratio can be greater than 1 and even reach the value 13 (extreme case of Mexico clays).

• **The water content** , denoted **w** , is defined by the ratio of the weight of water to the weight of solid particles in a given volume of soil. It is expressed as a percent. It is easily measurable in the laboratory.

w = 100.Ww/Ws ; sand: w = 1 to 15%, clay: w = 10 to 20%
The water content can exceed 100% and even reach several hundred percent.

• **The degree of saturation,** noted **Sr** , indicates in what proportion the voids are filled by water. It is defined as the ratio of the volume of water to the volume of voids. It is expressed in percent. Sr = 100.Vw/ Vv ; The degree of saturation can vary from 0% (dry soil) to 100% (saturated soil).

Among all the parameters defined above, the dimensionless parameters are the most important. They characterize the state in which the soil is found, that is to say the state of compactness of the skeleton as well as the quantities of water and air contained in the soil.

• **Relative density or density index,** noted **Id** , is defined by the expression:

$$I_d = \frac{e_{max} - e}{e_{max} - e_{min}}$$

e_{min}: is the void ratio corresponding to the most compact state.
e_{max}: is the void ratio corresponding to the loosest state.
e: is the void ratio of the soil in place.

The indication of the density index gives an idea of the state of compactness of a given soil:

I_d= 0 for the loosest state (e = e_{max}) and I_d=1 for the most compact state (e= e_{min}).

2.1.2 Relationships between parameters

Not all the parameters previously defined are independent. The most important relationships existing between these different parameters are given as follows:

[1] $n = \frac{Vv}{V}$	[5] $e = \frac{Vv}{Vs}$	[9] $w = \frac{Ww}{Ws}$
[2] $n = \frac{e}{1+e}$	[6] $e = \frac{n}{1-n}$	[10] $w = e.Sr.\frac{\gamma_w}{\gamma_s}$
[3] $n = 1 - \frac{\gamma_d}{\gamma_s}$	[7] $e = \frac{\gamma_s}{\gamma_d} - 1$	[11] $w = \frac{\gamma}{\gamma_d} - 1$
[4] $n = \frac{\gamma_{s-}\gamma_{sat}}{\gamma_{s-}\gamma_w}$	[8] $e = \frac{\gamma_{s-}\gamma_{sat}}{\gamma_{sat-}\gamma_w}$	[12] $w = Sr\,.\,\gamma_w(\frac{1}{\gamma_d} - \frac{1}{\gamma_s})$
[13] $Sr = \frac{Vw}{Vv}$	[14] $Sr = \frac{\gamma_s}{\gamma_w}.\frac{w}{e}$	[15] $Sr = \frac{w}{w_{sat}}$ (γ_d constant)
[16] $\gamma = (1 + w)(1 - n).\gamma_s$	[17] $\gamma = \frac{1+w}{1+e}.\gamma_s$	[18] $\gamma = (1 + w)\,\gamma_d$
[19] $\gamma = \gamma_d + n.Sr.\gamma_w$	[20] $\gamma = \frac{\gamma_s + e.Sr.\gamma_w}{1+e}$	[21] $\gamma = (1 - n).\gamma_s +$ $n.Sr.\gamma_w$
[22] $\gamma_d = (1 - n)\,\gamma_s$	[23] $\gamma_d = \frac{\gamma_s}{1+e}$	[24] $\gamma' = \gamma_{sat} - \gamma_w$
[25] $\gamma' = (1 - n)(\gamma_s - \gamma_w$	[26] $\gamma' = \frac{\gamma_s - \gamma_w}{1+e}$	[27] $\gamma' = \frac{\gamma_s - \gamma_w}{\gamma_s}.\gamma_d$

2.2 Granulometric characteristics

2.2.1 Particle size distribution

To properly describe a soil, it is therefore necessary to know its granulometry, that is to say the distribution of its particles according to their equivalent diameters. Two laboratory tests make it possible to establish the granulometry of soils (Fig. 2.2):

- granulometric analysis by sieving;

- granulometric analysis by sedimentation.

a- Granulometric analysis by sieving

The test consists of passing a representative sample of soil through superimposed sieves whose openings decrease from top to bottom. The largest particles therefore remain trapped on the highest sieves (rejects or retained), while the finer particles move towards the lower sieves (sieve or passing). When the masses retained on each sieve become constant, the sieving is finished and all the rejects are weighed. The mass of each reject is then compared to the total mass of the sample, which makes it possible to calculate the percentages of cumulative rejects and passing. The results are plotted on a semi-logarithmic graph or they construct a granulometric curve.

b- Granulometric analysis by sedimentation

In order to estimate the particle size distribution of silt and clay particles, a sedimentation particle size analysis is carried out. The method consists of measuring the sedimentation time in a water column, i.e. the rate of fall of the particles. From Stokes' law, the grain size is determined:

$$\boxed{v = \frac{9.8D^2\,(D_{rs} - D_{rl})}{3\eta} \Rightarrow D = \sqrt{\frac{3v\eta}{9.8\,(D_{rs} - D_{rl})}}}$$

Where:
D = diameter of the sphere (mm); v = falling speed of the sphere (cm/min);
Drs = relative density of the sphere;
Drl = relative density of the liquid;
= dynamic viscosity of the liquid (Pa.s).

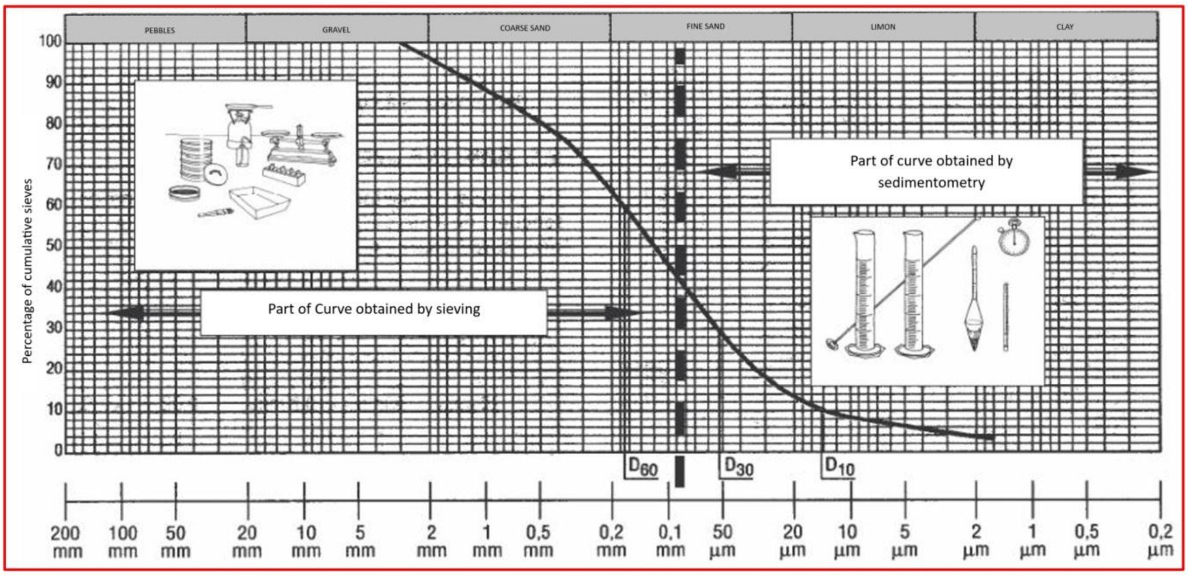

Fig.2.2: *Granulometric curve.*

2.2.2 Interpretation of granulometric curves

At first glance, a granulometric curve allows us to identify the types of soils that make up the sample analyzed. If we examine a granulometric curve of a sample made up of gravel, sand, silt and clay, we find the respective proportions of each type of soil expressed as percentages. When we know these proportions, it becomes possible to assign a well-defined name to the soil. Thus, if the soil is composed of 27% gravel, 38% sand, 29% silt and 6% clay for example, it is called gravelly silty sand with traces of clay.

The granulometry of a soil can be characterized by the uniformity coefficient (Hazen coefficient), and the curvature coefficient:

a- uniformity coefficient: allows the spread of the granulometric curve to be expressed: $C_u = \frac{D_{60}}{D_{10}}$

Or:

D_{60}= effective particle diameter which corresponds to 60% of the passer.
D_{10}= effective particle diameter which corresponds to 10% of the passer.
Depending on the value of the uniformity coefficient, five classes of particle size are recognized (Tab.2.2):

Tab. 2.2 *: Soil granulometry classes*

Uniformity coefficient	Grain size class
$C_u \leq 2$	Very tight grain size
$2 < C_u \leq 5$	Tight grain size
$5 < C_u \leq 20$	Semi-spread granulometry
$20 < C_u \leq 200$	Spread granulometry
$C_u > 200$	Very spread granulometry

b- curvature coefficient: allows to describe the shape of the granulometric curve:

$$C_c = \frac{(D_{30})^2}{D_{10} * D_{60}}$$

Where: D_{30}= effective particle diameter which corresponds to 30% of the passer.

When certain conditions on C_u and C_c are satisfied ($1 \leq C_u \leq 3$), the soil is said to be well graded, that is to say that its granulometry is well spread, without predominance of a particular fraction. When its granulometry is discontinuous ($1 > C_u > 3$), with a predominance of a particular fraction, it is said to be poorly graduated.

Well-graded soils are naturally dense deposits with high bearing capacity. They can be easily compacted into embankments and form stable slopes.

2.3 Consistency of fine soils (Atterberg limits)

2.3.1 Definition

Consistency, which can be defined as a state of firmness, is linked to the cohesion forces between the particles and therefore only concerns coherent soils. It has an influence on the resistance to deformation.
Consistency depends mainly on the distance separating the particles of a soil (the higher the void ratio, the greater the distance). Indeed, since the voids are filled with water, measuring the water content makes it possible to evaluate the void ratio and, consequently, the distance between the particles. « w ↗ ⇒ e ↗ ⇒ distant particles ⇒ soft consistency ⇒ soil will deform easily »

2.3.2 Consistency states: There are four consistency states (Fig.2.3):
a- Solid state: When the soil is in the solid state, its particles are in contact with each other, and the adsorbed water films are very thin and touch each other: there is no free water between the particles. Drying out the soil does not produce any shrinkage. The soil has very high shear resistance and, under the effect of a load, the deformations are small before it ruptures. In civil engineering, we speak of soil having fragile behavior, similar to that of brick.

b- Semi-solid state: A soil in the semi-solid state has a low water content, and the cohesion bonds between its particles are very strong. The adsorbed water films, although still thin, separate the particles slightly, so that drying out of the soil would cause shrinkage. It follows that deformations of the soil caused by loads are always accompanied by cracks.

c- Plastic state: When the soil is in the plastic state, its water content is greater and its particles are further apart from each other. The films of adsorbed water are much thicker, but they still touch each other; the cohesion of the soil is weaker than in the semi-solid state. Under small loads, the soil deforms without cracks. Its consistency varies from that of soft butter to that of firm solids: it is a soil that can be shaped by hand.

d- Liquid state: When the soil is in the plastic state, its water content is so high that there is practically no cohesion between the particles, which are surrounded by their film of adsorbed water and isolated from each other by free water. The soil can then behave like a viscous liquid with a consistency varying from that of pea soup to that of soft butter.

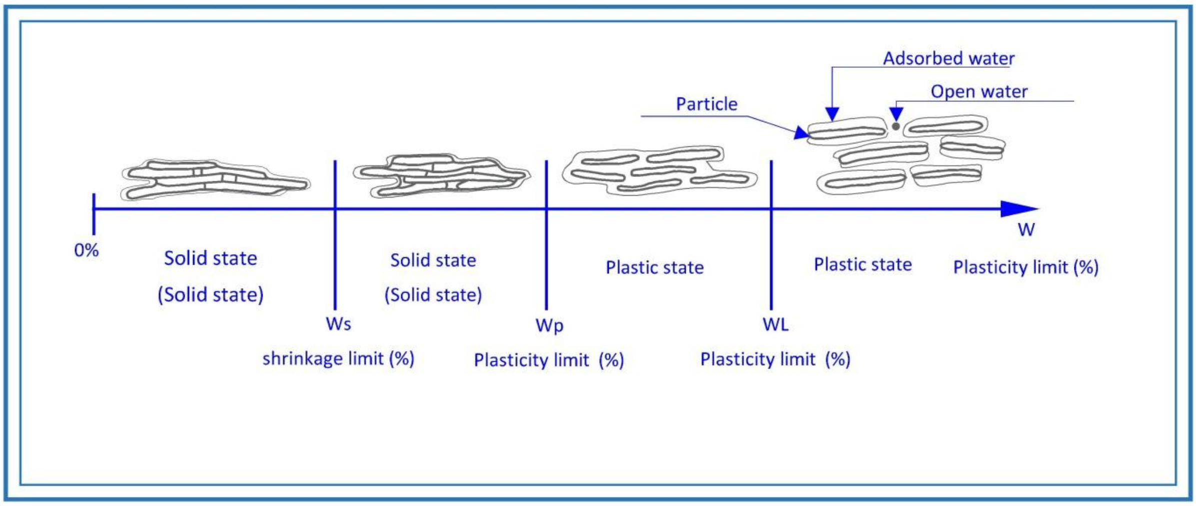

Fig.2.3: *Consistency states*

2.3.3 Atterberg limits

The water contents that delimit the four states of consistency are called consistency limits or Atterberg limits. These limits, which are expressed in percentages, are as follows:

- **Shrinkage limit (Wr):** is the maximum water content that the soil can have without changing volume. This limit separates the solid state from the semi-solid state.

- **Plastic limit (Wp):** it is defined as the water content of a soil that has lost its plasticity and cracks by deforming when subjected to low loads. This limit separates the plastic state from the semi-solid state. It varies from 0% to 100%, but it generally remains less than 40%.

- **Liquid limit (Wl):** is the water content that separates the liquid state from the plastic state. It can reach 1000% in the case of certain clays, but in most cases it does not exceed 100%. The plasticity and liquidity limits are used to identify and classify fine-grained soils. As for the shrinkage limit, it is used to study certain soils whose volume varies greatly due to changes in water content.

This is especially true in arid regions, where deposits of montmorillonite are found (shrinkage -swelling problem).

2.3.4 Plasticity and liquidity indices

- **Plasticity index (Ip),** which is expressed as a percentage, corresponds to the difference between the liquid limit and the plasticity limit: Ip = wl – wp . It is used to determine a zone in which this soil will be considered to be in a plastic state. (Fig.2.4).
- **Liquidity index (IL) ,** allows to know quickly if a soil is in the liquid, plastic, semi-solid or solid state. To establish this index, we compare the natural water content (in situ) (w) of a soil to its plasticity and liquidity limits:

$$I_L = \frac{W - W_P}{W_L - W_P} = \frac{W - W_P}{I_P}$$

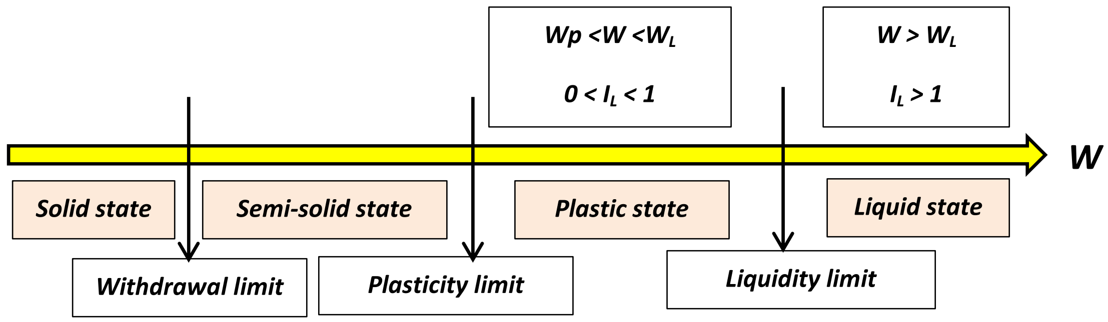

Fig. 2.4: *Atterberg Limits*

2.3.5 Clay activity

The activity of a clay (A) is equal to the ratio between its plasticity index and the fraction of clay present in a given soil:

$$A = \frac{I_P}{fraction\ d'argile}$$

The **clay fraction** corresponds to the percentage present in the soil of the weight of particles whose equivalent diameter is less than 0.002 mm. Observations demonstrate that the activity of a clay is constant and that each type of clay has its own activity.

2.3.6 The plasticity diagram

In 1932, Casagrande proposed a plasticity diagram (Fig.2.5) to identify fine-grained soils from the Atterberg limits. The diagram is divided into two zones by line A, each zone being further subdivided into three regions, depending on the plasticity of the soils.

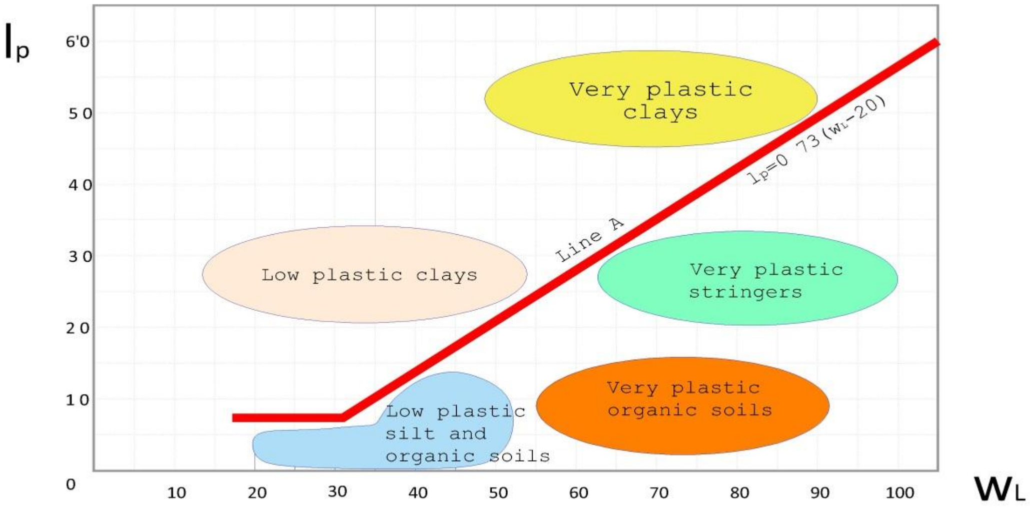

Fig.2.5: *Plasticity diagram (after Casagrande 1948)*

2.4 Geotechnical classification of soils

2.4.1 Principle of soil classifications

Soil classification systems arose from the need of civil engineers to have sufficiently reliable information on soil behavior to be able to make rapid and effective decisions, especially in the areas of road construction and infrastructure for airstrips or dams. The purpose of soil classification systems is to classify soils into families with the same or very similar geotechnical characteristics. They make it possible to group together a large number of samples collected during a survey campaign and to establish geotechnical sections of the terrain.

2.4.2 Triangular classification

Triangular classification systems take the form of triangles with scales on the sides representing the proportions of sand, silt and clay measured in a soil sample. (Fig.2.6).

These triangles are divided into zones with a conventional name according to the relative proportions of soil types. Only the scales of the proportions of clay, sand and silts (loam) appear on the triangle.

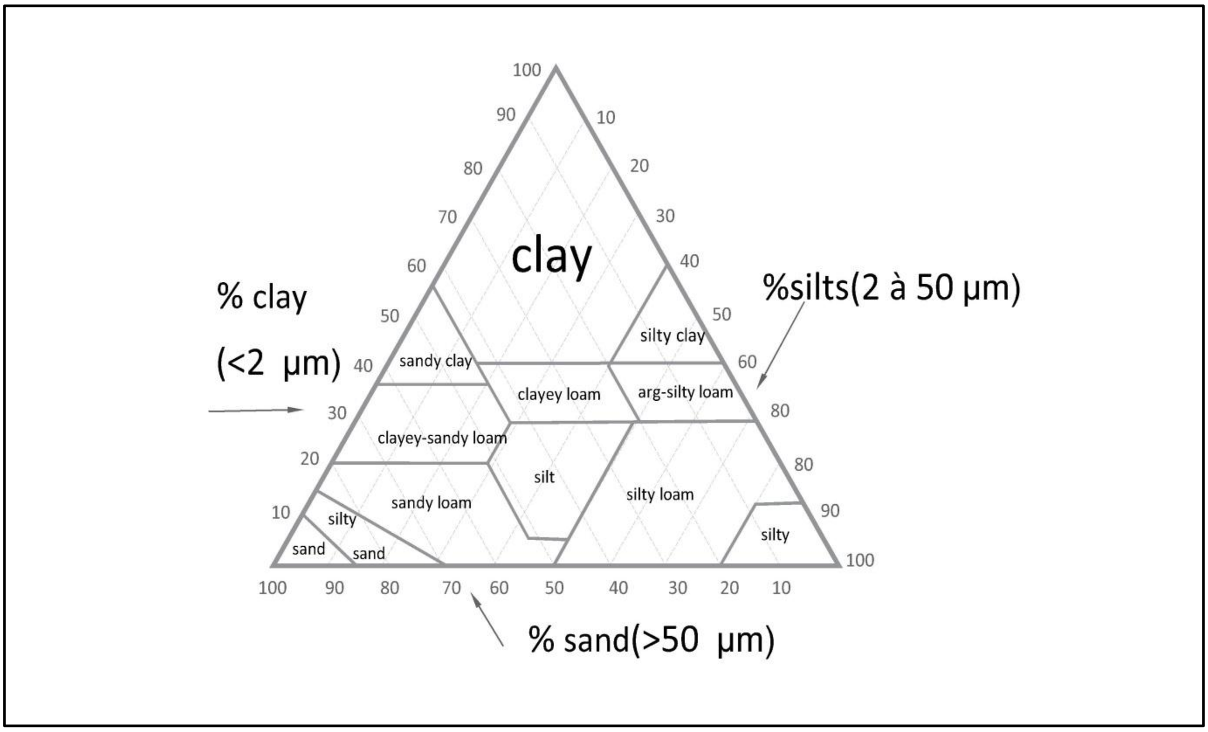

Fig.2.6: *Triangular classification of fine soils (containing less than 30% of elements with a diameter greater than 2 mm)*

Let us use the diagram in Figure 2.6 to classify a soil containing 41% clay, 42% sand, and 17% silt, i.e., a clayey sand with some silt. This is the point on the diagram that corresponds to these proportions, but it is located in the clay zone; this soil is therefore called clay, even though its clay fraction is not the largest. The triangular classification thus recognizes the importance of the clay matrix in the behavior of soils.

2.4.3 Classification (LPC/USCS) of soils

The LPC classification (1965) uses the results of classic soil identification tests:

▪Granulometric criteria:

— the percentages of gravel, sand and fine particles (2 mm and 0.08 mm sieve);
— the shape of the particle size curve: Cu and Cc (or Cz);

▪Plasticity characteristics W_L and, I_P, and line **A** of equation:

$$I_P = 0{,}73(W_L - 20) \quad \text{(Casagrande's report);}$$

▪The organic matter content.

a- ***Soil groups:*** The LPC classification system results in 15 soil types, each assigned a two-letter symbol, taken from the following three groups:

Tab.2.3: *Soil groups.*

Soil elements	Soil granularity	Soil plasticity
G : Gravel. Gravel is the main fraction **S** : Sand. Sand is the main fraction **L** : Silt or loamy **A** : Clay or clayey **T** : Peat **O** : Organic. Presence of organic matter	**b** : Well graduated **m** : Poorly graduated	**t** : Very plastic **p** : Little plastic

b- The classification procedure: The classification of coarse-grained soils is given in table **2.4** and figure **2.7** gives the classification of fine soils.

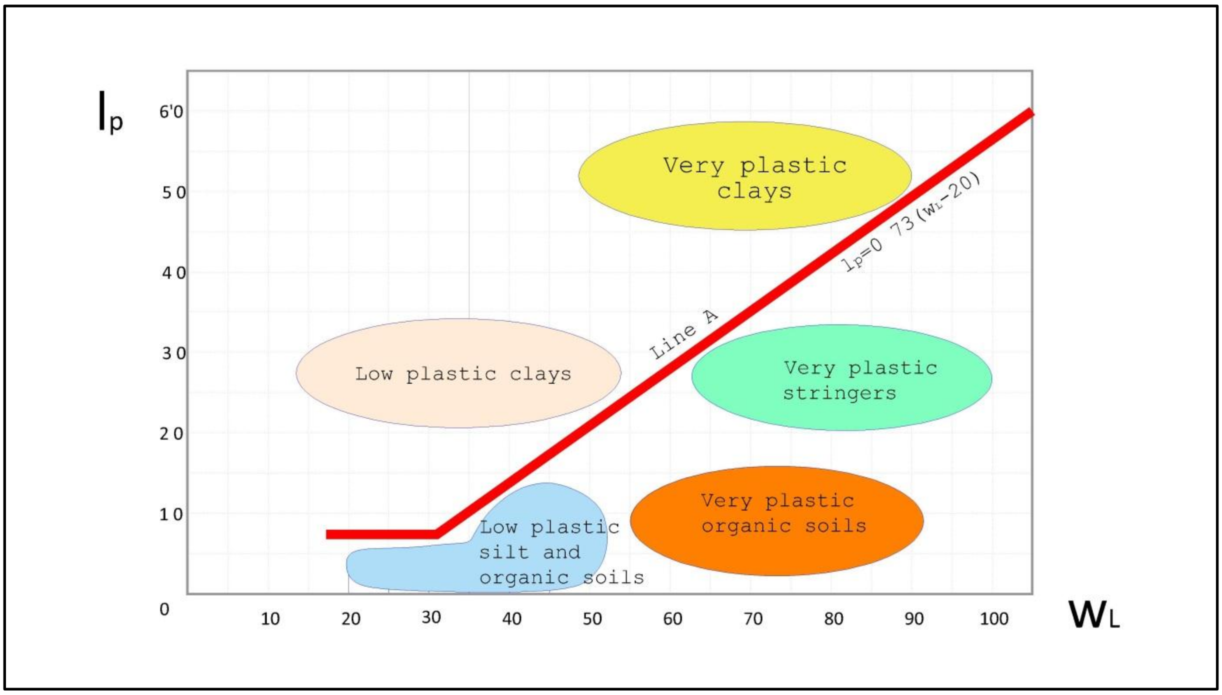

***Fig. 2.7** : LPC classification of fine soils in the laboratory. Plasticity diagram*

2.4.4 Modified LPC classification

The changes to the LPC/USCS classification relate exclusively to the description of organic soils (organic matter content greater than 3%). Only soils with less than 10% organic matter continue to be classified as fine soils. For higher organic matter contents, the emphasis is on the degree of decomposition (humification) of organic fibres, assessed using the Von Post Test.

Tab.2.4: *Classification of grainy soils – LPC/USCS*

<table>
<tr><th colspan="6">more than 50% of elements > 0.08 mm</th></tr>
<tr><th colspan="3">Definitions</th><th>Symbols</th><th>Terms</th><th>Appellations</th></tr>
<tr><td rowspan="4">SERIOUS</td><td rowspan="4">More than 50% of elements > 0.08 mm have a diameter > 2 mm</td><td rowspan="2">less than 5% of elements < 0.08 mm</td><td>Gb
(GW)</td><td>Cu > 4
and Cc between 1 and 3</td><td>well-graduated grave</td></tr>
<tr><td>Gm
(GP)</td><td>One of the conditions of Gb not satisfied</td><td>serious clean poorly graduated</td></tr>
<tr><td rowspan="2">more than 12% of elements < 0.08 mm</td><td>Gm
(GP)</td><td>Atterberg limit below A</td><td>silty gravel</td></tr>
<tr><td>GL
(GM)</td><td>Atterberg limit above A</td><td>clayey gravel</td></tr>
<tr><td rowspan="4">SANDS</td><td rowspan="4">More than 50% of elements > 0.08 mm have a diameter < 2 mm</td><td rowspan="2">less than 5% of elements < 0.08 mm</td><td>Sb
(SW)</td><td>Cu > 6
and Cc between 1 and 3</td><td>well-graduated sand</td></tr>
<tr><td>Sm
(SP)</td><td>One of the conditions of Sb not satisfied</td><td>poorly graded clean sand</td></tr>
<tr><td rowspan="2">more than 12% of elements < 0.08 mm</td><td>SL
(SM)</td><td>Atterberg limit below A</td><td>silty sand</td></tr>
<tr><td>SA
(SC)</td><td>Atterberg limit above A</td><td>clayey sand</td></tr>
<tr><td colspan="6">When 5% < % less than 0.08 mm < 12%: a double symbol is used.</td></tr>
</table>

The modified LPC classification results in twenty-two types of soils:
- coarse-grained soils: Gb, Gm , GA, GL, Sb, Sm , SA, SL

- fine soils: At , Ap , Lt , Lp

- weakly organic soils: fO-At , fO-Ap , fO-Lt , fO-Lp

- moderately organic soils: mO-a , mO-sf , mO -f

- very soils organic : tO -a, tO-sf , tO -f.

The symbols for coarse and fine soils are the same as in the LPC/USCS classification. For organic soils, the symbols: a, sf and f mean “amorphous organic matter”, “semi-fibrous organic matter” and “fibrous organic matter”.

2.4.5 GTR Classification

A classification of materials usable in the construction of embankments and subgrades of road infrastructures and given by the GTR (Setra LCPC;

1992).

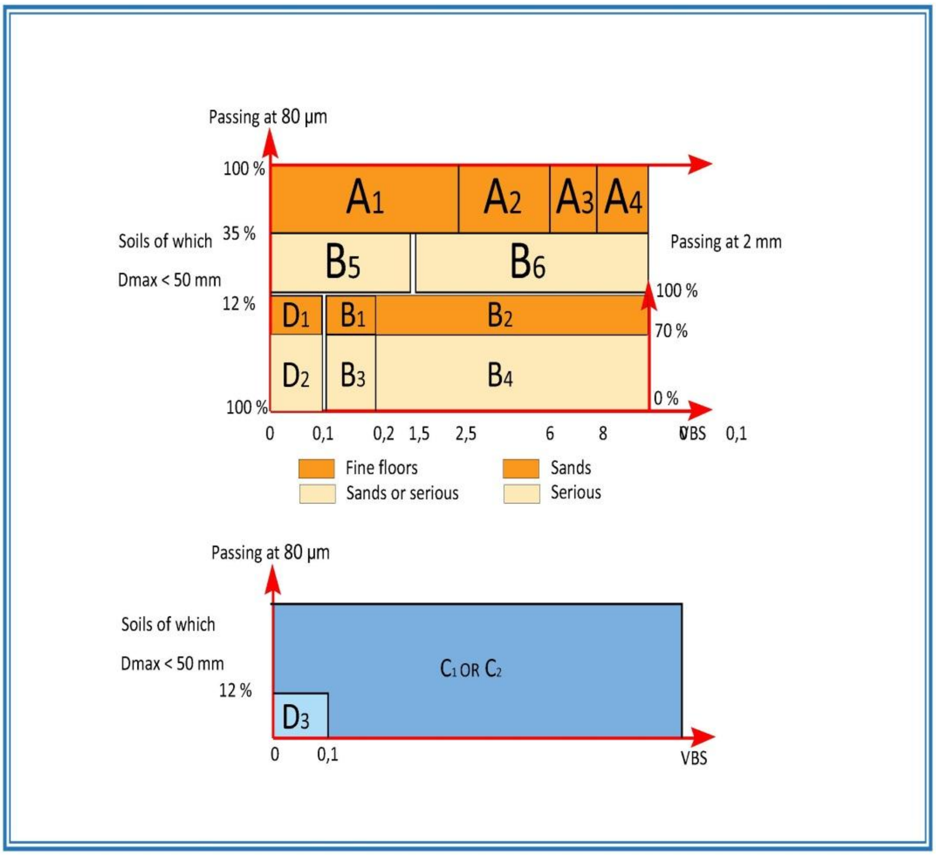

Fig.2.8: *General structure of the GTR classification for road earthworks*

The GTR classification allows soils to be classified, according to a certain number of parameters (passing at 80 µm, passing at 2 mm, the plasticity index Ip and the blue value Vbs), into:

- Class A - Fine floors

- Class B - Sandy and gravelly soils with fines

- Class C - Soils containing fine and large elements

- Class D - Floors insensitive to water.

The general principles of this classification are presented in Figure 2.8. The detailed classification is described in a technical guide from the LCPC and SETRA and in the NF P 11-300 standard.

2.5 EXERCISES

Exercise 1

We know for a soil:
- the volumetric weight = 14 kN/m3 - the water content = 40%
- the volumetric weight of solid particles = 27 kN/m3 Then
calculate the dry volumetric weight, the degree of saturation, the void ratio
, the porosity, the saturated volumetric weight as well as the
planed volumetric weight.

Solution:
Using the relationships between soil parameters, we will have:

$\gamma = \gamma d\ (1+w)$ gives $\gamma d = \gamma /(1+w) = 14/(1+0.4) = 10\ kN/m^3$:

$w = Sr\ \gamma w\ (1/\gamma d - 1/\gamma s)$, which gives : $Sr = 0.64 = 64\%$
$e = \gamma s / \gamma d - 1 = 1.7$
$n = e/(1+e) = 0.63$
$\gamma = \gamma d + nSr\ \gamma w$, which gives (with Sr = 1): $\gamma sat = \gamma d + n\gamma w = 16.3\ kN/m^3$

$\gamma' = \gamma sat - \gamma w = 6.3\ kN/m^3$

Exercise 2

Two samples of the same soil are taken: The first saturated (below the water table), its weight volume equal to 21 kN/m ^{3} . The second (above the water table), its weight volume equal to 18.7 kN/m^3 , γ_w =10 kN/m ^{3} .

Calculate the degree of saturation of the second sample knowing that the volumetric weight solid particles equal to 27 kN/m^3.

Solution :

For the same soil, we have for the same soil:

γ sat = 21 kN/m^3 , γ = 18.7 kN/m^3 , γ s = 27 kN/m^3 .

First, we calculate the value of the void index - (there remains the even above and below the water table, since it is the same soil and the same compactness condition) – and this using the following relationship:

$e = (\gamma s – \gamma sat) /(\gamma sat – \gamma w) = 0.55.$

Then the relationship following allows to determine Sr:

$\gamma = (\gamma s + e.Sr . \gamma_w) / (1+e)$ from which:

$Sr = [\gamma(1+e) - \gamma s] / e.\gamma_w = 0.36$

Sr = 36%.

Exercise 3

A soil sample was studied in the laboratory and the following results were obtained:

Unit weight (kN/m ^{3})	Water content (%)	Density of solid grains
18.5	35	2.7

It is assumed that the volumetric weight of water is equal to 10 kN/m ^{3}

Calculate dry density, degree of saturation, void ratio, porosity, saturated density and specific gravity planed.

Solution :

Knowing the density of the solid grains and the volumetric weight of water, we can calculate the volumetric weight of the solid grains γs :

$\gamma_s = 2.7.10 = 27$ kN/m ^{3}

- We have: $\gamma = \gamma_d (1+w)$, therefore: $\gamma_d = \gamma /(1+w)= 18.5 /(1+0.35)= 13.70$ kN/m^3

$w = Sr\, \gamma w\, (1/ \gamma_d – 1/ \gamma_s)$, hence: $Sr = w/ [\gamma w\, (1/ \gamma_d – 1/ \gamma_s)]$

Sr = 0.97 i.e.: Sr = 97%
$e = \gamma_s / \gamma_d -1 = 0.97$

$n = e /(1+e) = 0.49$

$\gamma = \gamma_d + nSr\, \gamma w$; at saturation (Sr = 1) we will have:

$\gamma sat = \gamma d + n\gamma w = 13.7 + 0.49.10 = 18.6$ kN/m^3

$\gamma ' = \gamma sat – \gamma w = 8.6$ kN/m^3 .

Exercise 4

Taking an intact sample from the center of a soft clay layer located below the The water table allowed the following measurements to be taken in the laboratory:

Total weight of (N)	Total volume (cm ^{3})	Dry weight (N)
1.41	93.9	0.774

We ask:

1) Determine the specific gravity and water content.
2) Determine the void ratio.
3) Check the degree of saturation, We give: $\gamma_w = 10$ kN/m ^{3} ; $\gamma_s = 27$ kN/m^3 .

Solution:

1) – Volumetric weight: $\gamma = W/V = 0.47$ N/3.13 10-5 m^3 = 15 kN/ m^3
- Water content: w = Ww / Ws with Ww = W-Ws

Ww = 1.41 – 0.774 = 0.636 N hence w= 0.822, i.e .: w% = 82.2%

2) – Void index: e = Vv / Vs

The sample having been taken from the water table, it is therefore saturated:

$Vv = Vw = Ww / \gamma w = (0.636.10^{-3}$ kN) / (10 kN/m ^{3}) = 6.36.10 $^{-5}$ m^3 and:

Vs = V- Vv = (9.39 – 6.36).10 $^{-5}$ m^3 = 3.03.10 $^{-5}$ m^3

From which: e = 6.36.10 $^{-5}$ m^3 /3.03. 10 $^{-5}$ m^3 = 2.10

3) – Checking the degree of saturation: Sr = Vw / Vv

Vs = Ws / γ s = (0.774.10 $^{-3}$ kN) / (27 kN/m^3) = 2.87.10 $^{-5}$ m^3

Vv = V-Vs= (9.39 – 2.87).10 $^{-5}$ m^3 = 6.52.10 $^{-5}$ m^3
Sr = Vw / Vv = 6.36.10 $^{-5}$ m^3 /6.51.10 $^{-5}$ m^3 = 0.9769 = 97.7%

We note that the value of the degree of saturation is very close to 100%, there is a slight loss of water saturation during the transport to the laboratory.

Exercise 5

A reconnaissance survey was carried out and two samples (clay) located below the water table were taken, on which usual measurements of weight and volume were carried out in the laboratory. The results obtained are grouped in the table below:

	Sample No. 1	Sample No. 2
Total soil weight (N)	0.96	1.36
Total soil volume (cm^3)	60	86
Dry weight (N)	0.6	0.8

It is assumed that the volumetric weight of solid grains is equal to 27 kN/m^3 and that the volumetric weight of water is 10 kN/m$^{3.}$

Determine for each sample:
1) the specific gravity and water content;
2) the void ratio,
3) the degree of saturation,
4) the relative change in volume during its collection and transport to the laboratory, knowing that γit was found equal to 27.5 kN/ m^3.

Solution :

For both samples, we have: W, V and Ws
W $_1$ = 0.96 N = 0.96.10 $^{-3}$ kN; W $_2$ = 1.36 N = 1.36.10 $^{-3}$ kN

V $_1$ = 60 cm^3 = 6.10 $^{-5}$ m^3 ; V $_2$ = 86 cm ^{3} = 8.6.10 $^{-5}$ m^3

Ws1 = 0.6 N = 0.6.10 $^{-3}$ kN; Ws2 = 0.8 N = 0.8.10 $^{-3}$ kN

1) - Volumetric weight: γ = W/V; we will therefore have:

γ1 = 0.96.10 $^{-3}$ kN /6.10 $^{-5}$ m^3 = 16 kN/m^3

$\gamma 2 = 1.36.10^{-3}$ kN $/8.6.10^{-5}$ m $^{3} = 15.81$ kN/m ^{3}

- Water content: w = Ww/Ws with Ww = W- Ws; therefore:

Ww1= 0.96 – 0.60 = 0.36 N and Ww2= 1.36 – 0.8 = 0.56 N. hence
: w1 = 0.36N/0.60 N = 0.6 = 60 % and: w2 = 0.56N/0.8N = 0.7 = 70%

2) Void index: e = Vv /Vs with Vs = Ws / γs and Vv = V-Vs; therefore:

Vs1= $0.6.10^{-3}$ kN / (27kN/m ^{3}) = $2.222.10^{-5}$ m ^{3} and Vs2 = $2.963.10^{-5}$ m ^{3}
Vv1= (6 - 2.222). 10^{-5} m ^{3} = $3,778.10^{-5}$ m ^{3} and Vv2 = $5,638.10^{-5}$ m ^{3}
and we will have: $e_1 = 3,778.10^{-5}$ m ^{3} $/2,222.10^{-5}$ m ^{3} = 1.70 and e_2 = 1.90

3) Degree of saturation: Sr = Vw / Vv with: Vw = Ww / γw

Vw1= 0.36 kN /(10kN/m ^{3})= $3.6.10^{-5}$ m ^{3} and Vw2=$5.6.10^{-5}$ m ^{3}
Then: Sr1 = $3.6.10^{-5}$ m ^{3} $/3.778.10^{-5}$ m ^{3} = 0.953 = 95.3% and Sr2 = 0.99= 99%.

Note that for both samples collected from the water table show a slight loss of water saturation during the transport to the laboratory.

4) The volumes V considered in our calculations are those determined in the laboratory and are slightly higher than those in situ which we will call V' because of the decompression.

We recalculate the values of Vs with γs = 27.5 kN/m^3, we find:

Vs1 = $0.6.10^{-3}$ kN / (27.5 kN/m^3) = $2,182.10^{-5}$ m^3 and Vs2 = $2,909.10^{-5}$ m^3
As the 2 samples are saturated in situ, it results: V' = Vs + Vw

V'1 = $5,782.10^{-5}$ m^3 and V'2 = $8,508.10^{-5}$ m^3
The relative volume variation is calculated by: $\Delta V/V' = (V-V')/V'$

$(\Delta V/V')_1 = (6 - 5.782).10^{-5}$ m^3 $/5.782.10^{-5}$ m ^{3} = 0.0377 = 3.8%

$(\Delta V/V')_2 = (8.6 - 8.508).10^{-5}$ m ^{3} $/8.508.10^{-5}$ m ^{3} = 0.0108 = 1.1%.

2.6 ADDITIONAL EXERCISES

2.6.1 Wet density: γ_h

Exercise 6

Give an expression for the wet density (γ_h) as a function of the porosity (n), the water content (ω) and the density of the solid grains (γ_s):

$$\gamma_h = f(n, \omega, \gamma_s) = (1+\omega)(1-n)\gamma_s$$

Solution:

Starting from the basic expression of wet density: $\gamma_h = \frac{W}{V}$

$$W = W_w + W_s$$

$$\gamma_h = \frac{W_s + W_w}{V}$$

$$\omega = \frac{W_w}{W_s} \Longrightarrow W_w = \omega W_s$$

$$\gamma_h = \frac{W_s + \omega W_s}{V}$$

$$\gamma_h = \frac{(1+\omega) W_s}{V}$$

$$\gamma_s = \frac{W_s}{V_s} \Longrightarrow W_s = \gamma_s * V_s$$

$$\gamma_h = \frac{(1+\omega)\gamma_s V_s}{V}$$

$$V = V_s + V_v \Longrightarrow V_s = V - V_v$$

$$\gamma_h = \frac{(1+\omega)(V - V_v)\gamma_s}{V}$$

$\gamma_h = (1+\omega)(\frac{V}{V} - \frac{V_v}{V})\gamma_s$, and as $n = \frac{V_v}{V}$, we will have in the end:

$$\boxed{\gamma_h = (1+\omega)(1-n)\gamma_s}$$

Exercise 7

Give an expression for the wet density (γ_h) as a function of the porosity (n), the water content (ω) and the density of the solid grains (γ_s):

$$\gamma_h = \mathrm{f}\,(e\,,\omega\,,\gamma_s)\ = \frac{(1+\omega)\gamma_s}{1+e}$$

Solution:

By definition, the wet density is given by the following relation: $\gamma_h = \frac{W}{V}$

$$W = W_w + W_s$$

$$\gamma_h = \frac{W_s + W_w}{V}$$

$$\omega = \frac{W_w}{W_s} \quad \Longrightarrow \quad W_w = \omega W_s$$

$$\gamma_h = \frac{W_s + \omega W_s}{V}$$

$$\gamma_s = \frac{W_s}{V_s} \quad \Longrightarrow \quad W_s = \gamma_s * V_s$$

$$V = V_s + V_v$$

$$\gamma_h = \frac{\gamma_s * V_s + \omega(\gamma_s * V_s)}{V_s + V_v}$$

$$\gamma_h = \frac{(1+\omega)(\gamma_s * V_s)}{V_s + V_v}$$

$$\gamma_h = \frac{(1+\omega)\gamma_s}{\frac{V_s + V_v}{V_s}}$$

$\gamma_h = \frac{(1+\omega)\gamma_s}{1+\frac{V_v}{V_s}}$, and knowing that : $e = \frac{V_v}{V_s}$, we will have in the end:

$$\boxed{\gamma_h = \frac{(1+\omega)\gamma_s}{1+e}}$$

Exercise 8

Establish the following expression which gives the wet density (γ_h) as a function of the water content (ω) and the dry density (γ_d):

$$\gamma_h = (1+\omega)\gamma_d$$

Solution:

The wet density is given by the following expression: $\gamma_h = \frac{W}{V}$

$W = W_w + W_s$

$$\gamma_h = \frac{W_s + W_w}{V}$$

$\omega = \frac{W_w}{W_s} \Longrightarrow W_w = \omega W_s$

$$\gamma_h = \frac{W_s + \omega W_s}{V}$$

$\gamma_s = \frac{W_s}{V_s} \Longrightarrow W_s = \gamma_s * V_s$

$$\gamma_h = \frac{\gamma_s * V_s + \omega(\gamma_s * V_s)}{V}$$

$$\gamma_h = \frac{(1+\omega)\gamma_s V_s}{V}$$

$\gamma_d = \frac{W_s}{V} \Longrightarrow V = \frac{W_s}{\gamma_d}$

$$\gamma_h = \frac{(1+\omega)\gamma_s V_s}{\frac{W_s}{\gamma_d}}$$

and as : $W_s = \gamma_s * V_s$, the expression is $\gamma_h$written:

$$\boxed{\gamma_h = (1+\omega)\gamma_d}$$

Exercise 9

Establish the expression of the volumetric weight (γ_h) as a function of the dry volumetric weight (γ_d), volumetric weight of solid grains (γ_s), the degree of saturation (S_r) and the volumetric weight of water (γ_w):

$$\gamma_h = f(\gamma_d, \gamma_s, S_r, \gamma_w) = \gamma_d + \left(\frac{\gamma_s - \gamma_d}{\gamma_s}\right) S_r \gamma_w$$

Solution:

$$\gamma_h = \frac{W}{V} = \frac{W_s+W_w}{V}$$

$$\gamma_d = \frac{W_s}{V} \quad \Longrightarrow \quad W_s = \gamma_d V$$

$$\gamma_w = \frac{W_w}{V_w} \quad \Longrightarrow \quad W_w = \gamma_w * V_w$$

$$\gamma_h = \frac{\gamma_d V+\gamma_w V_w}{V} = \gamma_d + \frac{\gamma_w V_w}{V}$$

$$n = \frac{V_v}{V} \quad \Longrightarrow \quad V = \frac{V_v}{n}$$

$$\gamma_h = \gamma_d + \frac{\gamma_w V_w}{\frac{V_v}{n}}$$

$$S_r = \frac{V_w}{V_v}$$

$$\gamma_h = \gamma_d + n\, S_r\, \gamma_w = \gamma_d + \frac{V_v}{V}\, S_r\, \gamma_w = \gamma_d + (\frac{V-V_s}{V})\, S_r \gamma_w$$

$$\gamma_d = \frac{W_s}{V} \quad \Longrightarrow \quad V = \frac{W_s}{\gamma_d}$$

$$\gamma_s = \frac{W_s}{V_s} \quad \Longrightarrow \quad V_s = \frac{W_s}{\gamma_s}$$

$$\gamma_h = \gamma_d + \frac{\frac{W_s}{\gamma_d} - \frac{W_s}{\gamma_s}}{\frac{W_s}{\gamma_d}}\, S_r\, \gamma_w = \gamma_d + \frac{W_s(\frac{1}{\gamma_d} - \frac{1}{\gamma_s})}{\frac{W_s}{\gamma_d}}\, S_r\, \gamma_w = \gamma_d + \frac{\frac{1}{\gamma_d} - \frac{1}{\gamma_s}}{\frac{1}{\gamma_d}}\, S_r\, \gamma_w$$

Finally, the expression of the volumetric weight (γ_h) as a function of the dry volumetric weight (γ_d), volumetric weight of solid grains (γ_s), the degree of saturation (S_r) and the volumetric weight of water (γ_w) is written in the following form:

$$\gamma_h = f(\gamma_d, \gamma_s, S_r, \gamma_w) = \gamma_d + \left(\frac{\gamma_s - \gamma_d}{\gamma_s}\right) S_r \gamma_w$$

Exercise 10

Establish the expression of the volumetric weight (γ_h) as a function of the dry volumetric weight (γ_d), the void ratio (e) and the volumetric weight of water (γ_w):

$$\gamma_h = f(\gamma_d, e, S_r, \gamma_w) = \gamma_d + \frac{e\, S_r}{1+e} \gamma_w$$

Solution:

$$\gamma_h = \frac{W}{V} = \frac{W_s+W_w}{V}$$

$\gamma_d = \frac{W_s}{V} \quad \Longrightarrow \quad W_s = \gamma_d V$

$\gamma_w = \frac{W_w}{V_w} \quad \Longrightarrow \quad W_w = \gamma_w * V_w$

$$\gamma_h = \frac{\gamma_d V + \gamma_w * V_w}{V} = \gamma_d + \frac{\gamma_w V_w}{V} = \gamma_d + \frac{\frac{V_w}{V_v}}{\frac{V_s + V_v}{V_v}} \gamma_w$$

$S_r = \frac{V_w}{V_v}$

$e = \frac{V_v}{V_s}$

$$\gamma_h = \gamma_d + \frac{S_r}{\frac{1}{e}+1} \gamma_w = \gamma_d + \frac{S_r}{\frac{1+e}{e}} \gamma_w = \gamma_d + \frac{e\, S_r}{1+e} \gamma_w$$

Finally, we will have:

$$\gamma_h = f(\gamma_d, e, S_r, \gamma_w) = \gamma_d + \frac{e\, S_r}{1+e} \gamma_w$$

Exercise 11

Establish the expression of the volumetric weight (γ_h) as a function of the porosity (n), the volumetric weight of the solid grains (γ_s), the degree of saturation (S_r) and the volumetric weight of water (γ_w):

$$\gamma_h = f(n, \gamma_s, S_r, \gamma_w) = (1-n)\gamma_s + n * S_r * \gamma_w$$

Solution:

$\gamma_h = \frac{W}{V}$

$\gamma_h = \frac{W_s + W_w}{V}$

$\gamma_d = \frac{W_s}{V} \quad \Longrightarrow \quad W_s = \gamma_d V;$

$\gamma_w = \frac{W_w}{V_w} \quad \Longrightarrow \quad W_w = \gamma_w * V_w$

$\gamma_h = \frac{\gamma_d V + \gamma_w V_w}{V} = \gamma_d + \frac{\gamma_w V_w}{V}$

$n = \frac{V_v}{V} \quad \Longrightarrow \quad V = \frac{V_v}{n}$

$\gamma_h = \gamma_d + \frac{\gamma_w V_w}{\frac{V_v}{n}}$

$S_r = \frac{V_w}{V_v}$

$\gamma_h = \gamma_d + n\, S_r\, \gamma_w$

but the dry density (γ_d) can take the following form:

$\gamma_d = (1-n)\gamma_s$, since:

$$\gamma_d = \frac{W_s}{V}$$

$\gamma_s = \frac{W_s}{V_s} \Longrightarrow W_s = \gamma_s * V_s$

$$\gamma_d = \frac{\gamma_s V_s}{V}$$

$V = V_v + V_s \Longrightarrow V_s = V - V_v$

$$\gamma_d = \frac{(V - V_v)\gamma_s}{V}$$

$\gamma_d = (\frac{V}{V} - \frac{V_v}{V})\gamma_s,$

and knowing that : $n = \frac{V_v}{V}$, the expression $\gamma_d$takes the following form:

$\gamma_d = (1-n)\gamma_s$

And we will finally have the expression of the volumetric weight (γ_h) as a function of the porosity (n), the volumetric weight of the solid grains (γ_s), the degree of saturation (S_r) and the volumetric weight of water (γ_w) which is written as:

$$\gamma_h = f(n, \gamma_s\ , S_r, \gamma_w)\ = (1-n)\gamma_s + n *\ S_r * \gamma_w$$

Exercise 12

Establish the expression of the saturated volumetric weight (γ_h) as a function of the volumetric weight of the solid grains (γ_s), the water content at saturation (ω), the degree of saturation (S_r) and the volumetric weight of the water (γ_w):

$$\gamma_h = f(\gamma_s, \omega, \gamma_w, S_r) = \frac{\gamma_s\ (1+\omega)}{1+\frac{\omega\,\gamma_s}{S_r \gamma_w}}$$

Solution:

$\gamma_h = \frac{W}{V}$; $W = W_s + W_w$;$V = V_s + V_v$

$$\gamma_h = \frac{W_s + W_w}{V_s + V_v}$$

$\gamma_s = \frac{W_s}{V_s}$ $\Longrightarrow$ $W_s = \gamma_s V_s$ And $V_s = \frac{W_s}{\gamma_s}$

$\omega = \frac{W_w}{W_s}$ $\Longrightarrow$ $W_w = \omega W_s = \omega\gamma_s V_s$

$S_r = \frac{V_w}{V_v}$ $\Longrightarrow$ $V_v = \frac{V_w}{S_r}$

$$\gamma_h = \frac{\gamma_s V_s + \omega\gamma_s V_s}{\frac{W_s}{\gamma_s} + \frac{V_w}{S_r}} = \frac{\gamma_s V_s (1+\omega)}{\frac{W_s}{\gamma_s} + \frac{V_w}{S_r}}$$

$\gamma_w = \frac{W_w}{V_w}$ $\Longrightarrow$ $V_w = \frac{W_w}{\gamma_w}$

$$\gamma_h = \frac{\gamma_s V_s (1+\omega)}{\frac{W_s}{\gamma_s} + \frac{W_w}{S_r\gamma_w}} = \frac{\gamma_s V_s (1+\omega)}{\frac{W_s S_r\gamma_w}{\gamma_s S_r \gamma_w} + \frac{W_w \gamma_s}{S_r\gamma_w \gamma_s}} = \frac{\gamma_s V_s \ (1+\omega)}{\frac{W_s S_r \gamma_w + W_w \gamma_s}{\gamma_s S_r\gamma_w}} = \frac{\gamma_s \ (1+\omega)}{\frac{W_s S_r \gamma_w + W_w \gamma_s}{V_s\gamma_s S_r\gamma_w}}$$

$\gamma_s = \frac{W_s}{V_s}$ $\Longrightarrow$ $\gamma_s V_s = W_s$

$$\gamma_h = \frac{\gamma_s \ (1+\omega)}{\frac{W_s S_r \gamma_w + W_w \gamma_s}{W_s S_r\gamma_w}} = \frac{\gamma_s \ (1+\omega)}{1+\frac{W_w \gamma_s}{W_s S_r\gamma_w}}$$

$$\frac{W_w}{W_s} = \omega$$

$$\gamma_h = \frac{\gamma_s \ (1+\omega)}{1+\frac{\omega \gamma_s}{S_r\gamma_w}}$$

Exercise 13

Establish the expression of the volumetric weight (γ_h) as a function of the volumetric weight of the solid grains (γ_s), the void ratio (e), the degree of saturation (S_r) and the volumetric weight of water (γ_w):

$$\gamma_h = f(\gamma_s, S_r, e, \gamma_w) = \frac{\gamma_s + S_r e \gamma_w}{1+e}$$

Solution:

$\gamma_h = \frac{W}{V}$

$W = W_s + W_w$

$V = V_s + V_v$

$\gamma_h = \frac{W_s+W_w}{V_s+V_v}$

$\gamma_s = \frac{W_s}{V_s} \Longrightarrow W_s = \gamma_s V_s$

$\gamma_w = \frac{W_w}{V_w} \Longrightarrow W_w = \gamma_w * V_w$

$\gamma_h = \frac{\gamma_s V_s + \gamma_w V_w}{V_s+V_v}$

$S_r = \frac{V_w}{V_v} \Longrightarrow V_w = S_r V_v$

$\gamma_h = \frac{\gamma_s V_s + \gamma_w S_r V_v}{V_s+V_v}$

Dividing the numerator and denumerator by V_s, we get:

$$\gamma_h = \frac{\frac{\gamma_s V_s}{V_s} + \frac{\gamma_w V_v}{V_s} \gamma_w S_r V_v}{\frac{V_s}{V_s} + \frac{V_v}{V_s}}$$

and as: $e = \frac{V_v}{V_s}$we obtain:

$$\gamma_h = \frac{\gamma_s + eS_r \gamma_w}{1+e}$$

Exercise 14

Establish the expression of the volumetric weight (γ_h) as a function of the dry volumetric weight (γ_d), the porosity (n), the degree of saturation (S_r) and the volumetric weight of water (γ_w):

$$\gamma_h = f(n, \gamma_d, S_r, \gamma_w) = \gamma_d + n S_r * \gamma_w$$

Solution:

$$\gamma_h = \frac{W}{V} = \frac{W_s+W_w}{V}$$

$$\gamma_d = \frac{W_s}{V} \Longrightarrow W_s = \gamma_d V;$$

$$\gamma_w = \frac{W_w}{V_w} \Longrightarrow W_w = \gamma_w * V_w$$

$$\gamma_h = \frac{\gamma_d V+\gamma_w V_w}{V} = \gamma_d + \frac{\gamma_w V_w}{V}$$

$$n = \frac{V_v}{V} \Longrightarrow V = \frac{V_v}{n}$$

$$\gamma_h = \gamma_d + \frac{\gamma_w V_w}{\frac{V_v}{n}}$$

$$S_r = \frac{V_w}{V_v}$$

$$\gamma_h = \gamma_d + n\, S_r\, \gamma_w$$

2.6.2 Dry volumetric weight: γ_d

Exercise 15

Give an expression for the dry density (γ_d) as a function of the porosity (n) and the density of the solid grains (γ_s):

$$\gamma_d = \text{f}\,(n\ ,\gamma_s)\ = (1-n)\gamma_s$$

Solution :

$$\gamma_d = \frac{W_s}{V}$$

$$\gamma_s = \frac{W_s}{V_s} \Longrightarrow W_s = \gamma_s * V_s$$

$$\gamma_d = \frac{\gamma_s V_s}{V}$$

$$V = V_v + V_s \Longrightarrow V_s = V - V_v$$

$$\gamma_d = \frac{(V - V_v)\gamma_s}{V}$$

$\gamma_d = (\frac{V}{V} - \frac{V_v}{V})\gamma_s,$

and knowing that : $n = \frac{V_v}{V}$, the expression $\gamma_d$takes the following form:

$$\boxed{\gamma_d = (1 - n)\gamma_s}$$

Exercise 16

Give an expression for the dry density (γ_d) as a function of the void ratio (e) and the density of solid grains (γ_s):

$$\gamma_d = \text{f}\,(e\ ,\gamma_s)\ = \frac{\gamma_s}{1+e}$$

Solution :

$\gamma_d = \frac{W_s}{V}$

$\gamma_s = \frac{W_s}{V_s} \quad \Longrightarrow \quad W_s = \gamma_s * V_s$

$\gamma_d = \frac{\gamma_s * V_s}{V}$

$V = V_v + V_s$

$\gamma_d = \frac{\gamma_s * V_s}{V_v + V_s}$

And dividing the numerator and denumerator by $V_s$we get:

$$\gamma_d = \frac{\gamma_s}{\frac{V_v+V_s}{V_s}} = \frac{\gamma_s}{1+\frac{V_v}{V_s}}$$

$$e = \frac{V_v}{V_s}$$

Hence the expression of the dry volumetric weight (γ_d) as a function of the void ratio (e) and the volumetric weight of solid grains (γ_s) is written as follows:

$$\boxed{\gamma_d = \frac{\gamma_s}{1 + e}}$$

Exercise 17

Establish the expression of the dry volumetric weight (γ_d) as a function of the wet volumetric weight (γ_h) and the water content (ω):

$$\gamma_d = f(\gamma_h \ , \omega) \ = \frac{\gamma_h}{(1+\omega)}$$

Solution :

$$\gamma_d = \frac{W_s}{V}$$

$$\gamma_s = \frac{W_s}{V_s} \quad \Longrightarrow \quad W_s = \gamma_s * V_s$$

$$\gamma_d = \frac{\gamma_s * V_s}{V}$$

$$\gamma_h = \frac{W}{V} \quad \Longrightarrow \quad V = \frac{W}{\gamma_h}$$

$$\gamma_d = \frac{\gamma_s V_s}{\frac{W}{\gamma_h}} = \frac{\gamma_h \gamma_s V_s}{W}$$

$$W = W_s + W_W$$

$$\gamma_d = \frac{\gamma_h \gamma_s V_s}{W_s + W_w}$$

$$\omega = \frac{W_W}{W_s} \quad \Longrightarrow \quad W_W = \omega W_s$$

$$\gamma_d = \frac{\gamma_h \gamma_s V_s}{W_s + \omega W_s} = \frac{\gamma_h \gamma_s V_s}{\gamma_s V_s + \omega(\gamma_s V_s)} = \frac{\gamma_h \gamma_s V_s}{(1+\omega)\gamma_s V_s}$$

From where the expression of dry density (γ_d) as a function of wet density (γ_h) and water content (ω):

$$\boxed{\gamma_d = f(\gamma_h \ , \omega) \ = \frac{\gamma_h}{(1+\omega)}}$$

Exercise 18

Write the expression for the dry density (γ_d) as a function of the density of the solid grains (γ_s), the degree of saturation (S_r), the water content (ω) and the density of water (γ_w):

$$\gamma_d = \frac{\gamma_w}{\frac{\gamma_w}{\gamma_s}+\frac{\omega}{s_r}}$$

Solution:

$$\gamma_d = \frac{w_s}{v_s + v_v} = \frac{w_s}{\frac{w_s}{\gamma_s} + \frac{v_w}{s_r}} = \frac{w_s}{\frac{w_s}{\gamma_s} + \frac{w_w}{s_r.\gamma_w}}$$

$$\gamma_d = \frac{w_s}{\frac{\gamma_w w_s}{\gamma_w \gamma_s} + \frac{w_s w_w}{w_s s_r.\gamma_w}} = \frac{\gamma_w}{\frac{\gamma_w}{\gamma_s} + \frac{w_w}{w_s s_r}} = \frac{\gamma_w}{\frac{\gamma_w}{\gamma_s} + \frac{\omega}{s_r}}$$

$$\gamma_d = \frac{\gamma_w}{\frac{\gamma_w}{\gamma_s} + \frac{\omega}{s_r}}$$

Exercise 19

Write the expression for the dry density (γ_d) as a function of the density of the solid grains (γ_s), the saturated water content (ω_{sat}) and the density of water (γ_w):

$$\gamma_d = \frac{\gamma_w}{\frac{\gamma_w}{\gamma_s} + \omega_{sat}}$$

Solution:

$$\gamma_d = \frac{w_s}{v_s + v_v} = \frac{w_s}{\frac{w_s}{\gamma_s} + \frac{v_w}{s_r}} = \frac{w_s}{\frac{w_s}{\gamma_s} + \frac{w_w}{s_r.\gamma_w}}$$

$$\gamma_d = \frac{w_s}{\frac{\gamma_w w_s}{\gamma_w \gamma_s} + \frac{w_s w_w}{w_s s_r.\gamma_w}} = \frac{\gamma_w}{\frac{\gamma_w}{\gamma_s} + \frac{w_w}{w_s s_r}} = \frac{\gamma_w}{\frac{\gamma_w}{\gamma_s} + \frac{\omega}{s_r}}$$

At saturation, we will have :$S_r = 1$; $\omega = \omega_{sat}$, hence the expression of the dry volumetric weight (γ_d) as a function of the volumetric weight of the solid grains (γ_s), the water content at saturation (ω_{sat}) and the volumetric weight of the water (γ_w):

$$\gamma_d = \frac{\gamma_w}{\frac{\gamma_w}{\gamma_s} + \omega_{sat}}$$

2.6.3 Specific weight of solid grains: γ_s

Exercise 20

Establish the expression of the volumetric weight of solid grains (γ_s) as a function of the wet volumetric weight (γ_h), the porosity (n) and the water content (ω):

$$\gamma_s = f(\gamma_h, n, \omega) = \frac{\gamma_h}{(1-n)(1+\omega)}$$

Solution :

$$\gamma_s = \frac{W_s}{V_s}$$

$$V = V_v + V_s \Longrightarrow V_s = V - V_v$$

$$\gamma_s = \frac{W_s}{V - V_v}$$

$$n = \frac{V_v}{V} \Longrightarrow V_v = nV$$

$$\gamma_s = \frac{W_s}{V - nV}$$

$$\gamma_s = \frac{W_s}{(1-n)V}$$

$$\gamma_h = \frac{W}{V} \Longrightarrow V = \frac{W}{\gamma_h}$$

$$\gamma_s = \frac{W_s}{(1-n)\frac{W}{\gamma_h}} = \frac{W_s\gamma_h}{(1-n)W} = \frac{\gamma_h}{(1-n)W\frac{1}{W_s}}$$

$$W = W_s + W_w$$

$$\gamma_s = \frac{\gamma_h}{(1-n)\frac{W_s+W_w}{W_s}} = \frac{\gamma_h}{(1-n)(1+\frac{W_w}{W_s})}$$

$$\omega = \frac{W_w}{W_s}$$

From where the expression of the density of solid grains (γ_s) as a function of the wet density (γ_h), the porosity (n) and the water content (ω) is written in the following form:

$$\boxed{\gamma_s = f(\gamma_h, n, \omega) = \frac{\gamma_h}{(1-n)(1+\omega)}}$$

Exercise 21

Establish the expression of the volumetric weight of solid grains (γ_s) as a function of the dry volumetric weight (γ_d) and the void ratio (e):

$$\gamma_s = f(\gamma_d, e) = (1+e)\gamma_d$$

Solution:

$\gamma_s = \frac{W_s}{V_s}$

$\gamma_d = \frac{W_s}{V} \Longrightarrow W_s = \gamma_d * V$

$\gamma_s = \frac{\gamma_d * V}{V_s}$

$V = V_v + V_s$

$$\gamma_s = \frac{\gamma_d(V_v+V_s)}{V_s} = \gamma_d\left(1+\frac{V_v}{V_s}\right)$$

$$e = \frac{V_v}{V_s}$$

From where the expression of the density of solid grains (γ_s) as a function of the dry density (γ_d) and the void ratio (e) takes the following form:

$$\boxed{\gamma_s = f(\gamma_d, e) = (1+e)\gamma_d}$$

Exercise 22

Establish the expression of the volumetric weight of solid grains (γ_s) as a function of the dry volumetric weight (γ_d) and the porosity (n):

$$\gamma_s = f(\gamma_d, n) = \frac{\gamma_d}{1-n}$$

Solution :

$\gamma_s = \frac{W_s}{V_s}$

$\gamma_d = \frac{W_s}{V} \Longrightarrow W_s = \gamma_d * V$

$\gamma_s = \frac{\gamma_d * V}{V_s}$

$V_s = V - V_v$

$$\gamma_s = \frac{\gamma_d * V}{V - V_v} = \frac{\gamma_d}{\frac{V - V_v}{V}} = \frac{\gamma_d}{(1 - \frac{V_v}{V})}$$

$$n = \frac{V_v}{V}$$

And finally, the expression of the density of solid grains (γ_s) as a function of the dry density (γ_d) and the porosity (n) is written as follows:

$$\boxed{\gamma_s = f(\gamma_d, n) = \frac{\gamma_d}{1-n}}$$

Exercise 23

Establish the expression of the volumetric weight of solid grains (γ_s) as a function of the dry volumetric weight (γ_d), the degree of saturation (S_r), the water content (ω) and the volumetric weight of water (γ_w):

$$\gamma_s = \frac{\gamma_w \gamma_d . S_r}{\gamma_w . S_r - \omega . \gamma_d}$$

Solution:

$$\gamma_s = \frac{w_s}{v_s} = \frac{w_s}{v - v_v} = \frac{w_s}{\frac{w_s}{\gamma_d} - \frac{v_w}{s_r}} = \frac{w_s . \gamma_d . S_r}{w_s . S_r - v_w . \gamma_d}$$

$$\gamma_s = \frac{\gamma_d . S_r}{\frac{w_s}{w_s} . S_r - \frac{w_w . \gamma_d}{\gamma_w . w_s}} = \frac{\gamma_d . S_r}{S_r - \frac{\omega . \gamma_d}{\gamma_w}} = \frac{\gamma_d . S_r}{\frac{\gamma_w . S_r - \omega . \gamma_d}{\gamma_w}}$$

Hence, the expression of the volumetric weight of solid grains (γ_s) as a function of the dry volumetric weight (γ_d), the degree of saturation (S_r), the water content (ω) and the volumetric weight of water (γ_w) is presented in the following form:

$$\gamma_s = \frac{\gamma_w \gamma_d . S_r}{\gamma_w . S_r - \omega . \gamma_d}$$

Exercise 24

Establish the expression of the volumetric weight of solid grains (γ_s) as a function of the dry volumetric weight (γ_d), the water content at saturation (ω_{sat}) and the volumetric weight of water (γ_w):

$$\gamma_s = \frac{\gamma_w \gamma_d}{\gamma_w - \omega_{sat}.\gamma_d}$$

Solution:

$$\gamma_s = \frac{w_s}{v_s} = \frac{w_s}{v - v_v} = \frac{w_s}{\frac{w_s}{\gamma_d} - \frac{v_w}{S_r}} = \frac{w_s.\gamma_d.S_r}{w_s.S_r - v_w.\gamma_d}$$

$$\gamma_s = \frac{\gamma_d.S_r}{\frac{w_s}{w_s}.S_r - \frac{w_w.\gamma_d}{\gamma_w.w_s}} = \frac{\gamma_d.S_r}{S_r - \frac{\omega.\gamma_d}{\gamma_w}} = \frac{\gamma_d.S_r}{\frac{\gamma_w.S_r - \omega.\gamma_d}{\gamma_w}} = \frac{\gamma_w\gamma_d.S_r}{\gamma_w.S_r - \omega.\gamma_d}$$

At saturation, we will have : $S_r = 1$; $\omega = \omega_{sat}$, and the expression of the volumetric weight of the solid grains (γ_s) as a function of the dry volumetric weight (γ_d), the water content at saturation (ω_{sat}) and the volumetric weight of water (γ_w) is written in the following form:

$$\gamma_s = \frac{\gamma_w \gamma_d}{\gamma_w - \omega_{sat}.\gamma_d}$$

2.6.4 Saturated density: $\boldsymbol{\gamma_{sat}}$

Exercise 25

Establish the expression of the saturated volumetric weight (γ_{sat}) as a function of the dry volumetric weight (γ_d), the porosity (n) and the volumetric weight of water (γ_w):

$$\gamma_{sat} = f(\gamma_d, n, \gamma_w) = \gamma_d + n\,\gamma_w$$

Solution:

$\gamma_h = \frac{W}{V} = \frac{W_s + W_w}{V}$

$\gamma_d = \frac{W_s}{V} \quad \Longrightarrow \quad W_s = \gamma_d V$

$\gamma_w = \frac{W_w}{V_w} \quad \Longrightarrow \quad W_w = \gamma_w * V_w$

$\gamma_h = \frac{\gamma_d V + \gamma_w V_w}{V} = \gamma_d + \frac{\gamma_w V_w}{V}$

$n = \frac{V_v}{V} \quad \Longrightarrow \quad V = \frac{V_v}{n}$

$\gamma_h = \gamma_d + \frac{\gamma_w V_w}{\frac{V_v}{n}}$

$S_r = \frac{V_w}{V_v}$

$$\gamma_h = \gamma_d + n\, S_r\, \gamma_w$$

and at saturation, we will have $S_r = 1$, And $\gamma_h = \gamma_{sat}$

Hence the expression of the saturated volumetric weight (γ_{sat}) as a function of the dry volumetric weight (γ_d), the porosity (n) and the volumetric weight of water (γ_w) takes the following form:

$$\boxed{\gamma_{sat} = f(\gamma_d, n, \gamma_w) = \gamma_d + n\,\gamma_w}$$

Exercise 26

Establish the expression of the volumetric weight (γ_{sat}) as a function of the porosity (n), the volumetric weight of the solid grains (γ_s), and the volumetric weight of the water (γ_w):

$$\gamma_{sat} = f(n, \gamma_s, \gamma_w) = (1-n)\gamma_s + n * \gamma_w$$

Solution:

$\gamma_h = \frac{W}{V} = \frac{W_s + W_w}{V}$

$\gamma_d = \frac{W_s}{V} \quad \Longrightarrow \quad W_s = \gamma_d V;$

$\gamma_w = \frac{W_w}{V_w} \quad \Longrightarrow \quad W_w = \gamma_w * V_w$

$\gamma_h = \frac{\gamma_d V + \gamma_w V_w}{V} = \gamma_d + \frac{\gamma_w V_w}{V}$

$n = \frac{V_v}{V} \quad \Longrightarrow \quad V = \frac{V_v}{n}$

$\gamma_h = \gamma_d + \frac{\gamma_w V_w}{\frac{V_v}{n}}$

$S_r = \frac{V_w}{V_v}$

$\gamma_h = \gamma_d + n\, S_r\, \gamma_w$

but the dry volumetric weight (γ_d) can take the following form:

$\gamma_d = (1-n)\gamma_s$, and this by following the steps below:

$\gamma_d = \frac{W_s}{V}$

$\gamma_s = \frac{W_s}{V_s} \quad \Longrightarrow \quad W_s = \gamma_s * V_s$

$\gamma_d = \frac{\gamma_s V_s}{V}$

$V = V_v + V_s \quad \Longrightarrow \quad V_s = V - V_v$

$\gamma_d = \frac{(V-V_v)\gamma_s}{V} = (\frac{V}{V} - \frac{V_v}{V})\gamma_s,$

and knowing that : $n = \frac{V_v}{V}$, the expression $\gamma_d$takes the following form:

$$\gamma_d = (1-n)\gamma_s$$

And we will finally have the expression of the volumetric weight (γ_{sat}) as a function of the porosity (n), the volumetric weight of the solid grains (γ_s), and the volumetric weight of water (γ_w) which is written as:

$$\gamma_{sat} = f(n, \gamma_s, \gamma_w) = (1-n)\gamma_s + n * \gamma_w$$

<u>Exercise 27</u>

Establish the following expression which gives the saturated volumetric weight (γ_{sat}) as a function of the water content at saturation (ω_{sat}) and the dry volumetric weight (γ_d):

$$\gamma_{sat} = (1 + \omega_{sat})\gamma_d$$

<u>Solution:</u>

The wet density is given by the following expression:$\gamma_h = \frac{W}{V}$

$W = W_w + W_s$

$\gamma_h = \dfrac{W_s + W_w}{V}$

$\omega = \frac{W_w}{W_s}$ $\Longrightarrow$ $W_w = \omega W_s$

$\gamma_h = \dfrac{W_s + \omega W_s}{V}$

$\gamma_s = \frac{W_s}{V_s}$ $\Longrightarrow$ $W_s = \gamma_s * V_s$

$\gamma_h = \frac{\gamma_s * V_s + \omega(\gamma_s * V_s)}{V} = \frac{(1+\omega)\gamma_s V_s}{V}$

$\gamma_d = \frac{W_s}{V}$ $\Longrightarrow$ $V = \frac{W_s}{\gamma_d}$

$\gamma_h = \frac{(1+\omega)\gamma_s V_s}{\frac{W_s}{\gamma_d}}$

and as : $W_s = \gamma_s * V_s$, the expression is $\gamma_h$written:

$$\gamma_h = (1+\omega)\gamma_d$$

At saturation, we will have: $\gamma_h = \gamma_{sat}$and$\omega = \omega_{sat}$

And so, the expression that expresses the saturated density (γ_{sat}) as a function of the saturated water content (ω_{sat}) and the dry density (γ_d) is written as follows:

$$\boxed{\gamma_{sat} = (1+\omega_{sat})\gamma_d}$$

Exercise 28

Establish the expression of the saturated volumetric weight (γ_{sat}) as a function of the dry volumetric weight (γ_d), the void ratio (e) and the volumetric weight of water (γ_w):

$$\gamma_{sat} = f(\gamma_d, e, \gamma_w) = \gamma_d + \frac{e}{1+e}\gamma_w$$

Solution:

$\gamma_h = \frac{W}{V}$

$\gamma_h = \frac{W_s + W_w}{V}$

$\gamma_d = \frac{W_s}{V} \quad \Longrightarrow \quad W_s = \gamma_d V$

$\gamma_w = \frac{W_w}{V_w} \quad \Longrightarrow \quad W_w = \gamma_w * V_w$

$$\gamma_h = \frac{\gamma_d V + \gamma_w * V_w}{V} = \gamma_d + \frac{\gamma_w V_w}{V} = \gamma_d + \frac{\frac{V_w}{V_v}}{\frac{V_s + V_v}{V_v}} \gamma_w$$

$S_r = \frac{V_w}{V_v}$

$e = \frac{V_v}{V_s}$

$$\gamma_h = \gamma_d + \frac{S_r}{\frac{1}{e}+1} \gamma_w = \gamma_d + \frac{S_r}{\frac{1+e}{e}} \gamma_w = \gamma_d + \frac{e\, S_r}{1+e} \gamma_w$$

at saturation, we will have $S_r = 1$, And $\gamma_h = \gamma_{sat}$, and therefore the expression of the saturated volumetric weight (γ_{sat}) as a function of the dry volumetric weight (γ_d), the void ratio (e) and the volumetric weight of water (γ_w) is written in the following form:

$$\boxed{\gamma_{sat} = f(\gamma_d, e, \gamma_w) = \gamma_d + \frac{e}{1+e} \gamma_w}$$

Exercise 29

Establish the expression of the saturated volumetric weight (γ_{sat}) as a function of the dry volumetric weight (γ_d), volumetric weight of solid grains (γ_s) and the volumetric weight of water (γ_w):

$$\gamma_{sat} = f(\gamma_d, \gamma_s, \gamma_w) = \gamma_d + \left(\frac{\gamma_s - \gamma_d}{\gamma_s}\right) \gamma_w$$

Solution:

$\gamma_h = \frac{W}{V}$

$\gamma_h = \frac{W_s + W_w}{V}$

$\gamma_d = \frac{W_s}{V} \quad \Longrightarrow \quad W_s = \gamma_d V;$

$\gamma_w = \frac{W_w}{V_w} \Longrightarrow W_w = \gamma_w * V_w$

$\gamma_h = \frac{\gamma_d V + \gamma_w V_w}{V} = \gamma_d + \frac{\gamma_w V_w}{V}$

$n = \frac{V_v}{V} \Longrightarrow V = \frac{V_v}{n}$

$\gamma_h = \gamma_d + \frac{\gamma_w V_w}{\frac{V_v}{n}}$

$S_r = \frac{V_w}{V_v}$

$$\gamma_h = \gamma_d + n\, S_r\, \gamma_w = \gamma_d + \frac{V_v}{V}\, S_r\, \gamma_w = \gamma_d + (\frac{V - V_s}{V})\, S_r \gamma_w$$

$\gamma_d = \frac{W_s}{V} \Longrightarrow V = \frac{W_s}{\gamma_d}$

$\gamma_s = \frac{W_s}{V_s} \Longrightarrow V_s = \frac{W_s}{\gamma_s}$

$$\gamma_h = \gamma_d + \frac{\frac{W_s}{\gamma_d} - \frac{W_s}{\gamma_s}}{\frac{W_s}{\gamma_d}}\, S_r\, \gamma_w = \gamma_d + \frac{W_s(\frac{1}{\gamma_d} - \frac{1}{\gamma_s})}{\frac{W_s}{\gamma_d}}\, S_r\, \gamma_w$$

$$\gamma_h = \gamma_d + \frac{\frac{1}{\gamma_d} - \frac{1}{\gamma_s}}{\frac{1}{\gamma_d}}\, S_r\, \gamma_w = \gamma_d + \left(\frac{\gamma_s - \gamma_d}{\gamma_s}\right) S_r \gamma_w$$

At saturation, ($S_r = 1$we will have : $\gamma_h = \gamma_{sat}$, and in this case the expression of the saturated volumetric weight (γ_{sat}) as a function of the dry volumetric weight (γ_d), volumetric weight of solid grains (γ_s) and the volumetric weight of water (γ_w):

$$\boxed{\gamma_{sat} = f(\gamma_d, \gamma_s, \gamma_w) = \gamma_d + \left(\frac{\gamma_s - \gamma_d}{\gamma_s}\right)\gamma_w}$$

Exercise 30

Establish the expression of the saturated volumetric weight (γ_{sat}) as a function of the volumetric weight of the solid grains (γ_s), the water content at saturation (ω_{sat}) and the volumetric weight of the water (γ_w):

$$\gamma_{sat} = f(\gamma_s,\ \omega_{sat}, \gamma_w) = \frac{\gamma_s\ (1 + \omega_{sat})}{1 + \omega_{sat}\frac{\gamma_s}{\gamma_w}}$$

Solution:

$\gamma_h = \frac{W}{V}$

$W = W_s + W_w$

$V = V_s + V_v$

$\gamma_h = \dfrac{W_s + W_w}{V_s + V_v}$

$\gamma_s = \frac{W_s}{V_s} \Longrightarrow W_s = \gamma_s\, V_s$ And $V_s = \frac{W_s}{\gamma_s}$

$\omega = \frac{W_w}{W_s} \Longrightarrow W_w = \omega\, W_s = \omega\gamma_s\, V_s$

$S_r = \frac{V_w}{V_v} \Longrightarrow V_v = \frac{V_w}{S_r}$

$$\gamma_h = \frac{\gamma_s\, V_s + \omega\gamma_s\, V_s}{\frac{W_s}{\gamma_s} + \frac{V_w}{S_r}} = \frac{\gamma_s\, V_s\,(1+\omega)}{\frac{W_s}{\gamma_s} + \frac{V_w}{S_r}}$$

$\gamma_w = \frac{W_w}{V_w} \Longrightarrow V_w = \frac{W_w}{\gamma_w}$

$$\gamma_h = \frac{\gamma_s\, V_s\,(1+\omega)}{\frac{W_s}{\gamma_s} + \frac{W_w}{S_r\gamma_w}} = \frac{\gamma_s\, V_s\,(1+\omega)}{\frac{W_s\, S_r\gamma_w}{\gamma_s S_r\,\gamma_w} + \frac{W_w\,\gamma_s}{S_r\gamma_w\,\gamma_s}}$$

$$\gamma_h = \frac{\gamma_s V_s\ (1+\omega)}{\frac{W_s S_r\,\gamma_w + W_w\,\gamma_s}{\gamma_s\, S_r\gamma_w}} = \frac{\gamma_s\ (1+\omega)}{\frac{W_s S_r\,\gamma_w + W_w\,\gamma_s}{V_s\gamma_s\, S_r\gamma_w}}$$

$\gamma_s = \frac{W_s}{V_s} \Longrightarrow \gamma_s\, V_s = W_s$

$$\gamma_h = \frac{\gamma_s\ (1+\omega)}{\frac{W_s S_r\,\gamma_w + W_w\,\gamma_s}{W_s\, S_r\gamma_w}} = \frac{\gamma_s\ (1+\omega)}{1+\frac{W_w\,\gamma_s}{W_s\, S_r\gamma_w}}$$

$\dfrac{W_w}{W_s} = \omega$

$$\gamma_h = \frac{\gamma_s\ (1+\omega)}{1+\dfrac{\omega\,\gamma_s}{S_r\gamma_w}}$$

At saturation, we will have: $S_r = 1$; $\gamma_h = \gamma_{sat}$ and $\omega = \omega_{sat}$, and therefore, the expression of the saturated density (γ_{sat}) as a function of the density of the solid grains (γ_s), the water content at saturation (ω_{sat}) and the density of water (γ_w) takes the following form:

$$\gamma_{sat} = \frac{\gamma_s \ (1 + \ \omega_{sat})}{1 + \ \omega_{sat} \ \frac{\gamma_s}{\gamma_w}}$$

Exercise 31

Establish the expression of the saturated volumetric weight (γ_{sat}) as a function of the volumetric weight of the solid grains (γ_s), the water content at saturation (ω_{sat}) and the volumetric weight of the water (γ_w):

$$\gamma_{sat} = f\,(\gamma_s,\ \omega_{sat}\,, \gamma_w) \ = \frac{\gamma_s + e\,\gamma_w}{1+e}$$

Solution:

$\gamma_h = \frac{W}{V}$

$W = W_s + W_w$

$V = V_s + V_v$

$\gamma_h = \frac{W_s + W_w}{V_s + V_v}$

$\gamma_s = \frac{W_s}{V_s} \Longrightarrow W_s = \gamma_s \, V_s$

$\gamma_w = \frac{W_w}{V_w} \Longrightarrow W_w = \gamma_w * V_w$

$\gamma_h = \frac{\gamma_s \, V_s + \gamma_w \, V_w}{V_s + V_v}$

$S_r = \frac{V_w}{V_v} \Longrightarrow V_w = S_r \, V_v$

$\gamma_h = \frac{\gamma_s \, V_s + \gamma_w \ S_r \, V_v}{V_s + V_v}$

Dividing the numerator and denumerator by V_s, we get:

$$\gamma_h = \frac{\frac{\gamma_s \, V_s}{V_s} + \frac{\gamma_w \ \ V_v}{V_s} \gamma_w \ S_r \, V_v}{\frac{V_s}{V_s} + \frac{V_v}{V_s}}$$

and as: $e = \frac{V_v}{V_s}$, we obtain:

$$\gamma_h = \frac{\gamma_s \ + eS_r \ \gamma_w}{1 + e}$$

At saturation, we will have: $S_r = 1$; $\gamma_h = \gamma_{sat}$, from which the expression of the saturated volumetric weight (γ_{sat}) as a function of the volumetric weight of the solid grains (γ_s), the void ratio (e) and the volumetric weight of water (γ_w) is written as follows :

$$\gamma_{sat} = \frac{\gamma_s \ + e \ \gamma_w}{1 + e}$$

2.6.5 Water content: ω

Exercise 32

Write the expression for the water content (ω)as a function of the porosity (n), the dry density (γ_d) and the density of water (γ_w) and the degree of saturation (S_r):

$$\omega = f\,(n, \gamma_d, \gamma_w, S_r) \ = \frac{nS_r\gamma_w}{\gamma_d}$$

Solution:

$\omega \ = \frac{W_w}{W_s}$

$\gamma_w = \frac{W_w}{V_w}$ $\Longrightarrow$ $W_w = \gamma_w * V_w$

$\gamma_d = \frac{W_s}{V}$ $\Longrightarrow$ $W_s = \gamma_d \ V$

$\omega \ = \frac{\gamma_w * V_w}{\gamma_d \ V}$

Dividing the numerator and denumerator by V_v, we get:

$$\omega \ = \frac{\gamma_w * \frac{V_w}{V_v}}{\gamma_d \frac{V}{V_v}} \ = \frac{\gamma_w * S_r}{\gamma_d * \frac{1}{n}}$$

$$\omega \ = \frac{n.S_r.\gamma_w}{\gamma_d}$$

Exercise 33

Give the expression of the water content (ω) as a function of the void ratio (e), the volumetric weight of the solid grains (γ_s), the degree of saturation (S_r)and the volumetric weight of water (γ_w):

$$\omega = f(\, e, \gamma_s, S_r, \gamma_w) \;= \frac{S_r \gamma_w\, e}{\gamma_s}$$

Solution:

$\omega = \frac{W_w}{W_s}$

$\gamma_w = \frac{W_w}{V_w} \Longrightarrow W_w = \gamma_w * V_w$

$\gamma_s = \frac{W_s}{V_s} \Longrightarrow W_s = \gamma_s * V_s$

$\omega = \frac{\gamma_w * V_w}{\gamma_s * V_s}$

$S_r = \frac{V_w}{V_v} \Longrightarrow V_w = S_r * V_v$

$$\omega = \frac{S_r\; V_v\; \gamma_w}{V_s\;\; \gamma_s}$$

Dividing the numerator and denumerator by V_s, we get: $\omega \;= \frac{S_r * \gamma_w * \frac{V_v}{V_s}}{\gamma_s \frac{V_s}{V_s}}$

$$\frac{V_v}{V_s} = \; e$$

$$\omega = \frac{S_r * \gamma_w * e}{\gamma_s}$$

Exercise 34

Establish the expression of the water content (ω) as a function of the volumetric weight of the solid grains (γ_s), the porosity (n), the degree of saturation (S_r) $et$the volumetric weight of the water (γ_w):

$$\omega = \frac{n \gamma_w S_r}{\gamma_s (1 - n)}$$

Solution:

$$\omega = \frac{w_w}{w_s} = \frac{v_w \gamma_w}{v_s \gamma_s} = \frac{s_r v_v \gamma_w}{\gamma_s (v - v_v)}$$

$$\omega = \frac{n v \gamma_w s_r}{v \gamma_s \left(1 - \frac{v_v}{v}\right)} = \frac{n \gamma_w s_r}{\gamma_s \left(1 - \frac{v_v}{v}\right)}$$

$$\omega = \frac{n \gamma_w s_r}{\gamma_s (1 - n)}$$

Exercise 35

Demonstrate the expression of water content (ω) as a function of dry density (γ_d), density of solid grains (γ_s), degree of saturation (S_r) and density of water (γ_w):

$$\omega = S_r \left(\frac{\gamma_w}{\gamma_d} - \frac{\gamma_w}{\gamma_s}\right)$$

Solution:

$$\omega = \frac{w_w}{w_s} = \frac{\gamma_w . v_w}{\gamma_d . v} = \frac{\gamma_w . s_r . v_v}{\gamma_d . v} = S_r \frac{\gamma_w . v_v}{\gamma_d . v} = S_r \frac{\gamma_w . (v - v_s)}{\gamma_d . v}$$

$$\omega = S_r \left(\frac{\gamma_w . v}{\gamma_d . v} - \frac{\gamma_w . v_s}{\gamma_d . v}\right) = S_r \left(\frac{\gamma_w}{\gamma_d} - \frac{\gamma_w . w_s}{\gamma_d \gamma_s . \frac{w_s}{\gamma_d}}\right)$$

Finally, the expression of the water content (ω) as a function of the dry density (γ_d), the density of the solid grains (γ_s), the degree of saturation (S_r) and the density of water (γ_w) is written as follows:

$$\omega = S_r \left(\frac{\gamma_w}{\gamma_d} - \frac{\gamma_w}{\gamma_s}\right)$$

Exercise 36

Give the expression of the water content (ω) as a function of the dry density (γ_d), the void ratio (e), the degree of saturation (S_r) and the density of water (γ_w):

$$\omega = \frac{S_r \gamma_w e}{\gamma_d (1 + e)}$$

Solution:

$$\omega = \frac{w_w}{w_s} = \frac{\gamma_w . v_w}{\gamma_d . v} = \frac{\gamma_w . S_r . v_v}{\gamma_d . v}$$

$$\omega = \frac{S_r \gamma_w . v_v}{\gamma_d (v_s + v_v)} = \frac{S_r \gamma_w . e . v_s}{\gamma_d (v_s + v_v)}$$

$$\omega = \frac{S_r \gamma_w e}{\gamma_d (\frac{v_s}{v_s} + \frac{v_v}{v_s})}$$

Finally, the expression of water content (ω) as a function of dry density (γ_d), void ratio (e), degree of saturation (S_r) and water density (γ_w) is presented in the following form:

$$\omega = \frac{S_r \gamma_w e}{\gamma_d (1 + e)}$$

2.6.6 Porosity:n

Exercise 38

Establish the expression of the porosity (n) as a function of the void ratio (e):

$$n = \frac{e}{1 + e}$$

Solution:

$$n = \frac{v_v}{v} = \frac{v_v}{v_s + v_v}$$

$$n = \frac{e v_s}{v_s \left(1 + \frac{v_v}{v_s}\right)} = \frac{e}{\left(1 + \frac{v_v}{v_s}\right)}$$

$$\frac{v_v}{v_s} = e$$

Hence the porosity (n) as a function of the void ratio (e) is written as follows:

$$n = \frac{e}{1 + e}$$

Exercise 39

Establish the expression of porosity (n) as a function of the volumetric weight of the solid grains (γ_s) and the dry volumetric weight (γ_d)

$$n = \frac{\gamma_s - \gamma_d}{\gamma_s}$$

Solution:

$$n = \frac{v_v}{v} = \frac{v_v}{\frac{w_s}{\gamma_d}} = \frac{v_v \gamma_d}{w_s}$$

$$n = \frac{(v - v_s)\gamma_d}{w_s} = \frac{\left(\frac{w_s}{\gamma_d} - v_s\right)\gamma_d}{w_s} = \frac{w_s - v_s \gamma_d}{\gamma_s v_s}$$

$$n = \frac{v_s \gamma_s - v_s \gamma_d}{\gamma_s v_s} = \frac{v_s(\gamma_s - \gamma_d)}{v_s \gamma_s}$$

Finally, the expression of porosity (n) as a function of the density of solid grains (γ_s) and the dry density (γ_d) takes the following form:

$$n = \frac{\gamma_s - \gamma_d}{\gamma_s}$$

Exercise 40

Establish the expression of porosity (n) as a function of the volumetric weight of the solid grains (γ_s), the water content (ω), the degree of saturation (s_r) and the volumetric weight of water (γ_w):

$$n = \frac{1}{\left(1 + \frac{s_r \gamma_w}{\gamma_s \omega}\right)}$$

Solution:

$$n = \frac{v_v}{v} = \frac{v_v}{v_v + v_s} = \frac{v_v}{v_v\left(1 + \frac{v_s}{v_v}\right)}$$

$$n = \frac{1}{\left(1 + \frac{v_s}{v_v}\right)} = \frac{1}{\left(1 + \frac{s_r w_s}{\gamma_s v_w}\right)} = \frac{1}{\left(1 + \frac{s_r w_w}{\omega \gamma_s v_w}\right)}$$

$$n = \frac{1}{\left(1 + \frac{S_r \gamma_w v_w}{\omega \gamma_s v_w}\right)}$$

Hence the expression of porosity (n) as a function of the volumetric weight of solid grains (γ_s), the water content (ω), the degree of saturation (s_r) and the volumetric weight of water (γ_w) is as follows:

$$n = \frac{1}{\left(1 + \frac{S_r \gamma_w}{\gamma_s \omega}\right)}$$

Exercise 41

Establish the expression of porosity (n) as a function of the volumetric weight of the solid grains (γ_s), the water content at saturation (ω_{sat}) and the volumetric weight of water (γ_w):

$$n = \frac{1}{\left(1 + \frac{\gamma_w}{\gamma_s \omega_{sat}}\right)}$$

Solution:

$$n = \frac{v_v}{v} = \frac{v_v}{v_v + v_s} = \frac{v_v}{v_v\left(1 + \frac{v_s}{v_v}\right)}$$

$$n = \frac{1}{\left(1 + \frac{v_s}{v_v}\right)} = \frac{1}{\left(1 + \frac{S_r W_s}{\gamma_s v_w}\right)} = \frac{1}{\left(1 + \frac{S_r W_w}{\omega \gamma_s v_w}\right)}$$

$$n = \frac{1}{\left(1 + \frac{S_r \gamma_w v_w}{\omega \gamma_s v_w}\right)}$$

At saturation, we will have :$S_r = 1$; $\omega = \omega_{sat}$, and the expression of porosity (n) as a function of the density of solid grains (γ_s), the water content at saturation (ω_{sat}) and the density of water (γ_w) is written in the following form:

$$n = \frac{1}{\left(1 + \frac{\gamma_w}{\gamma_s \omega_{sat}}\right)}$$

Exercise 42

Establish the expression of porosity (n) as a function of dry volumetric weight (γ_d), water content (ω), degree of saturation (s_r) and volumetric weight of water (γ_w):

$$n = \frac{\omega\gamma_d}{\gamma_w s_r}$$

Solution:

$$n = \frac{v_v}{v} = \frac{\frac{v_w}{s_r}}{\frac{w_s}{\gamma_d}} = \frac{\frac{v_w}{s_r}}{\frac{w_s}{\gamma_d}}$$

$$n = \frac{v_w\gamma_d}{w_s s_r} = \frac{\frac{w_w}{\gamma_w}\gamma_d}{\frac{w_w}{\omega}s_r}$$

$$n = \frac{w_w\gamma_d\omega}{w_w\gamma_w s_r}$$

Finally, the expression of porosity (n) as a function of water density (γ_w), dry density (γ_d), water content (ω) and degree of saturation (s_r) takes the following form:

$$n = \frac{\gamma_d\omega}{\gamma_w s_r}$$

Exercise 43

Establish the expression of porosity (n) as a function of dry volumetric weight (γ_d), saturated water content (ω_{sat}) and water volumetric weight (γ_w):

$$n = \frac{\omega_{sat}\gamma_d}{\gamma_w}$$

Solution:

$$n = \frac{v_v}{v} = \frac{\frac{v_w}{s_r}}{\frac{w_s}{\gamma_d}} = \frac{\frac{v_w}{s_r}}{\frac{w_s}{\gamma_d}}$$

$$n = \frac{v_w\gamma_d}{w_s s_r} = \frac{\frac{w_w}{\gamma_w}\gamma_d}{\frac{w_w}{\omega}s_r} = \frac{w_w\gamma_d\omega}{w_w\gamma_w s_r}$$

$$n = \frac{\gamma_d \omega}{\gamma_w s_r}$$

At saturation, we will have: $S_r = 1$; $\omega = \omega_{sat}$, and the expression of the porosity (n) as a function of the dry volumetric weight (γ_d), the water content at saturation is (ω_{sat}) and the volumetric weight of water (γ_w) takes the following form:

$$n = \frac{\omega_{sat}\gamma_d}{\gamma_w}$$

2.6.7 Void index:e

Exercise 44

Establish the expression of the void index (e) as a function of the porosity (n):

$$e = \frac{n}{1 - n}$$

Solution:

$$e = \frac{v_v}{v_s} = \frac{v_v}{v - v_v}$$

Dividing the numerator and denumerator by v, we get:

$$e = \frac{\frac{v_v}{v}}{\frac{v}{v} - \frac{v_v}{v}}$$

$$\frac{v_v}{v} = n$$

From where the expression of the void ratio (e) as a function of the porosity (n) is written in the following form:

$$e = \frac{n}{1 - n}$$

Exercise 45 Establish the expression of the void ratio (e) as a function of the volumetric weight of the solid grains (γ_s), the degree of saturation (S_r), the water content (ω) and the volumetric weight of the water (γ_w):

$$e = \frac{\gamma_s}{\gamma_w . S_r} \omega$$

Solution:

$$e = \frac{v_v}{v_s} = \frac{v_v}{\frac{w_s}{\gamma_s}}$$

$$e = \frac{v_v.\gamma_s}{w_s} = \frac{\gamma_s.v_w}{w_s.S_r}$$

$$e = \frac{\gamma_s.w_w}{\gamma_w.w_s.S_r}$$

$$\frac{w_w}{w_s} = \omega$$

Hence the expression of the void ratio (e) as a function of the volumetric weight of the solid grains (γ_s), the degree of saturation (S_r), the water content (ω) and the volumetric weight of water (γ_w) is written as follows:

$$e = \frac{\gamma_s}{\gamma_w.S_r}\ \omega$$

Exercise 46

Write the expression for the void ratio (e) as a function of the density of the solid grains (γ_s), the water content at saturation (ω_{sat}) and the density of water (γ_w):

$$e = \frac{\gamma_s}{\gamma_w}\ .\omega_{sat}$$

Solution:

$$e = \frac{v_v}{v_s} = \frac{v_v}{\frac{w_s}{\gamma_s}}$$

$$e = \frac{v_v.\gamma_s}{w_s} = \frac{\gamma_s.S_r.v_w}{w_s}$$

$$e = \frac{\gamma_s.S_r.w_w}{\gamma_w.w_s}$$

$$\frac{w_w}{w_s} = \omega$$

$$e = \frac{\gamma_s.S_r}{\gamma_w}\omega$$

At saturation, we will have : $S_r = 1$; $\omega = \omega_{sat}$, and the expression of the void ratio (e) as a function of the volumetric weight of the solid grains (γ_s), the water content at saturation (ω_{sat}) and the volumetric weight of the water (γ_w):

$$e = \frac{\gamma_s}{\gamma_w} \cdot \omega_{sat}$$

Exercise 47

Demonstrate the expression of the void ratio (e) as a function of the density of solid grains (γ_s) and the dry density (γ_d):

$$e = \frac{\gamma_s - \gamma_d}{\gamma_d}$$

Solution:

$$e = \frac{v_v}{v_s} = \frac{v - v_s}{\frac{w_s}{\gamma_s}} = \frac{\gamma_s(v - v_s)}{w_s} = \frac{v\gamma_s - \gamma_s v_s}{\gamma_d v}$$

$$e = \frac{v\gamma_s - \gamma_s \frac{w_s}{\gamma_s}}{\gamma_d v} = \frac{v\gamma_s - w_s}{\gamma_d v} = \frac{v\gamma_s - v\gamma_d}{\gamma_d v}$$

$$e = \frac{v(\gamma_s - \gamma_d)}{v\gamma_d}$$

And finally, the expression of the void ratio (e) as a function of the density of the solid grains (γ_s) and the dry density (γ_d):

$$e = \frac{\gamma_s - \gamma_d}{\gamma_d}$$

Exercise 48

Demonstrate the expression of the void ratio (e) as a function of the dry density (γ_d), the water content (ω), the degree of saturation (S_r) and the density of water (γ_w):

$$e = \frac{\omega\gamma_d}{\gamma_w s_r - \omega\gamma_d}$$

Solution:

$$e = \frac{v_v}{v_s} = \frac{v_w}{s_r v_s} = \frac{w_w}{\gamma_w s_r v_s}$$

$$e = \frac{\omega w_s}{\gamma_w s_r v_s} = \frac{\omega v \gamma_d}{\gamma_w s_r v_s} = \frac{\omega \gamma_d v}{s_r \gamma_w (v - v_v)}$$

$$e = \frac{\omega \gamma_d v}{v s_r \gamma_w \left(1 - \frac{v_v}{v}\right)} = \frac{\omega \gamma_d}{s_r \gamma_w \left(1 - \frac{\gamma_d v_w}{s_r w_s}\right)} = \frac{\omega \gamma_d}{s_r \gamma_w \left(1 - \frac{\omega \gamma_d v_w}{s_r w_w}\right)}$$

$$e = \frac{\omega \gamma_d}{s_r \gamma_w \left(1 - \frac{\omega \gamma_d v_w}{v_w\, s_r \gamma_w}\right)} = \frac{w \gamma_d}{s_r \gamma_w \left(1 - \frac{\omega \gamma_d}{s_r \gamma_w}\right)}$$

$$e = \frac{\omega \gamma_d}{s_r \gamma_w - \frac{s_r \gamma_w \omega \gamma_d}{s_r \gamma_w}}$$

And finally, the expression of the void ratio (e) as a function of the dry volumetric weight (γ_d), the water content (ω), the degree of saturation (S_r) and the volumetric weight of water (γ_w):

$$e = \frac{\omega \gamma_d}{\gamma_w s_r - \omega \gamma_d}$$

Exercise 49

Demonstrate the expression of the void ratio (e) as a function of the dry density (γ_d), the saturated water content (ω_{sat}) and the density of water (γ_w):

$$e = \frac{\omega_{sat}\, \gamma_d}{\gamma_w - \omega_{sat}\, \gamma_d}$$

Solution:

$$e = \frac{v_v}{v_s} = \frac{v_w}{s_r v_s} = \frac{w_w}{\gamma_w s_r v_s}$$

$$e = \frac{\omega w_s}{\gamma_w s_r v_s} = \frac{\omega v \gamma_d}{\gamma_w s_r v_s} = \frac{\omega \gamma_d v}{s_r \gamma_w (v - v_v)}$$

$$e = \frac{\omega\gamma_d v}{v s_r \gamma_w \left(1 - \frac{v_v}{v}\right)} = \frac{\omega\gamma_d}{s_r \gamma_w \left(1 - \frac{\gamma_d v_w}{s_r w_s}\right)} = \frac{\omega\gamma_d}{s_r \gamma_w \left(1 - \frac{\omega\gamma_d v_w}{s_r w_w}\right)}$$

$$e = \frac{\omega\gamma_d}{s_r \gamma_w \left(1 - \frac{\omega\gamma_d v_w}{v_w\ s_r \gamma_w}\right)} = \frac{w\gamma_d}{s_r \gamma_w \left(1 - \frac{\omega\gamma_d}{s_r \gamma_w}\right)} = \frac{\omega\gamma_d}{s_r \gamma_w - \frac{s_r \gamma_w \omega\gamma_d}{s_r \gamma_w}}$$

$$e = \frac{\omega\gamma_d}{\gamma_w s_r - \omega\gamma_d}$$

At saturation, we will have : $S_r = 1$; $\omega = \omega_{sat}$, and the expression of the void ratio (e) as a function of the dry volumetric weight (γ_d), the water content at saturation (ω_{sat}) and the volumetric weight of water (γ_w):

$$e = \frac{\omega_{sat}\,\gamma_d}{\gamma_w - \omega_{sat}\,\gamma_d}$$

2.6.8 Degree of saturation:S_r

Exercise 50

Establish the expression of the degree of saturation (S_r) as a function of the volumetric weight of the solid grains (γ_s), the water content (ω), the void ratio (e) and the volumetric weight of the water (γ_w):

$$S_r = f(\gamma_s, \omega, e, \gamma_w) = \frac{\omega\,\gamma_s}{e\,\gamma_w}$$

Solution:

$S_r = \frac{V_w}{V_v}$

$\gamma_w = \frac{W_w}{V_w} \Longrightarrow V_w = \frac{W_w}{\gamma_w}$

$e = \frac{V_v}{V_s} \Longrightarrow V_v = e\,V_s$

$$S_r = \frac{\frac{W_w}{\gamma_w}}{e\,V_s} = \frac{W_w}{e\,V_s\,\gamma_w}$$

$\omega = \frac{W_w}{W_s} \Longrightarrow W_W = \omega W_s$

$S_r = \frac{\omega\, W_s}{e\, V_s\, \gamma_w}$, *but:*

$\frac{W_s}{V_s} = \gamma_s$

Therefore, the expression of the degree of saturation (S_r) as a function of the density of the solid grains (γ_s), the water content (ω), the void ratio (e) and the density of water (γ_w):

$$\boxed{S_r = f(\gamma_s\ ,\omega, \mathrm{e}, \gamma_{\mathrm{w}}) = \frac{\omega\, \gamma_s}{e\, \gamma_w}}$$

Exercise 51

Establish the expression of the degree of saturation (S_r) as a function of the water content (ω), the volumetric weight of the solid grains (γ_s), the dry volumetric weight (γ_d), the void ratio (e) and the volumetric weight of water (γ_w):

$$S_r = f\,(\omega, \gamma_s, \gamma_d, \gamma_w) = \frac{\omega\, \gamma_s\ \gamma_d}{(\gamma_s - \gamma_d)\, \gamma_w}$$

Solution:

$S_r = \frac{V_w}{V_v}$

$\gamma_w = \frac{W_w}{V_w} \quad \Longrightarrow \quad V_w = \frac{W_w}{\gamma_w}$

$V = V_v + V_s \quad \Longrightarrow \quad V_V = V - V_S$

$S_r = \frac{W_w}{(V - V_s)\, \gamma_w}$

$\omega = \frac{W_w}{W_s} \quad \Longrightarrow \quad W_w = \omega W_s$

$W_s = \gamma_s * V_s \quad \Longrightarrow \quad V_s = \frac{W_s}{\gamma_s}$

$\gamma_d = \frac{W_s}{V} \quad \Longrightarrow \quad V = \frac{W_s}{\gamma_d}$

$$S_r = \frac{\omega\, W_s}{\left(\frac{W_s}{\gamma_d} - \frac{W_s}{\gamma_s}\right)\gamma_w} = \frac{\omega\, W_s}{\left(\frac{W_s\, \gamma_s\ - W_s\ \gamma_d}{\gamma_d}\right)\gamma_w}$$

$$S_r = \frac{\omega\, W_s}{W_s(\frac{\gamma_s - \gamma_d}{\gamma_d})\gamma_w}$$

And finally, the expression of the degree of saturation (S_r) as a function of the water content (ω), the density of solid grains (γ_s), the dry density (γ_d), the void ratio (e) and the density of water (γ_w) is written in the following form:

$$\boxed{S_r = f(\omega, \gamma_s, \gamma_d, \gamma_w) = \frac{\omega\, \gamma_s\, \gamma_d}{(\gamma_s - \gamma_d)\, \gamma_w}}$$

2.6.9 Water content at saturation: $\boldsymbol{\omega_{sat}}$

Exercise 52

Write the expression for the water content at saturation (ω_{sat})as a function of the porosity (n), the dry density (γ_d) and the density of water (γ_w):

$$\omega_{sat} = f(n, \gamma_d, \gamma_w) = \frac{n\gamma_w}{\gamma_d}$$

Solution:

$$\omega = \frac{W_w}{W_s}$$

$$\gamma_w = \frac{W_w}{V_w} \Longrightarrow W_w = \gamma_w * V_w$$

$$\gamma_d = \frac{W_s}{V} \Longrightarrow W_s = \gamma_d\, V$$

$$\omega = \frac{\gamma_w * V_w}{\gamma_d\, V}$$

Dividing the numerator and denumerator by V_v, we get:

$$\omega = \frac{\gamma_w * \frac{V_w}{V_v}}{\gamma_d \frac{V}{V_v}} = \frac{\gamma_w * S_r}{\gamma_d * \frac{1}{n}}$$

$$\omega = \frac{n\, S_r \gamma_w}{\gamma_d}$$

At saturation, we will have :$S_r = 1$; $\omega = \omega_{sat}$, hence the expression of the water content at saturation (ω_{sat}) as a function of the porosity (n), the dry volumetric weight (γ_d) and the volumetric weight of water (γ_w) is presented in the following form:

$$\omega_{sat} = f(n, \gamma_d, \gamma_w) = \frac{n\gamma_w}{\gamma_d}$$

Exercise 53

Give the expression of the water content at saturation (ω_{sat}) as a function of the void ratio (e), the density of the solid grains (γ_s) and the density of water (γ_w):

$$\omega_{sat} = f(e, \gamma_s, \gamma_w) = \frac{\gamma_w\, e}{\gamma_s}$$

Solution:

$\omega = \frac{W_w}{W_s}$

$\gamma_w = \frac{W_w}{V_w} \quad \Longrightarrow \quad W_w = \gamma_w * V_w$

$\gamma_s = \frac{W_s}{V_s} \quad \Longrightarrow \quad W_s = \gamma_s * V_s$

$\omega = \frac{\gamma_w * V_w}{\gamma_s * V_s}$

$S_r = \frac{V_w}{V_v} \quad \Longrightarrow \quad V_w = S_r * V_v$

$$\omega = \frac{S_r\, V_v\, \gamma_w}{V_s\;\gamma_s}$$

Dividing the numerator and denumerator by V_s, we get:

$$\omega = \frac{S_r * \gamma_w * \frac{V_v}{V_s}}{\gamma_s\, \frac{V_s}{V_s}}$$

$$\frac{V_v}{V_s} = e$$

$$\omega = \frac{S_r * \gamma_w * e}{\gamma_s}$$

At saturation, we will have :$S_r = 1$; $\omega = \omega_{sat}$, from which the expression of the water content at saturation (ω_{sat}) as a function of the void ratio (e), the volumetric weight of the solid grains (γ_s) and the volumetric weight of water (γ_w) is written as follows:

$$\omega_{sat} = f\,(e, \gamma_s, \gamma_w) = \frac{\gamma_w\, e}{\gamma_s}$$

Exercise 54

Establish the expression of the water content at saturation (ω_{sat}) as a function of the volumetric weight of the solid grains (γ_s), the porosity (n) and the volumetric weight of the water (γ_w):

$$\omega_{sat} = \frac{n\gamma_w}{\gamma_s(1-n)}$$

Solution:

$$\omega = \frac{w_w}{w_s} = \frac{v_w\gamma_w}{v_s\gamma_s}$$

$$\omega = \frac{s_r v_v \gamma_w}{\gamma_s(v - v_v)} = \frac{nv\gamma_w s_r}{v\gamma_s\left(1 - \frac{v_v}{v}\right)}$$

$$\omega = \frac{n\gamma_w s_r}{\gamma_s\left(1 - \frac{v_v}{v}\right)} = \frac{n\gamma_w s_r}{\gamma_s(1-n)}$$

At saturation, we will have :$S_r = 1$; $\omega = \omega_{sat}$, from which the expression of the water content at saturation (ω_{sat}) as a function of the volumetric weight of the solid grains (γ_s), the porosity (n) and the volumetric weight of the water (γ_w) takes the following form:

$$\omega_{sat} = \frac{n\gamma_w}{\gamma_s(1-n)}$$

Exercise 55

Demonstrate the expression of the water content at saturation (ω_{sat}) as a function of the dry density (γ_d), the density of the solid grains (γ_s) and the density of water (γ_w):

$$\omega_{sat} = \frac{\gamma_w}{\gamma_d} - \frac{\gamma_w}{\gamma_s}$$

Solution:

$$\omega = \frac{w_w}{w_s} = \frac{\gamma_w . v_w}{\gamma_d . v} = \frac{\gamma_w . s_r . v_v}{\gamma_d . v}$$

$$\omega = S_r \frac{\gamma_w . v_v}{\gamma_d . v} = S_r \frac{\gamma_w . (v - v_s)}{\gamma_d . v} = S_r (\frac{\gamma_w . v}{\gamma_d . v} - \frac{\gamma_w . v_s}{\gamma_d . v})$$

$$\omega = S_r (\frac{\gamma_w}{\gamma_d} - \frac{\gamma_w . w_s}{\gamma_d \gamma_s . \frac{w_s}{\gamma_d}}) = S_r (\frac{\gamma_w}{\gamma_d} - \frac{\gamma_w}{\gamma_s})$$

At saturation, we will have : $S_r = 1$; $\omega = \omega_{sat}$, and the expression for the water content at saturation (ω_{sat}) as a function of the dry volumetric weight (γ_d), the volumetric weight of the solid grains (γ_s) and the volumetric weight of the water (γ_w) takes the following form:

$$\omega_{sat} = \frac{\gamma_w}{\gamma_d} - \frac{\gamma_w}{\gamma_s}$$

Exercise 57

Demonstrate the expression of the water content at saturation (ω_{sat}) as a function of the dry density (γ_d), the void ratio (e) and the density of water (γ_w):

$$\omega_{sat} = \frac{\gamma_w e}{\gamma_d (1 + e)}$$

Solution:

$$\omega = \frac{w_w}{w_s} = \frac{\gamma_w . v_w}{\gamma_d . v} = \frac{\gamma_w . S_r . v_v}{\gamma_d . v}$$

$$\omega = \frac{S_r \gamma_w . v_v}{\gamma_d (v_s + v_v)} = \frac{S_r \gamma_w . e . v_s}{\gamma_d (v_s + v_v)}$$

$$\omega = \frac{S_r \gamma_w e}{\gamma_d (\frac{v_s}{v_s} + \frac{v_v}{v_s})} = \frac{S_r \gamma_w e}{\gamma_d (1 + e)}$$

At saturation, we will have: $S_r = 1$; $\omega = \omega_{sat}$, and the expression of the water content at saturation (ω_{sat}) as a function of the dry volumetric weight (γ_d), the void ratio (e) and the volumetric weight of water (γ_w) is written as follows:

$$\omega_{sat} = \frac{\gamma_w e}{\gamma_d (1 + e)}$$

<u>CHAPTER III</u>:

Soil Compaction

Chapter 3. Soil Compaction

3.1 Compaction Theory
3.2 Laboratory Compaction Tests (Proctor and CBR Tests)
3.3 Special In-Situ Compaction Equipment and Processes

3.4 Compaction Requirements and Control

3.1 Compaction theory

3.1.1 Introduction

Soils and road materials are made up of solids (soil grains, aggregates, sand, etc.), liquids (water, bitumen, emulsion, etc.) and air (trapped between solids and liquids).

The action of compaction results in the grains of soil coming closer together (settlement **T**) and an expulsion of air, and therefore we observe a reduction in the volume **V,** the weight of the solids **P** remaining identical, the apparent density ρ **= P/V** increases

3.1.2 Objectives of compaction

The main objectives are pursued when carrying out road works for earthworks, subgrades, road bases and wearing courses.
Table 3.1 summarizes the objectives envisaged by the compaction operations.

Tab. 3.1*: Objectives of compaction*

Delete the further deformations	Increase mechanical characteristics	Ensure waterproofing
Embankment settlements Differential settlements Deformations of Roadways Surface layer rutting	Increase lift and trafficability of layers of shape or fill. Increase the modulus of untreated foundations. Increase the resistance of treated seats and layers rolling. Allow materials to resist road traffic	Compaction is the first protections against water aggression. Important objective for the wearing course, avoiding the diaper disorders inferior

There are several methods to improve the properties of an existing soil, namely:

- *a-* ***Chemical process*** **:** By mixing or injecting chemicals into the ground such as Portland cement, lime , asphalt, calcium or sodium chloride, pulp and paper residues.
- *b-* ***Mechanical process:*** Mainly resolved by compaction and densification.
- *c-* ***Other processes:*** By lowering the water table to reduce interstitial pressures, or preloading and temporary loading to reduce settlements.

3.1.3 Proctor's theory

Proctor showed that compaction is a function of four parameters: the volumetric mass of the dry soil, the water content, compaction energy and soil type (granulometry, mineralogy, etc.).

When the water content is high, water absorbs a significant portion of the compaction energy without any benefit, whereas when the water content is low, water has an important lubricating role, and the dry density increases with the water content (Fig. 3.1).

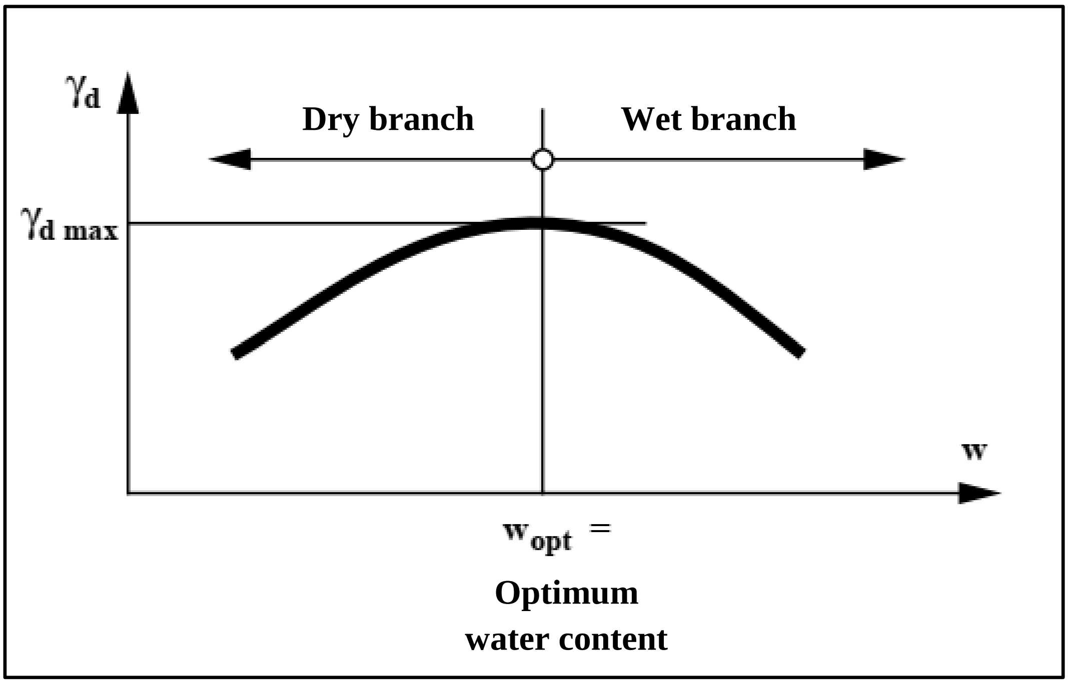

Fig. 3.1: *Compaction curve*

Compaction curves vary with the nature of the soil (Fig. 3.2). They are very flattened for sands, so their compaction is little influenced by the water content. Materials of this type constitute the best backfills.
When the compaction energy increases, the optimum volumetric weight increases and the optimum water content decreases (Fig. 3.3).

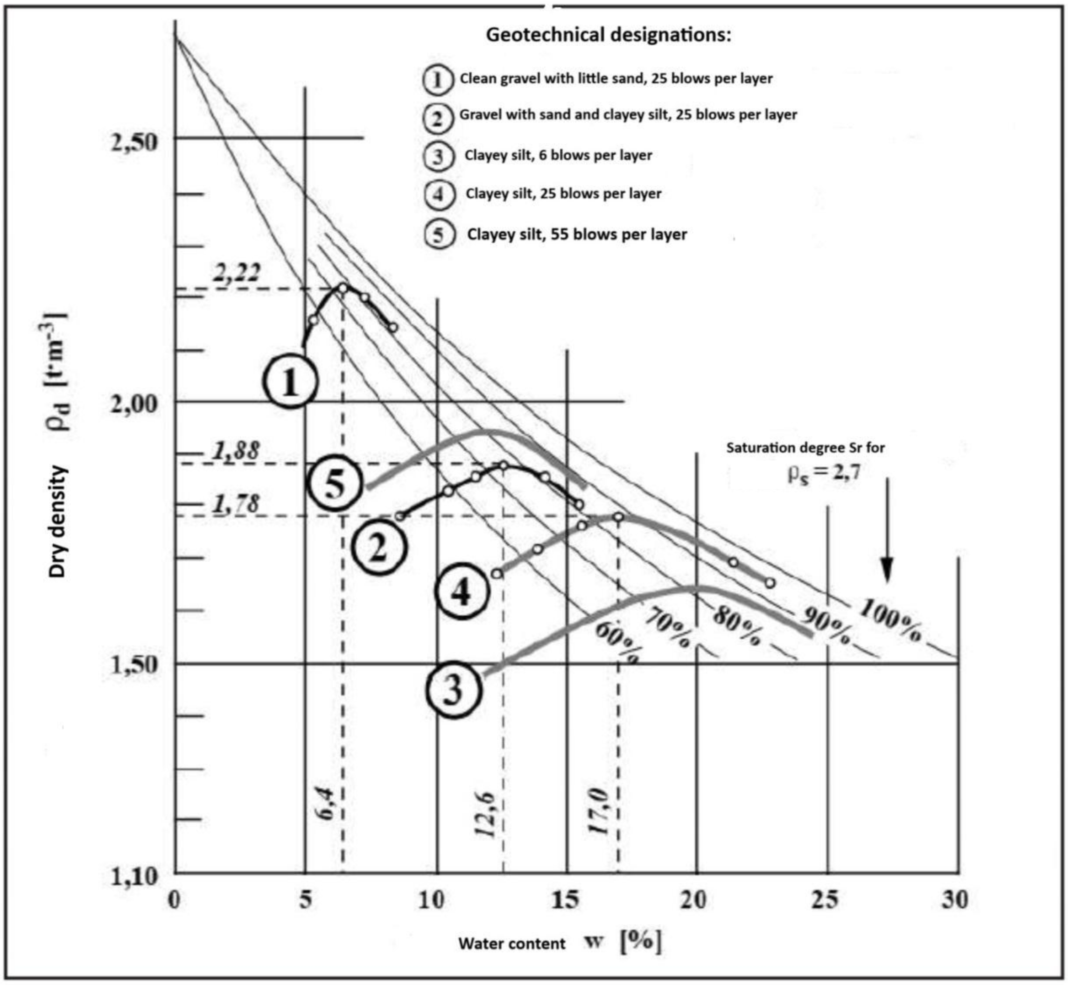

Fig. 3.2: *Influence of soil type on the shape of the compaction curve*

The compaction curves have as their envelope a curve called the saturation curve **,** which corresponds to the saturated state of the soil (Fig.3.4). The equation of this curve as a function of S_r is :

$\gamma_d = \frac{S_r \gamma_s \gamma_w}{\gamma_s \omega + S_r \gamma_w}$, and with $S_r = 1$ we will have: $\gamma_d = \frac{\gamma_s \gamma_w}{\gamma_s \omega + \gamma_w}$

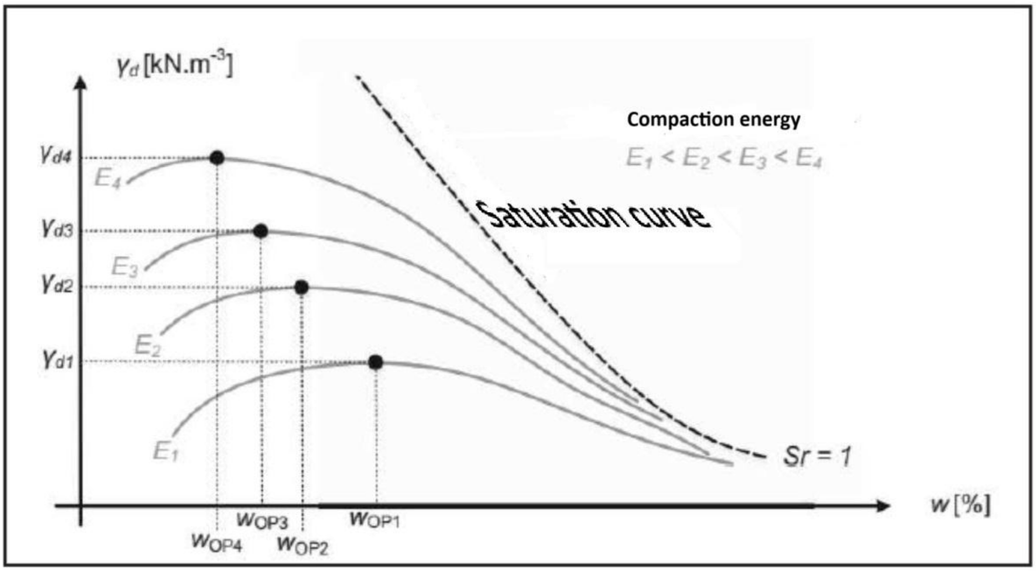

Fig. 3.3: *Influence of compaction energy*

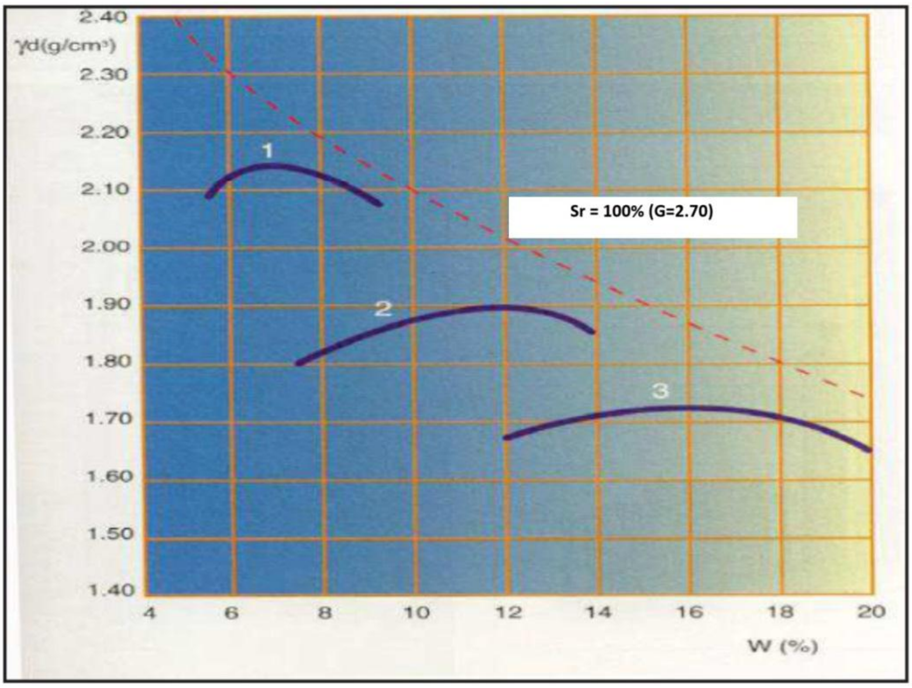

Fig. 3.4: *Saturation curve*

3.2 Laboratory compaction tests (Proctor and CBR tests)

3.2.1 Optimum Proctor

The Proctor test is used to determine the compaction characteristics of a soil. Ensuring sufficient compaction helps, among other things, to ensure good load-bearing capacity. The dry volumetric weight $\gamma_d$corresponds to the weight of the skeleton placed in a certain volume. It is therefore a good indicator of compactness. For a soil with fixed granularity and compaction energy, the dry volumetric weight $\gamma_d$reaches a maximum value for a certain water content value: the Proctor optimum ω_{opt}.

3.2.2 Principle of the method

The test consists of compacting in a standardized mold, using a standardized tamper, according to a well-defined process, the soil sample to be studied and measuring its water content and its dry specific weight after compaction.

The test is repeated several times in a row on samples brought to different water contents. Several points of a curve (γ_d, ω_{opt}) are thus defined; this curve is plotted, which represents a maximum whose abscissa is the optimum water content and the ordinate the optimum dry density.

For these tests, two types of molds can be used, depending on the fineness of the soil grains:
- The Proctor mold: ϕ_{interior} = 101.6 mm / H = 117 mm (without riser), and the volume of the Proctor mold will be: $V_{\text{mold proctor}}$ = 948 cm ^{3}

- The CBR mold: ϕ inner mold = 152 mm / H = 152 mm (without riser) including 25.4 mm thick spacer disk, i.e. a height Hutile = 126.6 mm (without riser), and the volume of the Proctor mold will be: Vmoule CBR = 2,296 cm ^{3}

With each of these molds, two types of test can be carried out (choice based on compaction energy):

- The PROCTOR NORMAL test,

- The MODIFIED PROCTOR test.

The choice of compaction intensity is made according to the overload that the structure will undergo during its lifespan:

Normal Proctor test: Relatively low desired resistance, of the type of unloaded or lightly loaded backfill,

Modified Proctor test: High resistance desired, such as motorway pavement. According to standard NF EN 13286-2, the equivalences between the results obtained with new and/or alternative equipment are based on the specific energy given by the equation:

$$E_S = mgH\frac{N}{V}$$

Or:

- E_S(J/m ^{3}): Specific compaction energy (Nm/m ^{3})... i.e. ... (Pa)
- m(kg): mass of the lady:
- g(N/kg): acceleration of gravity
- H(m): fall height
- N: number of strokes (Number of strokes per layer x number of layers).
- V: volume of the mold.

Table 3.2 summarizes the conditions of each test according to the mold selected (NF P 94-093):

Tab.3.2: *Normal and Modified Proctor test conditions*

Essay Proctor	Lady's Mass (Kg)	Fall height (Cm)	Number of strokes per layer	Number of layers	Compaction energy (Kj / m^3)
Normal	2.490	30.50	25 (Proctor mold)	3	587
			55 (CBR mold)	3	533
Amended	4.540	45.70	25 (Proctor mold)	5	2680
			55 (CBR mold)	5	2435

3.2.3 Materials required

CBR mould (possibly Proctor) - Normal or modified Proctor stamp - Levelling rule - Spacing disc - Homogenisation tanks for preparing the material - 5 and 20 mm sieves (checking and trimming the sample if necessary) - Trowel, spatula, brush - Graduated cylinder approximately 150 ml - Small containers (measurement of water content) - Balance capable of 20 kg, accuracy ± 5 g - Precision balance 200 g, accuracy ± 0.1 g - Oven 105°C ± 5°C - Oil burette.

3.2.4 Procedure

a/ Preparation of samples for testing:

Quantities to be taken: The following will be taken: 15 kg (Proctor mould); 33 kg (CBR).

Checking the sample for test feasibility:

- If D ≥ 20 mm, the soil must be sieved to 20 mm and the residue weighed: If the residue is < 25%, the test must be carried out in the CBR mould, but without integrating the residue (sample cut to 20 mm),

- If the refusal is > 25%, the PROCTOR test must not be carried out (hazardous compaction).

b / Preparation of the material

Choice of mold:

It depends on the size D of the large grains of the soil:

- If D ≤ 5 mm (and only in this case), the Proctor mold is authorized, but the CBR mold is recommended,

- If 5 < D ≤ 20 mm, use the CBR mold (soil kept intact with all its constituents),
- If D > 20 mm, but rejection ≤ 25%, the test is carried out in the CBR mold (soil trimmed to 20 mm),

Reminder: D > 20 mm, but refusal > 25%, the Proctor test cannot be done!

c / Execution of the test

For the NORMAL PROCTOR test, the filling is done in 3 layers.
For the MODIFIED PROCTOR test, the filling is done in 5 layers. The entire surface must be compacted for each layer as shown in Figure 3.5

1 - Assemble mold + base + spacer disk (if CBR mold) + paper disk at the bottom of the mold (facilitates demolding); then:

- Weigh the whole: either P1,
- Adjust the riser.

2 - Introduce the 1st layer and compact it, see figure 3.6. Place the mold on a concrete base of at least 100 kg, or on a concrete floor 25 cm thick, so that all the energy applied is applied to the sample. Tips: make scratches on the compacted surface (improves the bond with the next layer),
3 - Repeat the operation for each layer (3 for Normal compaction energy; 5 for Modified).

The approximate quantity of material to be used, for each layer, is mentioned in Table 3.3:

Tab.3.3: *Amount of material per layer of normal and modified Proctor tests*

Mold	Normal Proctor Test	Modified Proctor Test
Proctor	400 g	650 g
CBR	1,050 g	1,700 g

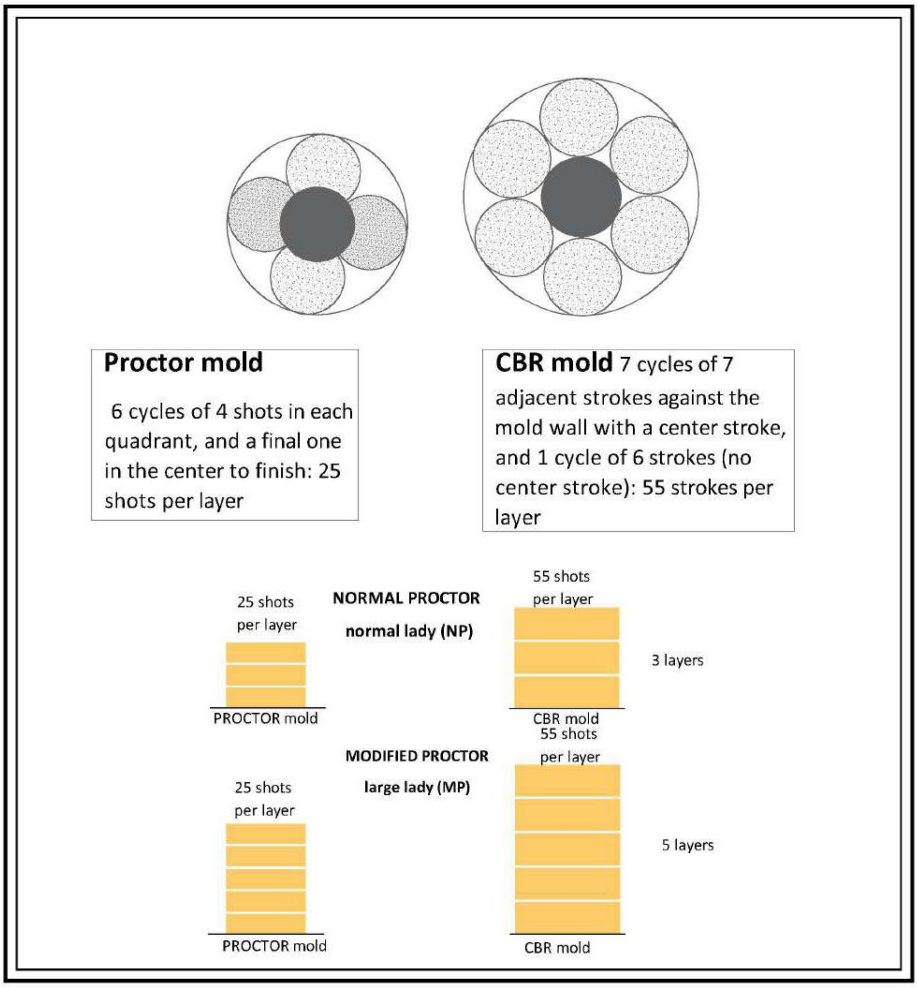

Fig. 3.5: *Number and compaction of layers according to the test and the type of mold*

4 - After compacting the last layer, remove the extension. The compacted soil should extend about 1 cm beyond the mold. If not, repeat the test,
5 - Level carefully from the center; during leveling, care should be taken not to create holes on the leveled surface,

6 - Weigh the whole just leveled (figure 3.6): either P2,

7 - Remove the base (and spacer disk if necessary) and take 2 samples from the sample, one at the top and the other at the bottom; determine the water content w; take the average of the two values obtained,

8 - Increase the water content ω of the initial sample by 2% and repeat the test 5 to 6 times, after having thoroughly cleaned the mold each time.

Note:

- PROCTOR mold: 2% ⇔ approximately 50 g of water for 2,500 g of soil,
- CBR mold: 2% ⇔approximately 110 g of water for 5,500 g of soil.

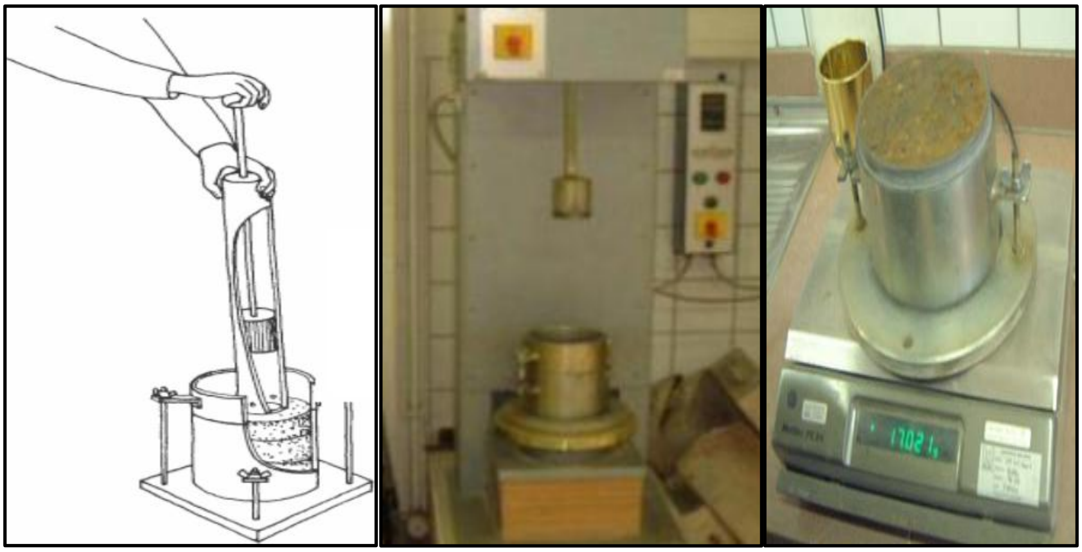

Fig. 3.6: Manual compaction – automatic compaction – weighing after compaction

3.2.5 Expression of results

Plot the curve γ_d= f(ω), with the following coordinates for each point on the curve:

- On the abscissa: ω, water content, - On the ordinate $\gamma_d$which is expressed:

$$\gamma_d = \frac{(P2 - P1)}{(1+\omega)Vmoule}$$

The coordinates of the Proctor optimum are deduced from the curve; they are expressed:
- For $\gamma_d$in kN/m ^{3} with 1 decimal place,

- For ω_{opt}% with 1 decimal place.

3.2.6 Calculation example for Modified Proctor test with CBR mold:

The results of a Modified Proctor test carried out on a road material (Adrar Tuff) as well as the calculations are shown in the two tables below.

The data of the CBR mold used in the test are as follows:

Weight of the mold without riser = 3842 grs .

Volume of the mold without riser = 2104 ^{cm3}

The first step in the calculations is to determine the wet density γ_h after each test, Table 3.4

Tab.3.4: *Final wet density after compaction - Modified Proctor*

Weight of added water	7%	9%	11%	13%	15%
Total wet weight (gr)	7882	8079	8275	8277	8220
Mold weight (gr)	3842				
Weight of wet soil (gr)	4040	4237	4433	4435	4378
Mold volume (cm^3)	2104				
Wet density (gr/cm^3)	**1.92**	**2.01**	**2.11**	**2.11**	**2.08**

The second step of the calculations (Table 3.5) involves determining the actual water content of each test ω, as well as the dry density. γ_d given by the following relation:

$$\gamma_d = \frac{\gamma_h}{1+\omega}$$

Tab.3.5: *Final water contents after compaction ω and dry densities γ_d*

Tare number	13	7	M3	21	15	22	A3	B11	3	B14
Total wet weight (gr)	162.35	161.14	138.71	137.10	130.15	130.80	139.45	137.15	139.04	137.85
Total dry weight (gr)	153.79	152.45	129.61	127.99	119.64	120.4	126.43	124.31	123.91	123.05
Tare weight (gr)	18.35	19.58	18.53	18.63	18.67	18.52	18.73	18.77	18.64	19.66

Water weight (gr)	8.56	8.69	9.10	9.11	10.51	10.40	13.02	12.84	15.13	14.80
Dry soil weight (gr)	135.44	132.87	111.08	109.36	100.97	101.88	107.7	105.54	105.27	103.39
Water content (%)	6.32	6.54	8.19	8.33	10.41	10.21	12.09	12.17	14.37	14.31
Average (%)	**6.43**		**8.26**		**10.31**		**12.13**		**14.34**	
Dry density	**1.80**		**1.86**		**1.91**		**1.88**		**1.82**	

Notes : Concerning the units, in MDS we generally confuse the weight with the mass (giving the unit of the gram to the weight). Also the density which is without unit, we take it in: gr/cm ^{3} – t/m ^{3} or even in kN /m ^{3}.

We admit the following equalities and equivalences, (and this by taking the value of gravity g=10 m/s ^{2}) :

$$1 \text{ gr/cm}^3 \equiv 1 \text{ kgf/dm}^3 = 1000 \text{ kgf/m}^3 = 1 \text{ tf/m}^3 \equiv 10 \text{ kN/m}^3$$

The graphical representation of the test is given in Figure 3.7

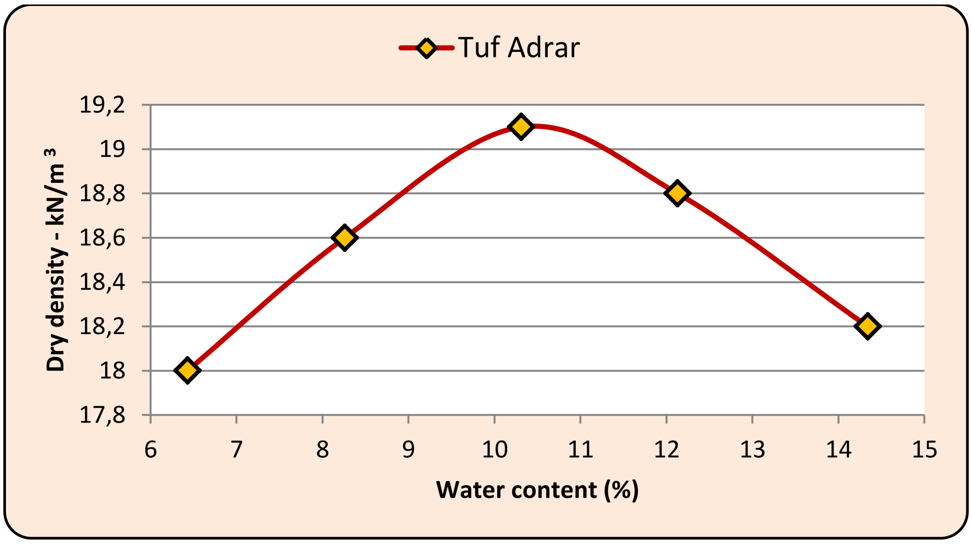

Fig. 3.7: *Variation of dry density* γ_d as a function of water content ω

3.3 In-situ compaction equipment and processes

For in-situ compaction, there are two distinct processes: common and special.

3.3.1 Compaction equipment and common compaction processes

Different families of machines are used during in situ compaction, we can cite the following: Pneumatic tire compactors (Pi) - smooth drum vibrating compactors (Vi) and padfoot compactors (VPi) - Static padfoot compactors (Spi) - Vibrating plates (PQi). The index (i) designates the class number, it increases with the efficiency of the compactor

Tire compactors (Pi)

For medium earthworks or for road materials "to be surfaced". Very mobile, they are used for sandy clay soils, fine and medium gravel. Everything is allowed (earthworks, subgrade, roadway, asphalt) but less effective than vibrators (figure 3.8). Maximum speed 6 km/h, average working speed between 3.5 and 5 km/h. Classification is a function of the wheel load (**CR**):

Fig. 3.8: *Tire compactor*

P1: 25kN < CR ≤ 40kN
P2: 40kN < CR ≤ 60kN
P3: CR > 60kN

Static compactors (with pad feet)

For large earthworks, often tandem, sometimes equipped with a blade (figure 3.9).
Maximum speed: 12 km/h. Average working speed: 6 km/h. The Classification is carried out according to the average static load per unit width (M1/L):
SP1: M1/L between 30 and 60 kg/cm
SP2: M1/L > 60 kg/cm and < 90 kg/cm

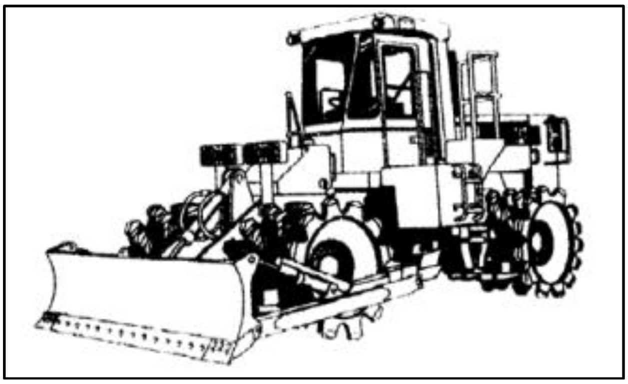

Fig. 3.9: *Static compactors (with pad feet)*

Compactors (smooth cylinder)
Less and less used except for rolling surface coatings and compacting very thin coatings < 4 cm. The tandem or tricycle morphologies (figure 3.10) are the most commonly encountered.

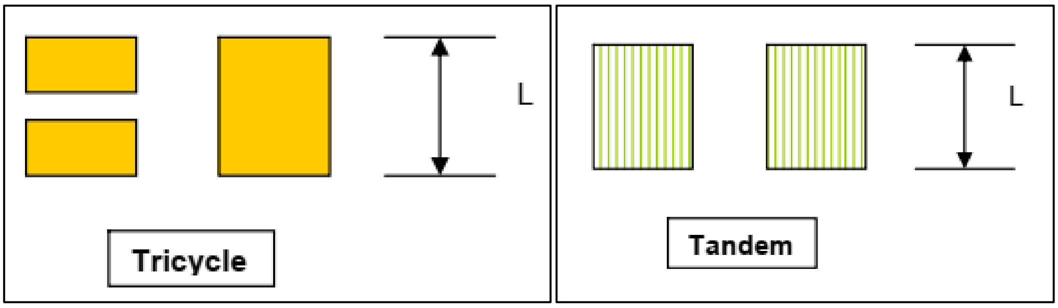

Fig. 3.10: *Tandem and tricycle smooth cylinder*

Vibrating compactors (Vi smooth cylinders or Vpi cylinders with pad feet)

Practically for any use: Highly buoyant materials with high objective - High thickness or granulometry materials (embankments, rockfill, subgrades) - Asphalts to be compacted quickly (temperature) - Surface coating. Maximum speed 12 km/h, average working speed between 3 and 5 km/h.
The classification is carried out from the static load **M** applied per cylinder width **L** and from the amplitude of the no-load vibration, figure 3.11.

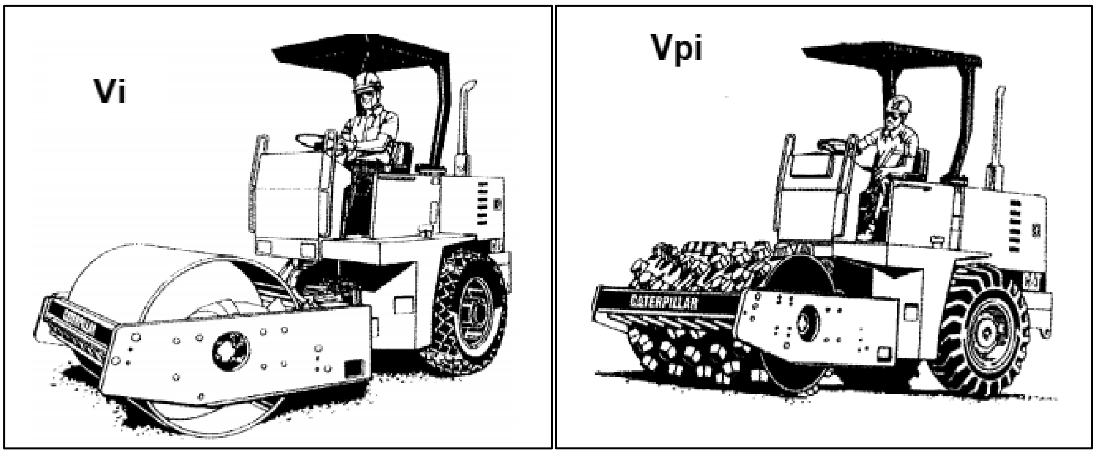

Fig. 3.11: *Vibrating compactors with smooth cylinders (Vi) and pad feet (Vpi)*

3.3.2 Special in situ compaction processes

For thick layers, dynamic compaction processes are used such as: *Explosive compaction* (Point explosives, Linear explosives), *Vibratory flotation compaction (* Vibrating tubes, Ballasted columns) and *Dynamic consolidation, the latter is* valid for all types of soil. It involves transmitting high-energy shocks to the surface of the soil to be treated (free fall of a mass of 10 to 30 tons.

3.4 Compaction requirements and control

Several factors mainly affect the efficiency of a machine on a given soil. Among these factors are those specific to the compacted ground (nature, water content), as well as factors characterizing the machine and the compaction sequence (number of passes, speed, contact pressure, frequency and intensity of vibration).

It should be noted that whatever the machines used, compaction on site must be carried out in thin layers of 20 to 30 cm (road works) or 10 to 15 cm (building works).

3.4.1. Influence of the speed of the machine

Depending on the machine used, there is an optimum speed, depending on the thickness of the layer and the nature of the material, to achieve maximum compaction. The stricter the quality requirements, the lower the translation speed. Generally, for most compactors, the speed is limited to 8km/h. In the case of vibrating compactors, the optimum speed is around 5km/h so that the vibrations can act effectively over the entire thickness of the layer.

3.4.2. Influence of the number of passes

Depending on the machine used and the quality parameters set beforehand, there is an optimal number of passes depending on the speed of the machine, the thickness of the layer and the nature of the material to achieve maximum compactness. The more stringent the quality requirements, the higher the optimal number of passes.

Typically, it takes 3 to 8 passes to compact a 30 cm thick layer of soil, but this number can easily reach 12 depending on the soil type, water content and compactor mass. If the desired compaction is not achieved after 12 passes under optimum moisture conditions, it is concluded that the compaction operations have not achieved their goal and that the compactor used is probably not adequate.

3.4.3. Effectiveness of compaction

The evaluation of the effectiveness of a compaction is carried out by comparing the volumetric weight of the dry soil on the site ($\gamma_{d\,chantier}$) with the maximum dry volumetric weight (Proctor optimum γ_{dmax}), the compactness $C$is therefore given using the following equation:

$$C = \frac{\gamma_{d\,chantier}}{\gamma_{dmax}}$$

Compaction is one of the criteria used to accept or reject compaction. It is expressed as a percentage, tending towards 100% when the value of ($\gamma_{d\,chantier}$) tends towards that of (γ_{dmax}). In general, the specifications require C≥ 95% (see 98%). The higher Cit is, the more effective the compaction has been.

3.4.4. Membrane densitometer

This is one of the devices used to evaluate the efficiency of in situ compaction, and this by deducing the value that the dry density will have after total drying.
The test is carried out in place immediately after compaction is completed.

The test consists of digging a cavity, collecting and weighing all of the extracted Wmaterial, determining its water content $\omega$and then measuring the volume of the cavity using a membrane densitometer. The device is equipped with a piston which, under the action of the operator, forces a volume of water into a flexible waterproof membrane which fits the shape of the cavity. A graduated rod allows the Vvolume $V$to be read directly, figure 3.12.

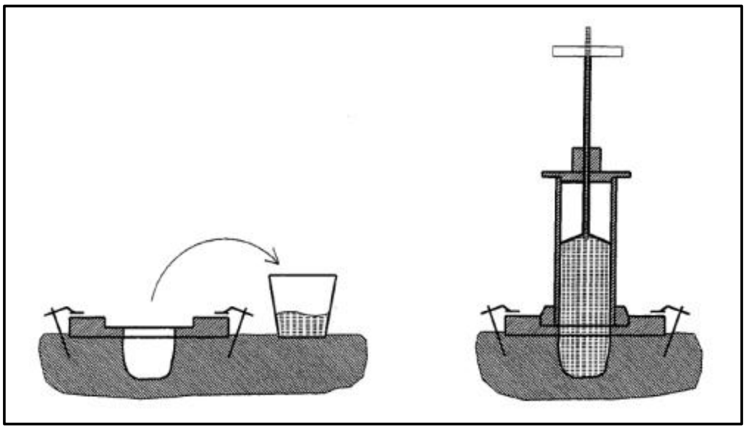
Fig. 3.12: *Membrane densitometer.*

The wet density is given by:

$$\gamma_h = \frac{W}{V}$$

And the dry density will be calculated by the following relation:

$$\gamma_{d\ chantier} = \frac{\gamma_h}{1+\omega}$$

Once $\gamma_{d\ chantier}$ is determined, we compare it with γ_{dmax}.

3.5 EXERCISES

Exercise 3.1: The results of the modified Proctor test carried out on a road material (Sample A1) are given. Carry out the necessary calculations and plot the curve of the variation of the dry density γ_d depending on the water content (ω %).

Determination of soil wet density γ_h	Total wet weight (gr)	7610	7751	7740
	Mold weight (gr)	3920	3920	3830
	Mold volume (cm ^{3})	2104		

Determination of soil water content ω %	Tare number:	537	I7	A3	569	1	2
	Total wet weight (gr)	138.05	126.8	104.71	115.72	109.8	100.31
	Total dry weight (gr)	131.25	120.3	98.25	108.43	99.97	91.22
	Tare weight (gr)	18.63	18.75	19.39	18.22	18.27	18.71

Solution:

- Determination of the wet density of the soil γ_h:

$$* \, Poids \; du \; sol \; humide \; P_h \; = \; Poids \; total \; humide \; - Mold \; weight$$

$$Poids \; du \; sol \; humide \; P_h \; = 7610 - 3920 = 3690$$

$$** \, Densit\acute{e} \; humide \; \gamma_h = \frac{P_h}{V} = \frac{3690}{2104} = \underline{1.75}$$

Weight of wet soil (gr)	3690	3831	3910
Wet density	**1.75**	**1.82**	**1.86**

- Determination of soil water content ω %:

$$\omega 1 \, \% = \frac{P_\omega}{P_s} = \frac{138{,}05 - 131{,}25}{131{,}25 - 18{,}63} = \frac{\mathbf{6{,}8}}{\mathbf{112{,}62}} = \mathbf{6{,}04}$$

$$\omega 2 \, \% = \frac{P_\omega}{P_s} = \frac{126{,}8 - 120{,}3}{120{,}3 - 18{,}75} = \frac{\mathbf{6{,}5}}{\mathbf{101{,}55}} = \mathbf{6{,}40}$$

Average: ω %= $\frac{\omega 1 \, \% + \omega 2 \, \%}{2} = \underline{6.22}$

Water weight (gr)	**6.8**	**6.5**	6.46	7.29	9.83	9.09
Dry soil weight (gr)	**112.62**	**101.55**	78.86	90.21	81.7	72.51
Water content (%)	**6.04**	**6.40**	8.19	8.08	12.03	12.54
Average (%)	**6.22**		**8.14**		**12.28**	

- Determination of the dry density of compacted soil γ_d:

$$\gamma_d = \frac{\gamma_h}{1 + \omega} = \frac{1{,}75}{1 + \frac{6{,}22}{100}} = \mathbf{1{,}65}$$

Water content (%)	**6.22**	**8.14**	**12.28**
Dry density	**1.65**	**1.68**	**1.66**

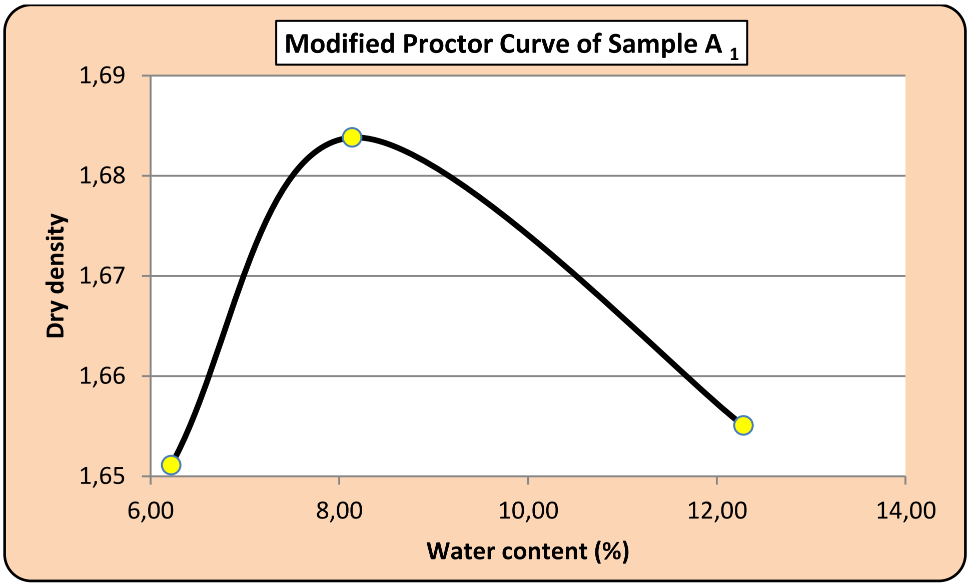

***Fig. 3.13:** Modified Proctor Curve of Sample A $_1$*

Exercise 3.2: The results of the modified Proctor test carried out on a road material (Sample A2) are given. Carry out the necessary calculations and plot the curve of the variation of the dry density γ_d depending on the water content (ω %).

Total wet weight (gr)	8300	8451	8410
Mold weight (gr)	3920	3830	3830
Mold volume (cm ^{3})	2104		

Tare number:	36	A4	B2	16	C5	11
Total wet weight (gr)	155.81	134.01	135.32	148.74	138.81	162.01
Total dry weight (gr)	148.34	126.68	125.96	138.53	127.41	147.92
Tare weight (gr)	18.89	17.97	20.13	19.21	18.7	14.38

Solution:

Weight of wet soil (gr)	4380	4621	4580
Wet density	**2.08**	**2.20**	**2.18**

Water weight (gr)	7.47	7.33	9.36	10.21	11.4	14.09
Dry soil weight (gr)	129.45	108.71	105.83	119.32	108.71	133.54
Water content (%)	5.77	6.74	8.84	8.56	10.49	10.55
Average (%)	**6.26**		**8.70**		**10.52**	

Water content (%)	6.26	8.70	10.52
Dry density	1.96	2.02	1.97

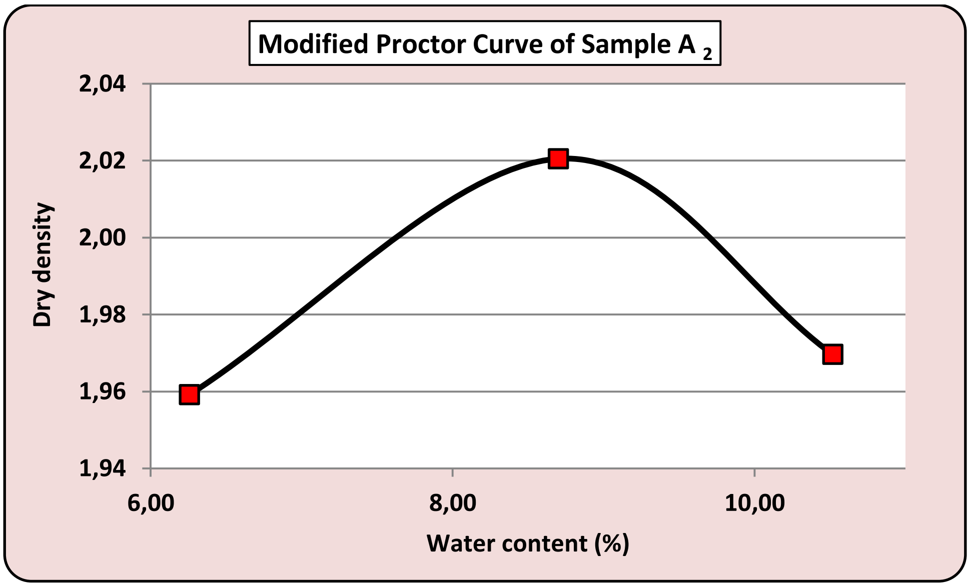

Fig. 3.14: *Modified Proctor Curve of Sample A $_2$*

Exercise 3.3: The results of the modified Proctor test carried out on a road material (Sample A3) are given. Carry out the necessary calculations and plot the curve of the variation of the dry density γ_d depending on the water content (ω %).

Total wet weight (gr)	8184	8369	8317
Mold weight (gr)	3920	3830	3830
Mold volume (cm ^{3})	2104		

Tare number:	537	I7	A3	569	1	2
Total wet weight (gr)	155.81	134.01	167.82	155.28	178.45	164.6
Total dry weight (gr)	148.34	126.68	155.99	144.52	163.25	150.45
Tare weight (gr)	18.89	17.97	18.71	18.68	18.04	18.9

Solution:

Weight of wet soil (gr)	4264	4539	4487
Wet density	2.03	2.16	2.13

Water weight (gr)	7.47	7.33	11.83	10.76	15.2	14.15
Dry soil weight (gr)	129.45	108.71	137.28	125.84	145.21	131.55
Water content (%)	5.77	6.74	8.62	8.55	10.47	10.76
Average (%)	**6.26**		**8.58**		**10.61**	

Water content (%)	**6.26**	**8.58**	**10.61**
Dry density	**1.91**	**1.99**	**1.93**

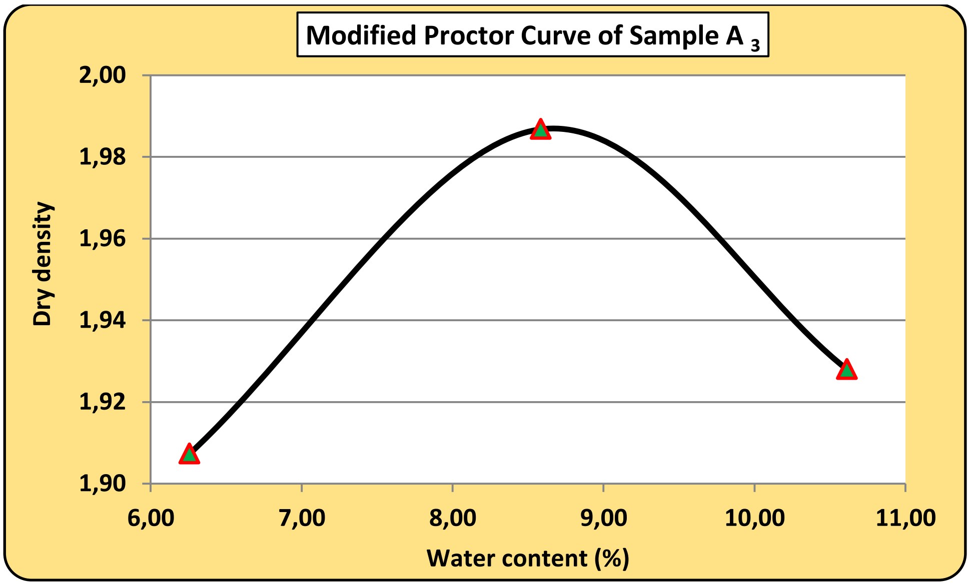

Fig. 3.15: *Modified Proctor Curve of Sample A $_3$*

Exercise 3.4: The results of the modified Proctor test carried out on a road material (Sample B1) are given. Carry out the necessary calculations and plot the curve of the variation of the dry density γ_d depending on the water content (ω %).

Total wet weight (gr)	8251	8403	8465	8445				
Mold weight (gr)	3830	3830	3830	3830				
Mold volume (cm ^{3})	2104							
Tare number:	564	504	504	505	M	K	K0	SD
Total wet weight (gr)	180.87	183.39	185.72	182.42	205.85	214.03	198.53	198.02
Total dry weight (gr)	171.51	173.94	173.22	169.99	188.58	196.05	179.1	178.53
Tare weight (gr)	19.04	18.74	19.12	17.69	18.44	18.94	19.09	18.25

Solution:

Weight of wet soil (gr)	4421	4573	4635	4615
Wet density	**2.10**	**2.17**	**2.20**	**2.19**

Water weight (gr)	9.36	9.45	12.5	12.43	17.27	17.98	19.43	19.49
Dry soil weight (gr)	152.47	155.2	154.1	152.3	170.14	177.11	160.01	160.28
Water content (%)	6.14	6.09	8.11	8.16	10.15	10.15	12.14	12.16
Average (%)	**6.12**		**8.14**		**10.15**		**12.15**	

Water content (%)	**6.12**	**8.14**	**10.15**	**12.15**
Dry density	**1.98**	**2.01**	**2.00**	**1.96**

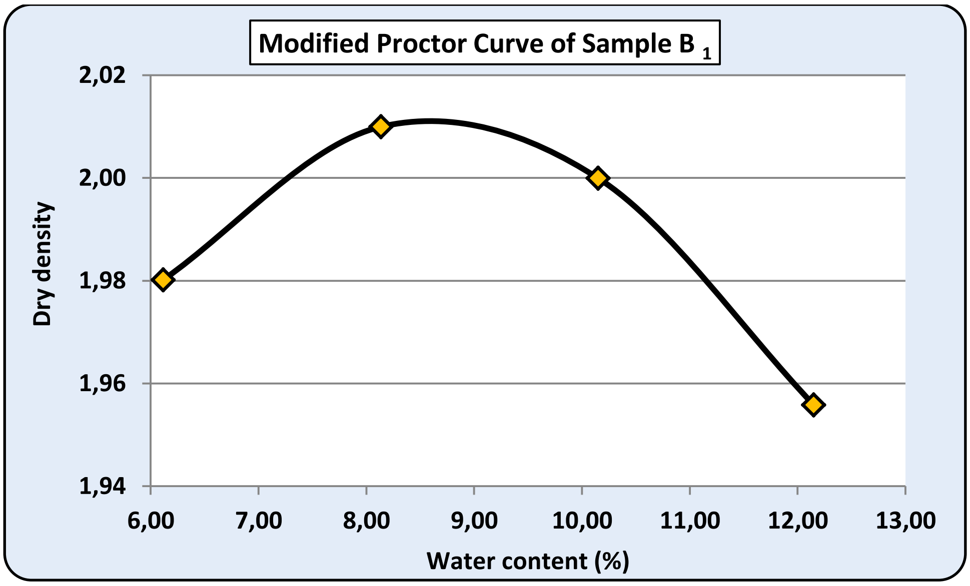

Fig. 3.16: *Modified Proctor Curve of Sample B $_1$*

Exercise 3.5: The results of the modified Proctor test carried out on a road material (Sample B2) are given. Carry out the necessary calculations and plot the curve of the variation of the dry density γ_d depending on the water content (ω %).

Total wet weight (gr)	7932	8035	8109	8017
Mold weight (gr)	3920	3830	3830	3833
Mold volume (cm ^{3})	2104			

Tare number:	A0	W0	X1	11	f0	f11	SD	LM
Total wet weight (gr)	145.06	147.97	144.8	144.54	164.65	160.58	156.22	225.23
Total dry weight (gr)	133.37	135.92	130.96	130.83	146.41	142.88	137.03	196.26
Tare weight (gr)	19.12	17.69	18.44	18.94	19.09	18.25	18.4	18.3

Solution:

Weight of wet soil (gr)	4012	4205	4279	4184				
Wet density	**1.91**	**2.00**	**2.03**	**1.99**				
Water weight (gr)	11.69	12.05	13.84	13.71	18.24	17.7	19.19	28.97
Dry soil weight (gr)	114.25	118.23	112.52	111.89	127.32	124.63	118.63	177.96
Water content (%)	10.23	10.19	12.3	12.25	14.33	14.20	16.18	16.28
Average (%)	**10.21**		**12.28**		**14.26**		**16.23**	

Water content (%)	**10.21**	**12.28**	**14.26**	**16.23**
Dry density	**1.73**	**1.78**	**1.78**	**1.71**

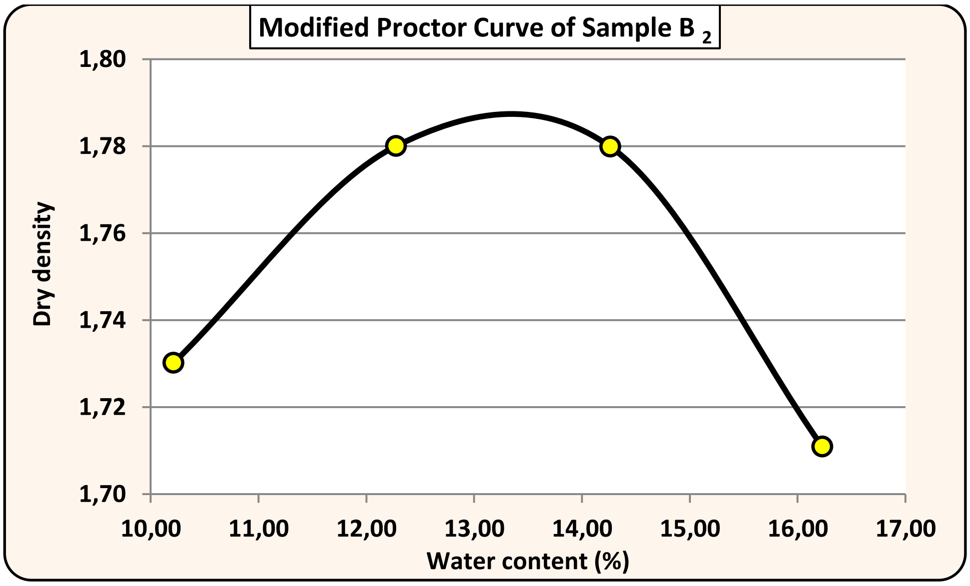

Fig. 3.17: *Modified Proctor Curve of Sample B $_2$*

Exercise 3.6: The results of the modified Proctor test carried out on a road material (Sample B3) are given. Carry out the necessary calculations and plot the curve of the variation of the dry densityγ_d depending on the water content (ω %).

Total wet weight (gr)	7962	8082	8089
Mold weight (gr)	3830	3830	3830
Mold volume (cm^3)	2104		

Tare number:	F2	X1	X2	Z3	T1	H2
Total wet weight (gr)	192.84	192.71	186.33	191.17	200.31	198.42
Total dry weight (gr)	176.64	176.33	168.11	172.3	177.53	176.04
Tare weight (gr)	18.44	18.83	18.61	18.5	18.33	18.24

Solution:

Weight of wet soil (gr)	4132	4252	4259
Wet density	**1.96**	**2.02**	**2.02**

Water weight (gr)	16.2	16.38	18.22	18.87	22.78	22.38
Dry soil weight (gr)	158.2	157.5	149.5	153.8	159.2	157.8
Water content (%)	10.24	10.40	12.19	12.27	14.31	14.18
Average (%)	**10.32**		**12.23**		**14.25**	

Water content (%)	**10.32**	**12.23**	**14.25**
Dry density	**1.78**	**1.80**	**1.77**

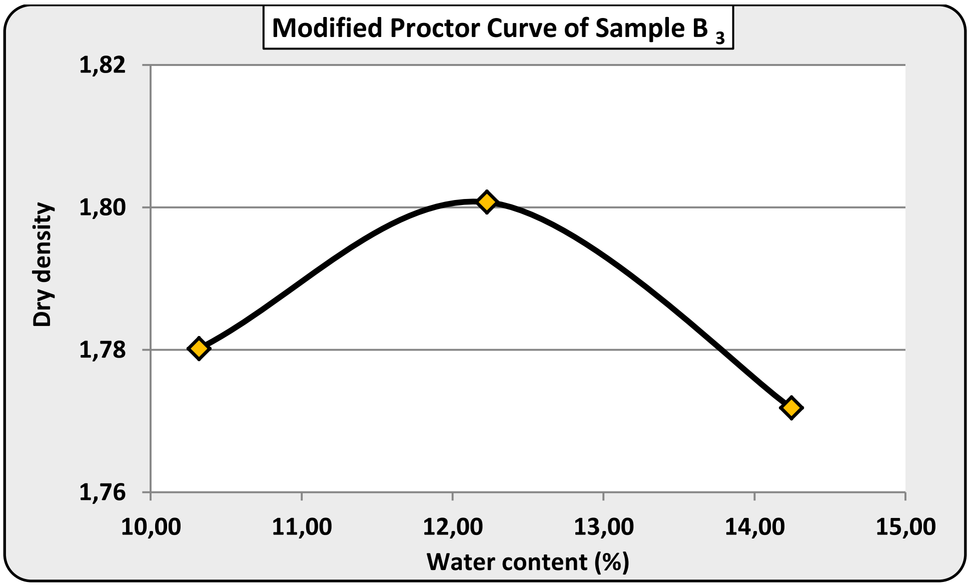

Fig. 3.18: *Modified Proctor Curve of Sample B* $_3$

Exercise 3.7: The results of the modified Proctor test carried out on a road material (Sample B4) are given. Carry out the necessary calculations and plot the curve of the variation of the dry density γ_d depending on the water content (ω %).

Total wet weight (gr)	8210	8460	8396
Mold weight (gr)	3830	3920	3830
Mold volume (cm^3)	2104		

Tare number:	S0	R9	Z7	C10	12	N4
Total wet weight (gr)	2691	2691	2222.3	2222.3	2323	2323
Total dry weight (gr)	2543.6	2543.6	2077.2	2077.2	2137.4	2137.4
Tare weight (gr)	125	125	280	280	304	304

Solution:

Weight of wet soil (gr)	4380	4540	4566
Wet density	**2.08**	**2.16**	**2.17**

Water weight (gr)	147.4	147.4	145.1	145.1	185.6	185.6
Dry soil weight (gr)	2418.6	2418.6	1797.2	1797.2	1833.4	1833.4
Water content (%)	6,0944	6,0944	8,0737	8,0737	10,123	10,1233
Average (%)	**6.09**		**8.07**		**10.12**	

Water content (%)	**6.09**	**8.07**	**10.12**
Dry density	**1.96**	**2.00**	**1.97**

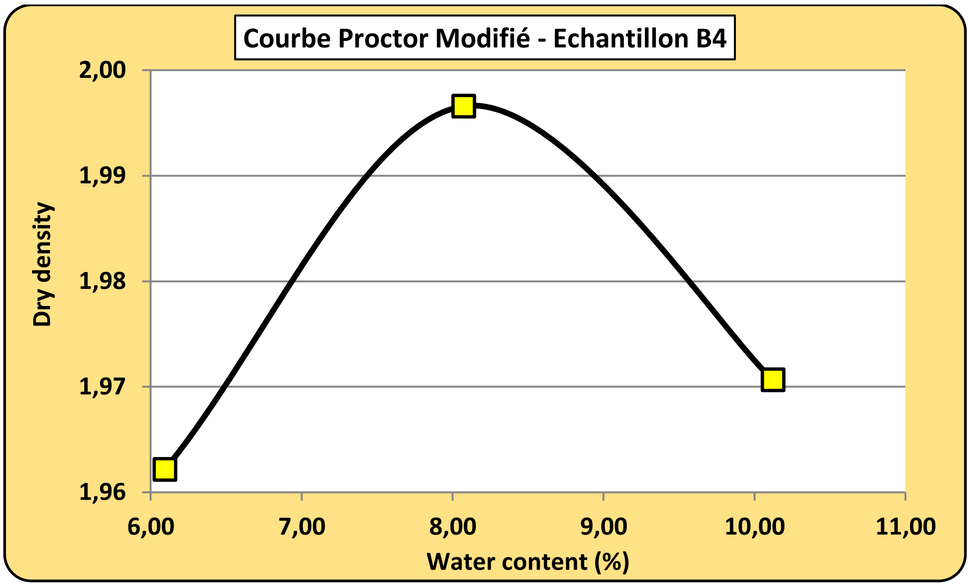

Fig. 3.19: *Modified Proctor Curve of Sample B $_4$*

Exercise 3.8: The results of the modified Proctor test carried out on a road material (Sample B5) are given. Carry out the necessary calculations and plot the curve of the variation of the dry density γ_d depending on the water content (ω %).

Total wet weight (gr)	8671	8766	8694
Mold weight (gr)	3920	3830	3830
Mold volume (cm ^{3})	2104		

Tare number:	569	F19	MZ	N4	MZ	I29
Total wet weight (gr)	224.3	176	181	140	144.3	152.9
Total dry weight (gr)	209.7	164	165.9	128.6	130.5	137.8
Tare weight (gr)	27.9	25.2	18.6	18.4	18.9	16.8

Solution:

Weight of wet soil (gr)	4751	4936	4864
Wet density	**2.26**	**2.35**	**2.31**

Water weight (gr)	14.6	12	15.1	11.4	13.8	15.1
Dry soil weight (gr)	181.8	138.8	147.3	110.2	111.6	121
Water content (%)	8.03	8.65	10.25	10.34	12.37	12.48
Average (%)	**8.34**		**10.30**		**12.42**	

Water content (%)	8.34	10.30	12.42
Dry density	2.08	2.13	2.06

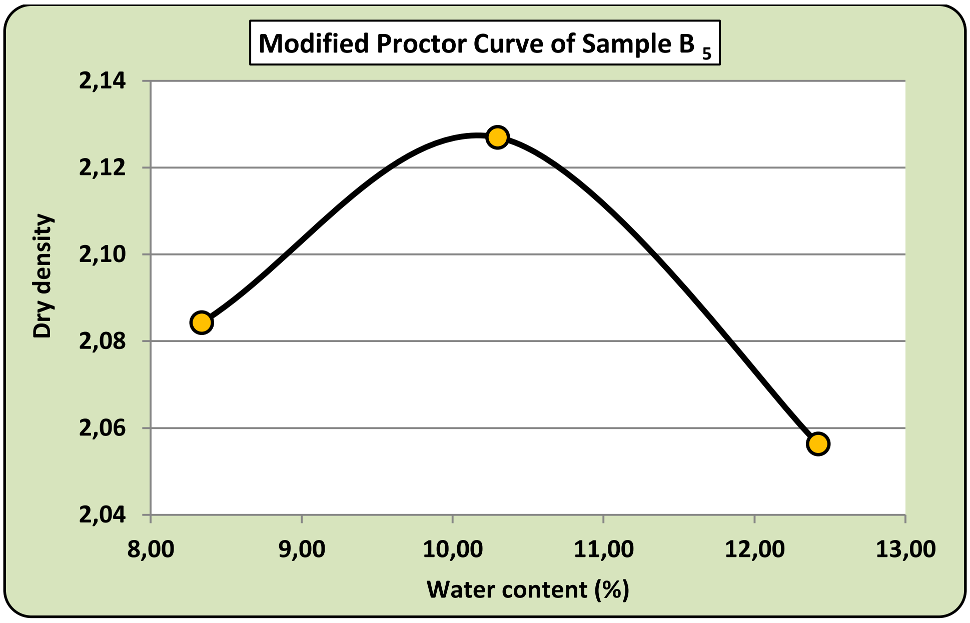

Fig. 3.20: *Modified Proctor Curve of Sample B* $_5$

<u>Exercise 3.9</u>: The results of the modified Proctor test carried out on a road material (Sample C1) are given. Carry out the necessary calculations and plot the curve of the variation of the dry density γ_d depending on the water content (ω %).

<u>Solution:</u>

Total wet weight (gr)	7842	8120	8116
Mold weight (gr)	3830	3830	3831
Mold volume (cm ^{3})	2104		

Tare number:	AZ	EY	XXL	D10	XX	D11
Total wet weight (gr)	142.53	152.71	183.92	182.21	180.75	176.24
Total dry weight (gr)	132.89	142.38	168.55	167.13	162.73	158.83
Tare weight (gr)	18.66	17.02	18.32	19.01	18.61	18.5

Weight of wet soil (gr)	4012	4290	4285
Wet density	1.91	2.04	2.04

Water weight (gr)	9.64	10.33	15.37	15.08	18.02	17.41
Dry soil weight (gr)	114.23	125.36	150.23	148.12	144.12	140.33
Water content (%)	8.44	8.24	10.23	10.18	12.50	12.41
Average (%)	**8.34**		**10.21**		**12.45**	

Water content (%)	**8.34**	**10.21**	**12.45**
Dry density	**1.76**	**1.85**	**1.81**

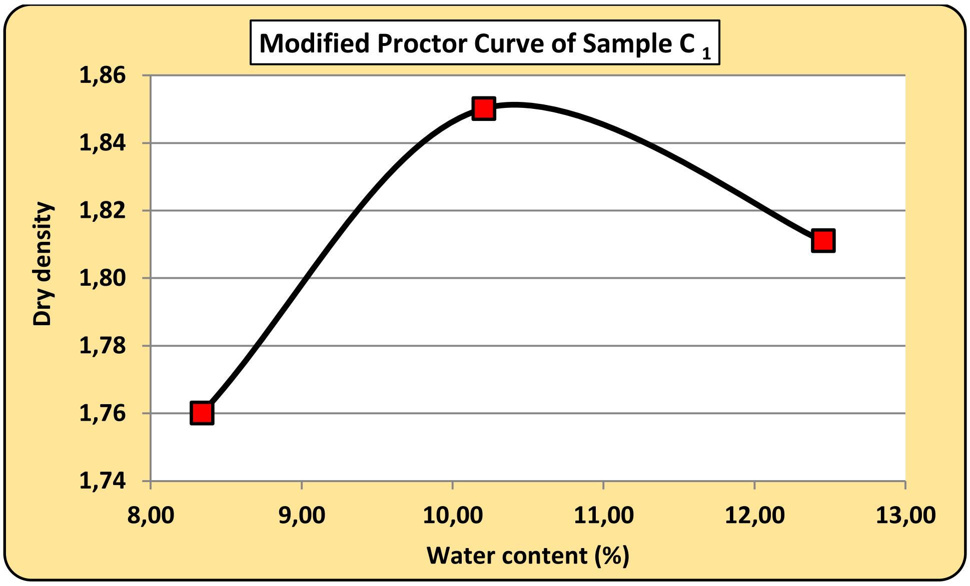

Fig. 3.21: *Modified Proctor Curve of Sample C_1*

<u>Exercise 3.10</u>: The results of the modified Proctor test carried out on a road material (Sample C2) are given. Carry out the necessary calculations and plot the curve of the variation of the dry density γ_d depending on the water content (ω %).

Total wet weight (gr)	8358	8395	8371
Mold weight (gr)	3919	3830	3833
Mold volume (cm^3)	2104		

Tare number:	A9	A5	B2	Z2	1	I7
Total wet weight (gr)	180.13	183.02	186.18	182.67	205.55	214.15
Total dry weight (gr)	170.77	173.57	173.68	170.24	188.5	196.17
Tare weight (gr)	18.3	18.37	19.58	17.94	18.36	19.06

Solution:

Weight of wet soil (gr)	4439	4565	4538
Wet density	**2.11**	**2.17**	**2.16**

Water weight (gr)	9.36	9.45	12.5	12.43	17.05	17.98
Dry soil weight (gr)	152.47	155.2	154.1	152.3	170.14	177.11
Water content (%)	6.14	6.09	8.11	8.16	10.02	10.15
Average (%)	**6.11**		**8.14**		**10.09**	
Water content (%)	**6.11**	**8.14**	**10.09**			
Dry density	**1.99**	**2.01**	**1.96**			

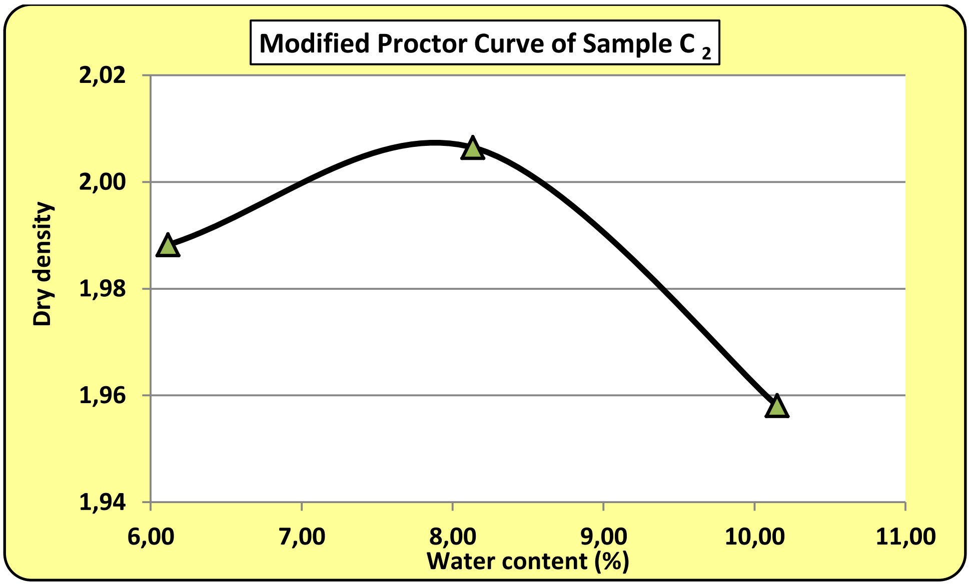

Fig. 3.22: *Modified Proctor Curve of Sample C* $_2$

3.6 ADDITIONAL EXERCISES

Exercise 3.11: Based on the results of the modified Proctor test carried out on a road material (Adrar airport road tuff), plot the curve of the variation of the dry density γ_d depending on the water content . Deduce the compaction parameters $(\omega_{opt};\ \gamma_{dmax})$ for this material.

Sample	Tuff road to Adrar airport				
$\omega\%$	6.17	7.12	8.64	10.83	11.93
γ_d	1.99	2.03	2.06	2.02	1.98

Exercise 3.12: Based on the results of the modified Proctor test carried out on Sable crushed, plot the curve of the variation of the dry density γ_d depending on the water content. Deduce the compaction parameters $(\omega_{opt};\ \gamma_{dmax})$ for this sand.

Sample	Crushed sand				
ω%	4.59	5.73	7.63	8.85	9.42
γ_d	2.06	2.21	2.16	2.10	2.01

<u>**CHAPTER IV**</u>**:**

Water in the soil

Chapter 4. Water in the soil

4.1 Types of water in soil

4.2 Darcy's Law

4.3 Permeability coefficient (K)

4.4 Exercises

Introduction

The presence of water in the soil plays a very important role, we are interested in the free water which can circulate between the grains, this water saturating a mass of land builds an underground water table, most often on the free surface or sometimes located between two impermeable formations, this is the captive water table.

Once the presence of water in a terrain has been recognised, it will be necessary to address the problems it poses, which practically all come down to either its elimination (excavation exhaustion) or a reduction in its load (drainage). (CAMBFORT, 1980)

Drainage and temporary or permanent lowering of the water table are often essential for the construction of structures and for their stability.

Theoretical knowledge of the laws of water flow in the soil, as well as that of the mechanical action which results from it, will be necessary for the designer, they will allow him to understand the physical and mechanical principles which are not fundamentally called into question by the results of practice, as well as to predict and explain the particular behavior of the massifs.

4.1 Types of water in soil

Water in soils is subject to several forces. Water molecules first experience a reciprocal attraction constituting the cohesion of water, and allowing them to remain grouped together.

We distinguish between water of constitution, water in the vapor phase, hygroscopic water, pellicular water, capillary water and free or gravitational water.

4.2 Darcy's law

Darcy (1856) proposed the following relation to describe unidirectional flows:

$$\frac{Q}{S} = k\frac{h_1 - h_2}{l}$$

Or h_1 Andh_2 are the piezometric heights measured at the two ends of the sample (Figure 4.1).

He notes that the flow rate per unit area Q is proportional to ($h_1 - h_2$) and inversely proportional to the length l of the flow, as long as $\frac{h_1-h_2}{l}$ is not too strong. It is this relationship that made it possible to express Darcy's law:

$$v = -ki$$

With : Discharge speed (this is the flow rate passing through a unit section): i : Hydraulic gradient or pressure loss per unit length in the direction of flow;

k : Coefficient of permeability of the soil which has the dimension of a speed, it characterizes both the soil and the filtering liquid.

Consider a cylinder of soil with section **S** (figure 4.1) and suppose that a flow occurs from M to N.

Either Q the flow rate through the section S. By definition, the speed of water $V = \frac{Q}{S}$

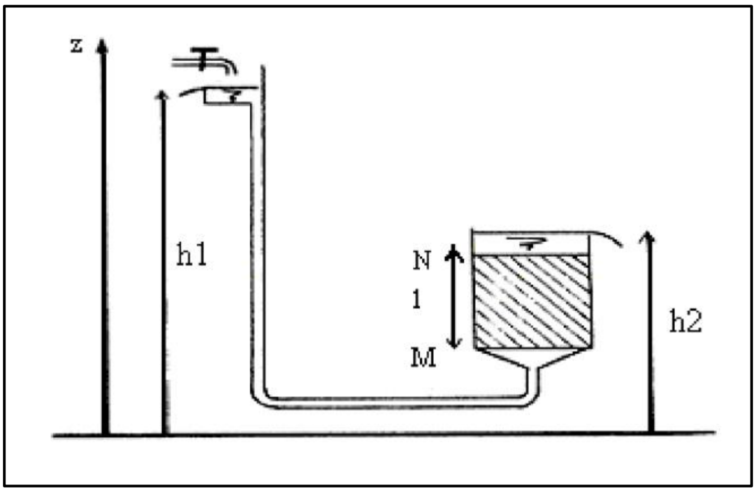

Fig. 4.1: *Darcy's Experience*

This is an apparent speed because, on the one hand, water only flows through the pores and the actual available section is only $S.n$,(n : porosity); on the other hand, the pores are not straight and the water makes many detours, this is what we call the tortuosity of the medium.

4.2.1 Hydraulic load

The hydraulic load H conditions the energy of a point of the water table. As the water moves from the high energy point to the low energy point, it is imperative to know well H, if possible the field of H. In the laboratory, pressure is generally measured using manometers, while in the field piezometer tubes are used. It is recommended to install several piezometers in the same place, each open at different depths. Hydraulic heads corresponding to different depths can also be measured.

In hydrodynamics, the load in M designates the quantity:

$$h_1 = z_1 + \frac{u_1}{\gamma_w} + \frac{v_1^2}{2g}$$

It is expressed in meters of water. This charge corresponds to the total energy of a water particle of unit mass:

z_1 : is the altitude of point M relative to a horizontal reference plane (position energy).

u_1: is the pore water pressure in M ($\frac{u_1}{\gamma_w}$) pressure energy.

v : the speed of the water.

In soils, speeds are low (v<10 cm/s) and the quantity $\frac{v_1^2}{2g}$ which represents the kinetic energy is completely negligible, so the previous formula is written:

$$h_1 = z_1 + \frac{u_1}{\gamma_w}$$

According to Bernoulli's theorem:

– Yesh_1 = h_2= constant, there is no flow, the water table is in equilibrium.

- On the other hand, if h_1 > h_2, there is flow from M to N and the pressure loss (h_1 - h_2) corresponds to the energy lost in friction. The pressure loss is both the driver and the consequence of the flow.

The hydraulic gradient is the quantity:

$$i = \frac{h_1 - h_2}{l}$$

l : distance MN.

DARCY's law which governs flow phenomena in soils is expressed by the formula:

$$v = k.i$$

4.3 Permeability coefficient (K)

DARCY's law therefore establishes the proportionality of the discharge speed and the hydraulic gradient. The proportionality coefficient k The dimension of a speed is the coefficient of permeability, it depends on both the porous medium and the fluid. It is generally expressed in m/s or cm/s. Below (Table IV-1) is an approximate scale of the values of this coefficient of permeability. k depending on the nature of the land.

Tab. 4.1: *Order of magnitude of the permeability coefficient soils in cm/s (according to COSTET et al. 1983)*

Nature of the land	$\boldsymbol{k}$**(m/s)**
Gravel	$10^{-1} < k < 10^{2}$
Sand	$10^{-3} < k < 10^{-1}$
Silt and clayey sand	$10^{-7} < k < 10^{-3}$
Clay	$10^{-11} < k < 10^{-7}$
Apparently uncracked rocks	$10^{-10} < k < 10^{-8}$

4.3.1 Laboratory measurement of the permeability coefficient

The permeability coefficient of a soil is an intrinsic characteristic of the soil and which depends on the granulometry of the soil, its nature and its structure.

The finer the soil, the smaller the pores, and the greater the friction, and therefore the pressure losses, the smaller the coefficient of permeability. It is sometimes said for simplification that clays are impermeable, in fact they have a very low permeability.

The more compact a soil is, the lower the porosity and the less space in which water can circulate, so the less permeable the soil will be.

Two methods which are direct applications of DARCY's law are used in the laboratory:

1- Measurement under constant load for highly permeable soils.
2- Measurement under variable load for poorly permeable soils.

4.3.1.1 Constant head permeameter

A permeameter consists of a sealed enclosure in which a soil sample of section is placed (S) and width l. The two ends of the sample are connected to two tubes by means of porous stones. In the constant head permeameter (figure 4.2), the head difference is maintained using overflows (h) between the two faces of the sample constant and the quantity of water is measured (Q) which has passed during a given time (t).

$V = Q.\Delta t$ Knowing that : $Q = v.s$

According to DARCY's law, we have : $v = k.i = k\frac{h}{l}$

$V = k\frac{h}{l}.s.\Delta t$ from where : $k = \frac{V.l}{s.h.\Delta t}$

4.3.1.2 Variable head permeameter

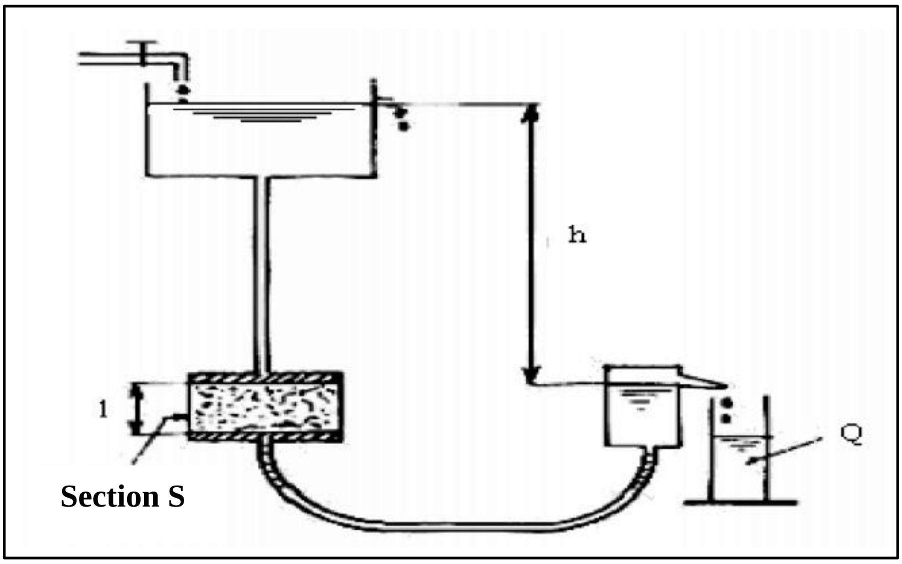

Fig. 4.2: *Constant head permeameter*

In the variable head permeameter , the tube (figure 4.3) is filled with water and the drop in its level is monitored as a function of time, i.e. s is the section of this tube. For a time dt, the amount of water flowing is:

$$Q = -sdh$$

But also $$Q = v.S.dt = k.i.S.dt$$

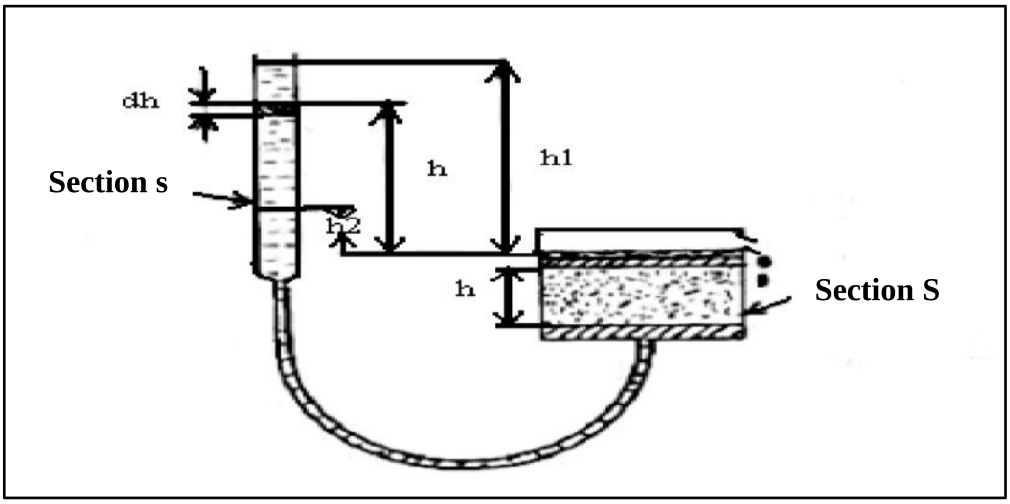

Fig. 4.3: *Variable head permeameter*

$$Q = \frac{k.h.S.dt}{l}$$

Let, by equalizing, the two expressions of Q:

$$k.dt = \frac{-s.l.dh}{S.h}$$

and by integrating between two instants we find:

$$k = \frac{-s.l(lnh_1 - lnh_0)}{S(t_1 - t_0)}$$

$$k = 2{,}3\frac{s.l.log\frac{h_0}{h_1}}{S(t_1 - t_0)}$$

h_0 : the charge difference over time t_0

h_1 : the charge difference over time t_1

4.3.2 Equivalent average permeability of stratified soils

In the case of stratified soils, each layer has its own permeability which influences the overall permeability of the massif. The permeable ground of total thickness H and composed of (n)

successive layers of thickness (h_1,h_2, ……, h_n) and permeability coefficient (k_1, k_2, ….. k_n) respectively (SCHNEEBELI.1978).

4.3.2.1 Case of horizontal flow: In this case the hydraulic gradient $i$is the same at the crossing of each layer. So:

$v_1 = k_1 i$, $v_2 = k_2 i$..., $v_n = k_n i$.

Considering the flow rate which crosses a slice of ground of width (b) unit, we see that it is equal to the sum of the unit flow rates.

$$Q = q_1 + q_2 + q_3 + \cdots + q_n$$
$$vH = v_1 h_1 + v_2 h_2 + v_3 h_3 + \cdots + v_n h_n$$
$$v = \frac{v_1 h_1 + v_2 h_2 + v_3 h_3 + \cdots + v_n h_n}{H}$$
$$v = \frac{i(k_1 h_1 + k_2 h_2 + k_3 h_3 + \cdots + k_n h_n)}{H}$$

With v : equivalent average speed.

On the other hand $v = K_H . i$ Or K_H: Equivalent permeability coefficient of the massif. Therefore:

$$K_H = \frac{k_1 h_1 + k_2 h_2 + k_3 h_3 + \cdots + k_n h_n}{H}$$
$$K_H = \frac{1}{H}(k_1 h_1 + k_2 h_2 + k_3 h_3 + \cdots + k_n h_n)$$

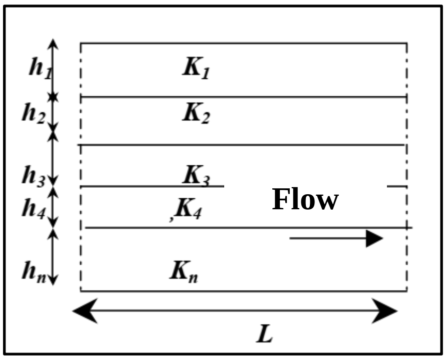

Fig. 4.4: *Horizontal flow*

4.3.2.2 Case of vertical flow

By the principle of continuity, the discharge speed is the same when crossing the different layers, we therefore have:

$$v = k_1 i_1 = k_2 i_2 = k_3 i_3 = \cdots = k_n i_n = k_\mathrm{v} i$$

The hydraulic gradient i is also equal to:

$$i = \frac{h_1 + h_2 + \cdots + h_n}{H}$$

Or h_1, h_2, ...h_n are the pressure losses through different layers and H the total pressure loss therefore:

$$v = k_\mathrm{v} i = k_\mathrm{v} \frac{h_1 + h_2 + \cdots + h_n}{H}$$

$$v = k_1 \frac{h_1}{H_1} = k_2 \frac{h_2}{H_2} = \cdots = k_n \frac{h_n}{H_n} = \frac{h_1}{\frac{H_1}{k_1}} = \cdots = \frac{h_1 + h_2 + \cdots h_n}{\frac{H_1}{k_1} + \frac{H_2}{k_2} + \cdots \frac{H_n}{k_n}}$$

and therefore:

$$k_\mathrm{v} = \frac{H}{\frac{H_1}{k_1} + \frac{H_2}{k_2} + \cdots \frac{H_n}{k_n}}$$

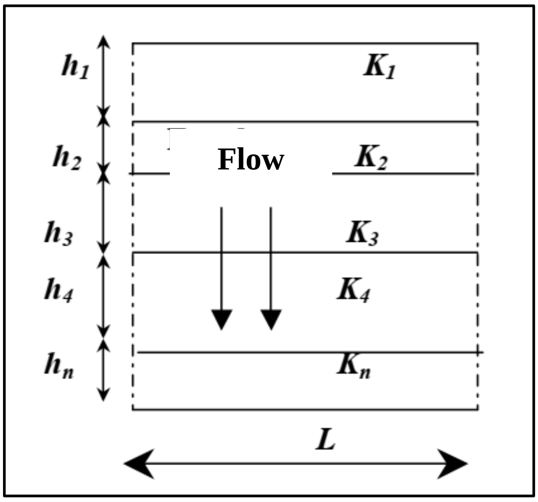

Fig. 4.5: *Vertical flow*

4.4 Exercises

Exercise 1

Calculate the hydraulic gradient and permeability contrast $\frac{k_1}{k_2}$

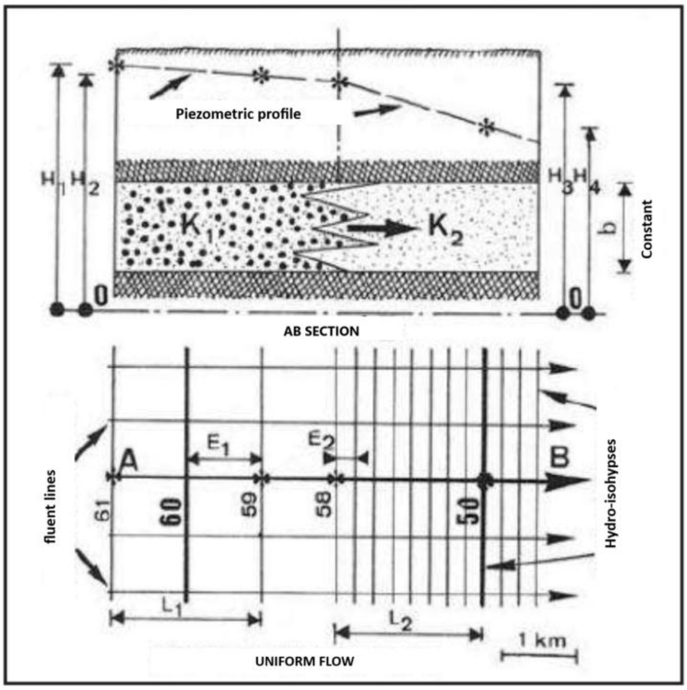

Fig. 4.6: *Uniform flow in a confined aquifer of constant thickness*

$$i_1 = \frac{H_1 - H_2}{L_1} = \frac{2}{2000} = 0{,}001; \quad i_2 = \frac{H_3 - H_4}{L_1} = \frac{8}{2000} = 0{,}004$$

$$\frac{k_1}{k_2} = \frac{i_2}{i_1} = \frac{0{,}004}{0{,}001} = 4 = \frac{E_1}{E_2} = 4$$

Uniform flow in a confined aquifer of constant thickness. A decrease in the coefficient of permeability, due to a lateral variation in facies, leads to an increase in the hydraulic gradient and a decrease in the spacing modulus E. The ratio of the spacing moduli allows the ratio of the permeability coefficients to be calculated directly.

Exercise 2

Determine the flow rate of a confined water table well taking into account the following information:
- Difference in piezometric heights of 2.5 m between two piezometers located respectively 10 and 30 m from the center of the well.

- Thickness of the water table of 30 m.

- Hydraulic conductivity 0.0001 m/s.

$$Q = 2\pi Ke\frac{H_1 - H_2}{ln\frac{x_1}{x_2}} = 0{,}0428\ \mathrm{m}^3/s\ = 43\ \mathrm{l}/s$$

Exercise 3

Determine the coefficient of permeability K in a confined water table if the drawdown measurements in two piezometers located respectively 20 and 150 m from the well are, in order, 3.3 and 0.3 m. The thickness of the water table is 30 m and the flow rate is 0.2 m ^{3}/s.

$$K = \frac{Q.\,ln\frac{x_1}{x_2}}{2\pi(H_1 - H_2)e} = 7{,}1.\,10^{-4}\ \mathrm{m}/s$$

BIBLIOGRAPHICAL REFERENCES

Olivari G. (1983) Mécanique des sols appliquée aux calculs d'ouvrages. Tome 2. ENTPE. Lyon.

Plumelle C. (2004-2005) Cour Géotechnique C.N.A.M- Géotechnique B1/B6.

Reiffsteck P. (2010) Cour de fondation des ouvrages. Laboratoire Central des Ponts et Chaussées Division Mécanique des Sols des Roches et de la Géologie de l'Ingénieur.

CEN (2005) Eurocode 7 : calcul géotechnique - Partie 1 : règles générales, NF EN 1997-1 (P94-251-1), 175 pages

CEN (2006) Eurocode 7 : calcul géotechnique - Partie 2 : Reconnaissance des terrains et essais, NF EN 1997-2 (P94-252), 209 pages

Mekerta Belkacem, (2017), « Mécanique des sols avancée» Université Ahmed Draïa – Adrar

Guettouche Amar, (2015), « Mécanique des sols: Cours et exercices », Support de cours - Université Ferhat Abbas- Sétif-1.

Khaled Meftah, (2008), « Cours et exercices de mécanique des sols », Support de cours-Septembre 2008.

Khelifa Harichane, (2013), « Mécanique des sols 1 », Support de cours -Université Hassiba Ben-Bouali de Chlef.

Michel Dysli, (1997), «Mécanique des sols », Ecole Polytechnique Fédérale de Lausanne 3ème Edition.

Gérard Degoutte, Paul Royet (2009), «Aide -Mémoire de Mécanique des sols »

Clément Cesodt, **Philippe Reiffsteck** (2015), «Exercices et problèmes corrigés de mécanique des sols, avec rappels de cours ».

Norme EN 13286-2, (2004), «Méthodes d'essai de détermination en laboratoire pour la masse volumique de référence et la teneur en eau - Compactage Proctor. ».

Fascicule 62 Titre V (1993), Règles techniques de calcul et de conception des fondations des ouvrages de génie civil, CCTG, Ministère de l'Équipement, du Logement et des Transports, Texte officiel N° 93-3, 182 pages

Fondations profondes pour le bâtiment (1992), Norme NF-P11-212 AFNOR, Paris, 72 pages (DTU n°13.2 CSTB),

Frank R., (1999) Calcul des fondations superficielles et profondes, Techniques de l'ingénieur et Presses des Ponts et Chaussées, 141 pages.

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كلية العلوم والتكنولوجيا
قسم الهندسة المدنية

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نحن رئيس المجلس العلمي لقسم الهندسة المدنية بكلية العلوم والتكنولوجيا

بناءًا على المحضر المؤرخ بتاريخ 03 نوفمبر 2024

نشهد ان الأستاذ: **د. أقاسم مصطفى**

أستاذ محاضر أ – جامعة أدرار

قام بإنتاج مطبوعة بيداغوجية بعنوان:

« Soil Mechanics 1: Course Support and Tutorials »

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